# HOMEWORK 1

Obligatory exercises are 1,2 and 3.

**Exercise 1.** Show that $SS^k = S^{k+1}$. *Hint:* Consider the map $f\colon S^k \times I \to S^{k+1}$, where

$$f(x,t) = ((\sqrt{1-(2t-1)^2})x, 2t-1)$$

**Exercise 2.** Let $f\colon X \to Y$. Consider the mapping cylinder $M_f$.

(1) Prove that the inclusion $\iota_X\colon X \hookrightarrow M_f$ is a cofibration.
(2) Show that this gives a possibility to factor every map $f\colon X \to Y$ as

$$f = r \circ \iota_X$$

where $\iota_X$ is a cofibration and $r\colon M_f \to Y$ is a homotopy equivalence.

**Exercise 3.** Prove: If $X$ is a Hausdorff space, then its diagonal

$$\Delta = \{(x,x) \in X \times X\}$$

is a closed subspace of $X \times X$.

**Exercise 4.** Let $X$ be a Hausdorff space and $A \subseteq X$ be its retract. Prove that $A$ is closed. *Hint:* Use the previous exercise and the map $X \to X \times X\colon x \mapsto (x, r(x))$ where $r\colon X \to A$ is a retraction.

**Exercise 5.** The consequence of the previous exercise is: If $X$ is Hausdorff and $A \hookrightarrow X$ is a cofibration, then $A$ is closed. *Hint:* $X \times I$ is Hausdorff.