

### Listening 3

<http://ocw.mit.edu/OcwWeb/Mathematics/18-03Spring-2006/VideoLectures/detail/embed01.htm>

$$\frac{d^2 y}{dx^2} = \frac{p}{K} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$y'' = \frac{p}{K} \sqrt{1 + (y')^2}$$

a) What is a differential equation?

What kinds of differential equations do you know?

b) Listen to the first part of the lecture and answer the following Qs.

- 1) What is the lecturer assuming?
  - a].....
  - b].....
- 2) Where can you study differential equations or modeling in case you need some explanations?
- 3) Which acronyms is the lecturer going to use?
- 4) What is the difference between two equations the lecturer wrote on the board?.
- 5) What does the blue color indicate?
- 6) What is the aim of this first lecture?

## Transcript - Lecture 1 -- 18.03

**Listen for the second time and try to fill in the missing words.**

OK, let's get started. I'm assuming that, A, you went recitation yesterday, B, that even if you didn't, you know how to 1) \_\_\_\_\_ variables, and you know how to 2) \_\_\_\_\_ simple models, solve physical problems with differential equations, and possibly even solve them. So, you should have learned that either in high school, or 18.01 here, or, yeah. So, I'm going to start from that point, assume you know that. I'm not going to tell you what differential equations are, or what modeling is. If you still are uncertain about those things, the book has a very long and good explanation of it. Just read that 3) \_\_\_\_\_ . So, we are talking about first order ODEs.

ODE: I'll only use two acronyms. ODE is ordinary differential equations. I think all of MIT knows that, whether they've been taking the course or not. So, we are talking about first-order ODEs, which in standard form, are written, you 4) \_\_\_\_\_ the derivative of  $y$  with respect to,  $x$ , let's say, on the left-hand side, and on the right-hand side you write everything else. You can't always do this very well, but for today, I'm going to assume that it has been done and it's 5) \_\_\_\_\_. So, for example, some of the ones that will be considered either today or in the problem set are things like  $y' = x / y$ .

That's pretty simple. The problem set has  $y' = x - y^2$ . And, it also has  $y' = y - x^2$ . There are others, too. Now, when you look at this, this, of course, you can solve by separating variables. So, this is solvable. This one is-- and neither of these can you separate variables. And they look extremely similar. But they are extremely dissimilar. The most dissimilar about them is that this one is 6) \_\_\_\_\_ solvable. And you will learn, if you don't know already, next time next Friday how to solve this one

This one, which looks almost the same, is unsolvable in a certain sense. Namely, there are no elementary functions which you can write down, which will give a solution of that differential equation. So, right away, one 7) \_\_\_\_\_ the most significant fact that even for the simplest possible differential equations, those which only involve the first derivative, it's possible to write down extremely looking simple 8) \_\_\_\_\_ .

I'll put this one up in blue to indicate that it's bad. Whoops, sorry, I mean, not really bad, but 9) \_\_\_\_\_. It's not solvable in the ordinary sense in which you think of an equation is solvable. And, since those equations are the rule rather than the exception, I'm going about this first day to not solving a single differential equation, but indicating to you what you do when you meet a blue equation like that.

What do you do with it? So, this first day is going to be 10) \_\_\_\_\_ to geometric ways of looking at differential equations and numerical. At the very end, I'll talk a little bit about numerical ways. And you'll work on both of those for the first problem set.

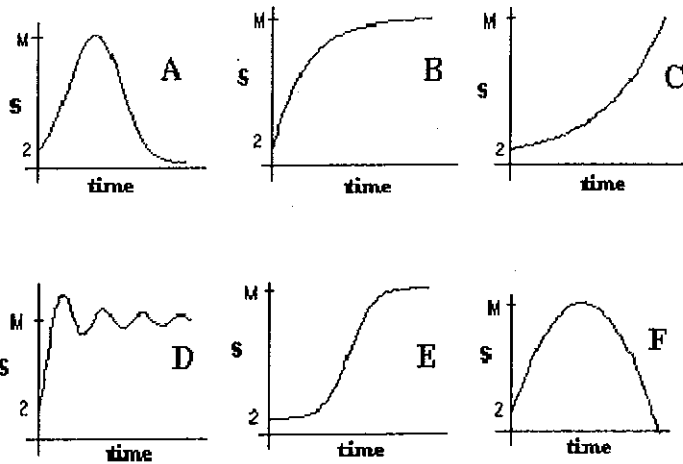
**The spread of a rumor**

Suppose two students at your school start a rumor. How could we describe the spread of the rumor throughout the school population? Could we determine a function  $S$  such that  $S(t)$  approximates the number of people that know the rumor at a time arbitrary time  $t$ , where  $t$  is measured in, say, hours?

We'll begin by trying to decide what the graph of  $S$  might look like. Assume that  $M$  is the population of your school and that  $M$  is sufficiently large that it makes sense to model discrete numbers of students with a continuous function. Thus, if  $S(3) = 127.8$ , we'll predict that the number of students who know the rumor after 3 hours is approximately 128.

1. Study the six graphs below. For each graph, decide whether or not it could be the graph of the function  $S$ . In each case, give the reasons for your decision.

**Possible Graphs of  $S$**



3. Describe three conditions that  $dS/dt$ , the rate of spread of the rumor, should satisfy. Keep in mind that we are describing the rate of change of the number of students who know the rumor. Suppose for example, that you know the number of "rumor-aware" students at two o'clock. What factors might determine the number of rumor-aware students at three o'clock? Consider the nature of the rumor itself, conditions at your school, and at least one condition that changes as the rumor spreads.

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Now consider the following Qs. In case you are not sure, study the following text.

1. What is calculus?
2. What is the difference between real and complex analysis?
3. What is the relation between the Riemann hypothesis and complex analysis?
4. What does it mean when a system is described as „dynamic“?
5. What does the chaos theory study?
6. What does the functional analysis study?

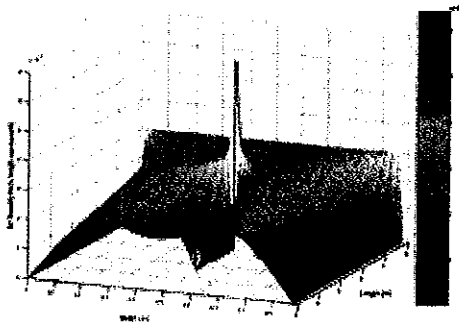
Understanding and describing change is a common theme in the natural sciences, and calculus was developed as a powerful tool to investigate it. Functions arise here, as a central concept describing a changing quantity. The rigorous study of real numbers and real-valued functions is known as real analysis, with complex analysis the equivalent field for the complex numbers. The Riemann hypothesis, one of the most fundamental open questions in mathematics, is drawn from complex analysis. Functional analysis focuses attention on (typically infinite-dimensional) spaces of functions. One of many applications of functional analysis is quantum mechanics. Many problems lead naturally to relationships between a quantity and its rate of change, and these are studied as differential equations. Many phenomena in nature can be described by dynamical systems; chaos theory makes precise the ways in which many of these systems exhibit unpredictable yet still deterministic behavior.

## Differential equation

From Wikipedia, the free encyclopedia

7. Have a look at the text and try to fill in the missing words. First, try to guess, than consult the list of words.

A **differential equation** is a mathematical equation for an unknown function of one or several a) .....that relates the values of the function itself and its derivatives of various orders.



Visualization of airflow into a duct modelled using the Navier-Stokes equations, a set of partial differential equations.

Differential equations arise in many areas of science and technology: whenever a deterministic relationship involving some continuously varying quantities (b) ..... by functions and their rates of change in c)..... and/or time (expressed as derivatives) is known or postulated. This is illustrated in classical mechanics, where the d)..... of a body is

described by its position and e)..... as the time varies. Newton's Laws allow one to relate the position, velocity, f)..... and various forces acting on the body and state this relation as a differential equation for the unknown position of the body as a function of g)..... In some cases, this differential equation (called an equation of motion) may be h)..... explicitly.

- a) variables      derivatives      equations
- b) described      modelled      draw ↻
- c) time      force      space
- d) state      position      motion
- e) acceleration      velocity      position
- f) velocity      acceleration      time
- g) time      space      gravity
- h) counted      solved      guessed

**WORD STUDY:** Prefixes are important means of creating new words, usually the opposites. There are some words from the text, try to supply prefixes forming new expressions.

- dependent .....
- proportional .....
- known .....
- natural .....
- changing .....
- predictable .....
- partial .....
- significant .....
- real .....
- continuous .....
- finite .....
- important .....