HOMEWORK 2 – 2017

Exercise 1. Given the short exact sequence of Abelian groups

$$0 \longrightarrow A \xrightarrow{i} B \xrightarrow{j} C \longrightarrow 0$$

the following conditions are equivalent:

(1) There exists $p\colon C \to B$ such that $j \circ p = id_C$.
(2) There exists $q\colon B \to A$ such that $q \circ i = id_A$.
(3) There are p, q as above such that $i \circ q + p \circ j = id_B$

We have shown that (2) $\Rightarrow$ (1) and (3). Prove that (1) $\Rightarrow$ (2) and (3)

Exercise 2. For the short exact sequence od chain complexes

$$0 \longrightarrow A_* \xrightarrow{f} B_* \xrightarrow{g} C_* \longrightarrow 0$$

there is a long exact sequence of homology groups

$$\dots \longrightarrow H_{n+1}(C_*) \xrightarrow{\partial_*} H_n(A_*) \xrightarrow{f_*} H_n(B_*) \xrightarrow{g_*} H_n(C_*) \xrightarrow{\partial_*} H_{n-1}(A_*) \longrightarrow \dots$$

(1) We have defined connecting homomorphism ∂_* by the prescription

$$\partial_*([c]) = [a], \quad \text{where } \ \partial c = 0, \ f(a) = \partial b, \ g(b) = c.$$

Prove that the definition is independent of the choice of c in the homology class in $H_n(C_*)$.

(2) Prove the exactness in $H_n(C_*)$.
(3) Prove the exactness in $H_n(A_*)$.