Study of the plasma decay with microwaves

Contents

1 Introduction

Plasma is a state of matter, which, apart from neutrals, also contains charged particles: ions and electrons. Naturally the concentration of these particles is one of the fundamental plasma characteristics. In order to be sustained, plasma needs the energy to be continuously supplied - when the energy flow is interrupted, plasma starts to decay. The decay process manifests as a gradual drop in the concentration of charged particles. The drop is caused by two principles:

- **diffusion** and following recombination on the walls
- volume **recombination**

2 Diffusion

Starting with the continuity equation for electron density

$$\frac{\partial n}{\partial t} + \mathrm{div}\vec{\Phi} = 0 \tag{1}$$

while the term $\mathrm{div}\vec{\Phi}$ can be rewritten as

$$\mathrm{div}\vec{\Phi} = -\mathrm{div}\ \mathrm{grad}(Dn) = -\nabla^2(Dn) \tag{2}$$

with D being the *diffusion coefficient*. Therefore

$$\frac{\partial n}{\partial t} - \nabla^2(Dn) = 0 \tag{3}$$

Assuming that D is constant in the whole volume ($D \neq f(x, y, z)$), the solution can be expressed in terms of eigen-functions n_j of the $D\nabla^2 n$ operator. The concentration then equals

$$n(\vec{r}, t) = \sum_j a_j(t) n_j(\vec{r}) \tag{4}$$

Eigen-functions must satisfy the relation

$$\nabla^2 n_j + \frac{1}{D\tau_j} = 0 \tag{5}$$

Substituting into the original equation, the solution can be expressed as the superposition of so called *diffusion modes*, where each diffusion mode has certain relaxation time τ_j

$$n(\vec{r}, t) = \sum_j n_j(\vec{r}) \exp\left(-\frac{t}{\tau_j}\right) \tag{6}$$

If only the first diffusion mode is considered (i.e. the slowest mode of decay) we get

$$n(\vec{r}, t) = n_0(\vec{r}) \exp\left(-\frac{D}{\Lambda^2} t\right) \tag{7}$$

where $\Lambda^2 = \tau_0 D$ is the *diffusion length*. Furthermore, if the shape of the discharge tube is cylindrical, and the length-to-radius ratio is many times larger than one, it can be treated as a one-dimensional problem.

$$n(x, t) = n_0(x) \exp\left(-\frac{D}{\Lambda^2} t\right) \tag{8}$$

In this case, the radial profile of concentration is

$$n_0(x) = \text{const} \cdot J_0\left(\frac{x}{\Lambda}\right) \tag{9}$$

and the diffusion length is directly proportional to the radius of the discharge tube r_0

$$\Lambda \approx \frac{r_0}{2.405} \tag{10}$$

where the numerical factor 2.405 is the first order of a Bessel function of the first kind J_0.

3 Volume recombination

The number of particles, that recombine per unit of volume and time is proportional to the product of electron and positive ion concentrations

$$n_r = \alpha \cdot n_i \cdot n \tag{11}$$

The linear factor α is known as the *recombination coefficient*. The temporal change of concentration in neutral plasma ($n_i = n$) is then

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -\alpha n^2 \tag{12}$$

4 Determining the dominant process

Generally speaking, the recombination losses take over at higher pressures, while the diffusion losses are significant at lower pressures. Which process is actually dominant, can be determined from the electron concentration profile in time $n = f(t)$.

- *Diffusion-* losses are characterised by the time development.

$$n(t) = n_0 \exp\left(-\frac{D}{\Lambda^2}t\right) \tag{13}$$

  in other words, the function $\ln n = f(t)$ is linear; the diffusion constant D can be determined from its linear coefficient.

- In the case of *recombination*, the temporal evolution of charged particles can be expressed as

$$\frac{1}{n(t)} = \frac{1}{n_0} + \alpha t \tag{14}$$

  meaning, that the inverse of charged particle concentration $\frac{1}{n} = f(t)$ is linear here; where the recombination coefficient α can be obtained from its linear coefficient.

Using this approach, allows us to either determine the recombination coefficient or the diffusion coefficient, but never both (the other effect is neglected altogether, which can lead to inaccurate results). Given that both processes exist simultaneously and both are taken into account, we get the new differential equation for the time evolution of concentration:

$$\frac{\mathrm{d}n(t)}{\mathrm{d}t} = -\frac{D}{\Lambda^2}n(t) - \alpha n^2(t) \tag{15}$$

Solving this differential equation yields:

$$n(t) = \frac{1}{c \cdot \exp\left(tD/\Lambda^2\right) - \alpha\Lambda^2/D} \tag{16}$$

The measured data can be ultimately fitted with this improved equation, getting the parameters D, α and the constant c. Adding the initial condition $n(0) = n_0$, we may calculate the n_0 as well.

5 Resonator method for the electron concentration determination

This method was originally introduced by Slater in 1946 and is based on the measurement of the complex high-frequency conductivity of plasma. Filling of the resonator with plasma induces changes both in the resonator eigenfrequency ω and the quality of the resonator Q. The calculation of these changes (as functions of the degree of resonator filling) was modelled by the *perturbation method*, which yields the result:

$$\triangle\left(\frac{1}{Q}\right) - 2i\frac{\triangle\omega}{\omega} = \frac{1}{\varepsilon_0\omega}\frac{\int_{V'}\sigma E^2 \mathrm{d}V}{\int_V E^2 \mathrm{d}V} \tag{17}$$

where V is the volume of the whole cavity, V' is the volume of the plasma-filled part, σ the conductivity of plasma and E is the amplitude of the electric field. The electrical conductivity is in general a complex quantity

$$\sigma = \frac{ne^2}{m}\left(\frac{\nu}{\omega^2+\nu^2} - i\frac{\omega}{\omega^2+\nu^2}\right) \tag{18}$$

For the electron concentration n determination, the imaginary part of the equation (17) with the eigenfrequency shift $\triangle\omega$ term is used. Provided that the collision frequency is lower than the angular frequency of the resonator, the imaginary part of conductivity is

$$\nu < \omega \Rightarrow \nu^2 \ll \omega^2 \Rightarrow \sigma_i = -\frac{ne^2}{m}\frac{1}{\omega} \tag{19}$$

The shift in resonance frequency (with and without plasma) is then

$$\frac{2\triangle\omega}{\omega} = \frac{e^2}{\varepsilon_0 m\omega^2}\frac{\int_{V'} n(\vec{r})E^2(\vec{r})\mathrm{d}V}{\int_V E^2(\vec{r})\mathrm{d}V} \tag{20}$$

In the cylindrical resonator, usually the mode TM_{010} is excited:

$$\begin{aligned} E_r &= 0 \\ E_\varphi &= 0 \\ E_z(r) &= E_0 \cdot J_0(2.405\ r/R) \end{aligned}$$

where E_0 is the amplitude of high-frequency electric field at the axis of the resonator with radius R and the factor 2.405 is the first order of the Bessel function of the first kind J_0.

Since the discharge tube region occupies only the vicinity of the resonator axis, the $E(r,\varphi,z)$ can be replaced with constant $E = 0.8 \cdot E_0$ where 0.8 is an appropriate numerical factor. Taking the E_0 from the numerator out of the integral, we get

$$\frac{2\triangle\omega}{\omega} = \frac{e^2}{\varepsilon_0 m\omega^2}\frac{(0.8)^2E_0^2\int_{V'} n(r,z,\varphi)\mathrm{d}V}{E_0^2\int_V J_0^2(2.405\ r/R)\mathrm{d}V} \tag{21}$$

The integral in the numerator can be expressed as

$$\int_{V'} n(r,z,\varphi)\mathrm{d}V = \bar{n}V' \tag{22}$$

where $\bar{n}$ is the mean value of the electron concentration inside the discharge tube volume V'. Following with the denominator integration

$$\int_V J_0^2\left(2.405\frac{r}{R}\right)\mathrm{d}V = \int_0^{2\pi}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_0^R J_0^2\left(2.405\frac{r}{R}\right) r\mathrm{d}r\,\mathrm{d}z\,\mathrm{d}\varphi = 2\pi h\cdot I(R) \tag{23}$$

where h is the discharge tube length and

$$I(R) = \int_0^R \left[J_0(2.405\frac{r}{R})\right]^2 r\,\mathrm{d}r = \frac{R^2}{2}\{\underbrace{[J_0(2.405)]^2}_{0} + \underbrace{[J_1(2.405)]^2}_{0.271}\} = \frac{R^2}{2}\cdot 0.271 \tag{24}$$

This yields following relation of the eigenfrequency shift and the eigenfrequency of the unperturbed case.

$$\frac{2\triangle\omega(t)}{\omega} = \frac{0.64}{0.271}\frac{e^2}{\varepsilon_0 m\omega^2}\frac{V'\bar{n}(t)}{V} \tag{25}$$

From this equation, the mean electron density (in the discharge tube with radius R') as a function of time t can be expressed

$$\bar{n}(t) = \frac{0.271}{0.64}\frac{V}{V'}\frac{2\triangle\omega(t)}{\omega}\frac{\varepsilon_0 m\omega^2}{e^2} = \frac{0.271}{0.64}\frac{R^2}{R'^2}\triangle f(t)\frac{8\pi^2\varepsilon_0 m f}{e^2} \tag{26}$$

6 Experimental set-up

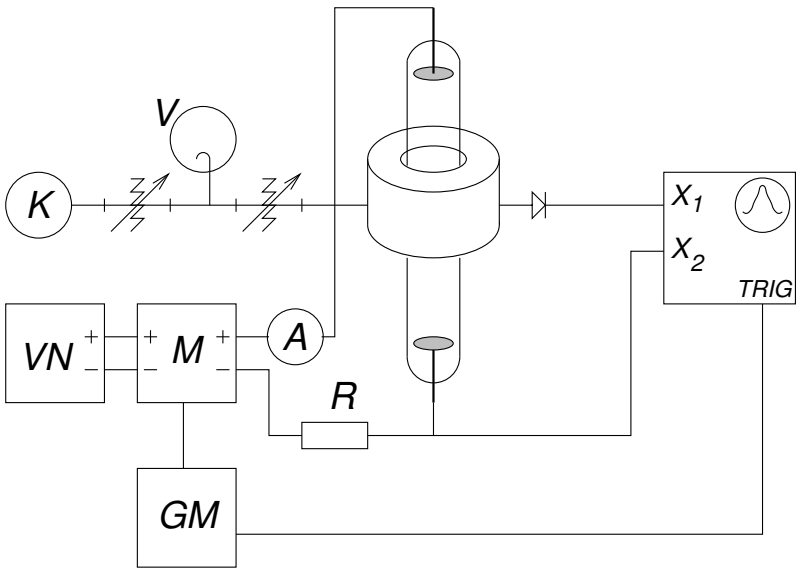

Figure 1: The diagram of the experimental set-up in this laboratory.

The high-frequency energy from the source K with tunable frequency passes through the attenuator and precision wave meter V to the resonator (R =40 mm). The discharge

tube (R'=9 mm) passes through the resonator, aligned with the axis. The transmitted signal is rectified with diode and guided to the input of an oscilloscope.

The high voltage from the source VN is keyed via the electronic pentode trigger M and applied to the discharge tube through the cathode resistance R. The pentode is controlled by the rectangular pulse generator GM, which is also used as the trigger for the oscilloscope. The discharge current is measured with an ampermeter and the voltage across the discharge tube can be observed at the second channel of the oscilloscope.

A combination of the rotary vane and the diffusion pump is used to evacuate the discharge tube, with the pressure measured by the Pirani gauge. The pressure of the working gas in the discharge tube (helium) can be adjusted with a dispenser which consists of two valves with a reservoir in between.

In reality the whole experimental set-up is needs a lot of space which might cause confusion during measurement. For better orientation and reference please see the guiding photo Fig.2.

6.1 Measurement

Let the empty resonator have eigenfrequency f_0. This value increases to f_1 after the ignition of a discharge. As soon as the discharge is turned off, the plasma starts to decay and the eigenfrequency gradually decreases towards the initial value f_0. This sequence is periodically repeated and thus, with appropriate synchronisation, the extinction of plasma can be observed on the oscilloscope,.

The resonator eigenfrequency decreases with time elapsed from the moment of turning the discharge off. At certain moment t' the fixed frequency f' of the high-frequency source will be equal to the eigenfrequency, which will manifest as a maximum in the transmission of the high-frequency energy to the resonator and therefore a peak in the signal detected by the oscilloscope.

The frequency f' is tuned in the range $(f_0; f_1)$. From the oscillograph, the time t' is determined and from the difference $\triangle f = f' - f_0$ the electron concentration $n_e(t)$ can be calculated. Afterwards these functions need to be plotted:

- $1/n = f(t)$
- $\ln n = f(t)$

in order to fit parameters α, D and determine, which process (recombination/diffusion) is dominant. Furthermore both coefficients and the value of n_0 will be determined also by fitting the equation (16). Finally results of the former and the latter approach should be compared. The measurement is to be performed at various pressures (200 Pa, 1000 Pa a 5000 Pa).

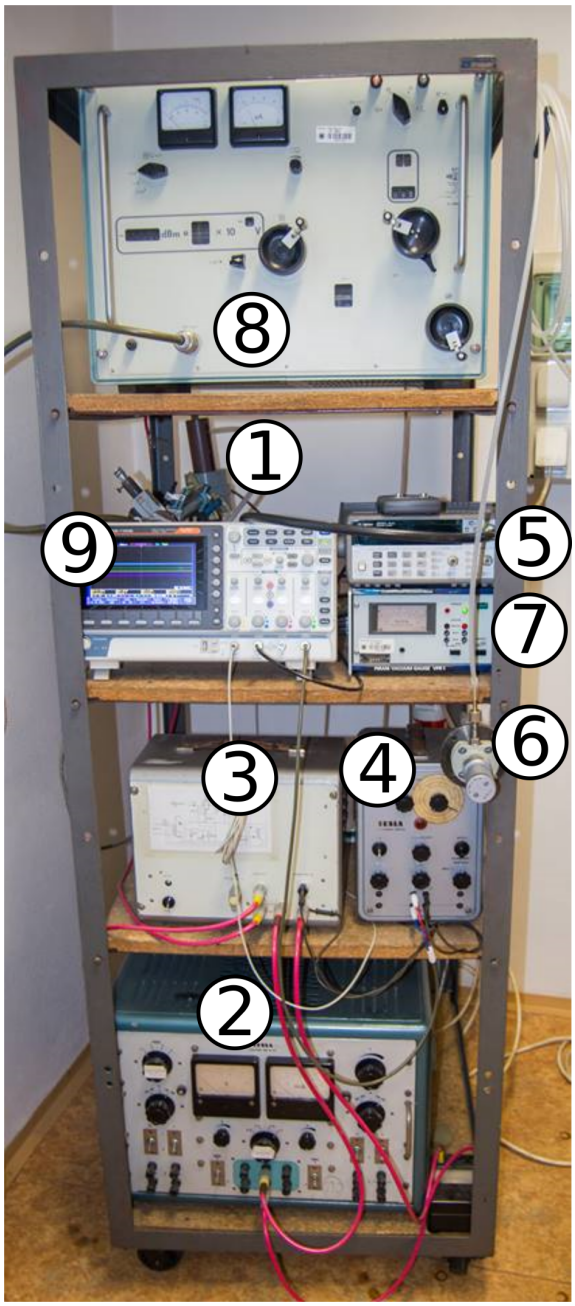

Figure 2: The actual appearance of the resonator method experiment. 1 - the resonator with discharge tube. 2 - the VN source, 3 - the pentode trigger, 4 - the shunt resistor, 5 - the rectangular pulse generator, 6 - the pressure dispenser, 7 - Pirani gauge, 8 - the high frequency source K with the the wave meter, 9 - oscilloscope. The rotary vane and the diffusion pump are not visible.

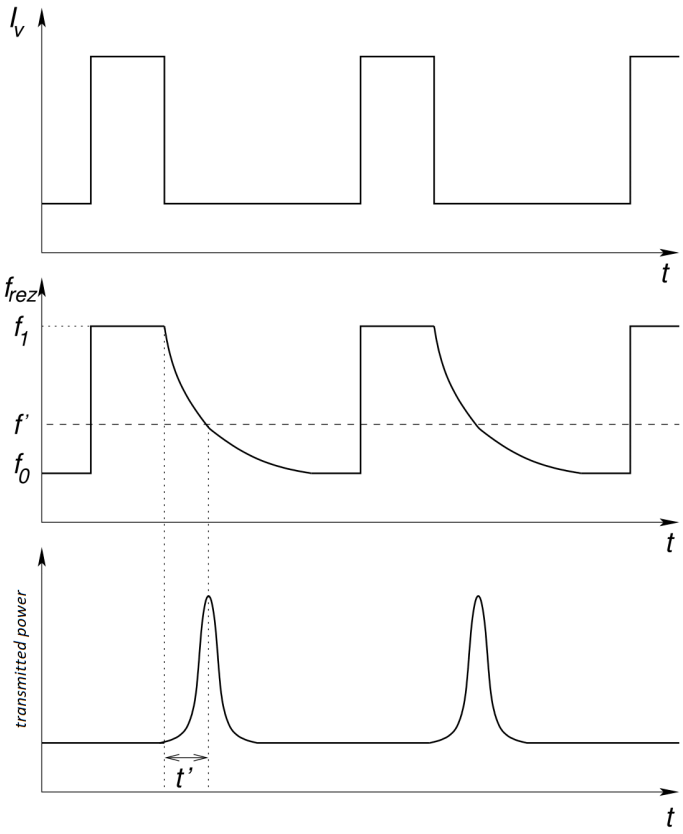

Figure 3: The discharge current, resonator eigenfrequency and the magnitude of the signal transmitted through the resonator as a function of time.