

## HOMEWORK 7

- (1) Compute the Euler characteristic  $\chi(X)$  of all nonorientable 2-dim connected compact varieties
- (2) Let  $f : \mathbb{R}P^n \rightarrow \mathbb{R}P^n$  where  $n$  is even. Prove that  $f$  has a fixed point (using  $L(f)$ ).
- (3) Suppose that  $S^1$  is covered by two open sets  $U_1, U_2$ . Prove that for some  $U_i$  there exists a point  $x$  such that  $x, -x \in U_i$ . (Hint: Use Borsuk-Ulam theorem and the distance map  $\text{dist}(U)$ ) Show that this generalizes to any covering of  $S_n$  by  $n + 1$  open sets.
- (4) Prove that no subset of  $R^n$  is homeomorphic to  $S^n$ .