# Task Based Course - Population Dynamics

## 1. Introduction

We have all heard about the famous (or maybe infamous) model of Thomas Malthus which predicts exponential growth of a population. Even though the model provides oversimplified description of the changes in population size, it has revolutionized the way we look at the population dynamics. Currently, models with ideas rooted in population dynamics are used to describe interactions between different species, to determine the maximum harvest for agriculturists, to understand dynamics of diseases, to model demographic transition, and to predict the limits of population growth.

## 2. Tasks

**Population Growth Models**

Population growth refers to the change of a population over time. As we know from the introduction, the simplest model was introduced by Thomas Malthus, and therefore, it is called the Malthusian growth model. However, there are other more complex and accurate ways to describe population growth. The first possible extension is to introduce so called carrying capacity of a biological species in an environment. This extension ultimately leads to the well know logistic curve with the population size approaching the carrying capacity. Nevertheless, population growth models go beyond simple logistic curve, for instance, Richards growth model or Gompertz growth model.

> *Task: Compare and contrast Malthusian, Logistic (Verhulst), Richards, and Gompertz growth models. That is, pick a country of your choice and estimate population model parameters based on historical data of population growth. How well does each model correspond to the historical data? Choose the best model and make predictions about the population size in 5, 10, 20, 50, and 100 years from now. Interpret the results.*

**Useful links:**

Population models:

http://www.iba.muni.cz/res/file/ucebnice/hrebicek-uvod-do-matematickeho-modelovani.pdf

Annual growth rates by country:

http://data.worldbank.org/indicator/SP.POP.GROW

Population size by country:

http://data.worldbank.org/indicator/SP.POP.TOTL

**Predator-Prey Interaction**

Population growth of species depends on many factors including the carrying capacity of an environment, potential presence of a disease in the population, or the predation. Since the

introduction of population models, the predation has been an active subject of study for scientists in numerous fields. The main difference between a classic population growth model and a predator-prey model is that the model describes population dynamics of two species which interact. Probably the most pronounced model is a pair of first-order, nonlinear differential equations commonly known as the Lotka-Volterra system of equations.

> *Task: Pick a population of your choice that is predominantly hunted by a single predator, for instance the populations of fox and hare. Estimate the parameters of the Lotka-Volterra model and describe the population dynamics of the predator and the pray. How well does each model correspond to the historical data? Make predictions about the population dynamics in 5, 10, 20, 50, and 100 years from now. Will the sizes of both populations reach an equilibrium? Interpret the results.*

**Useful links:**

Predator-prey models:

http://www.iba.muni.cz/res/file/ucebnice/hrebicek-uvod-do-matematickeho-modelovani.pdf

http://download.springer.com/static/pdf/307/art%253A10.1007%252FBF00276925.pdf?auth66=1425889992_d6375173a5184586765e2cfdcc8fc75d&ext=.pdf


**Age-Structured Population Models**

Let's now take a one step back and think about possible ways to improve a model of a single population. The models that we often think about are those which describe a population as a single unit with homogenous birth rate and death rate. However, it is a great simplification to assume a homogenous population. A 45 years old woman is less likely to have a child than a 25 years old one. This is exactly the kind of refinement which we often consider, when we model a single population. The commonly used approach is to divide a population into cohorts by age, 0 - 15, 16 - 30, 31 - 45, and 45+ for instance. Sometimes, we even go beyond the age and distinguish between sexes as well. One of the most popular model in population ecology, which builds on the aforementioned premise, is the Leslie matrix model invented by Patrick H. Leslie.

> *Task: Use the Leslie matrix to model a population in country of your choice. Estimate the model parameters based on historical data. How well does the model correspond to the historical data? Make predictions about the population size and its age distribution in 5, 10, 20, 50, and 100 years from now. Will the demographic pyramid in your country of choice reach an equilibrium? Interpret the results and present them in form of demographic pyramids.*

**Useful links:**

Leslie matrix:

http://web.stanford.edu/~jhj1/teachingdocs/Jones-Leslie1-050208.pdf

Population by age, sex, and urban/rural residence:

## Population Dynamics of Fisheries

Since the human population on the planet Earth has been rapidly growing over the past decades, sustainable fish yields are growing concern of fisheries scientist. Therefore, precise restrictions on the amount of fish one can harvest in given areas are needed. One of the ways to find the right gap for harvesting is to make a model. In other words, our goal will be to describe the way in which given population grows and shrinks over time with controlled birth and harvest rates and a given death rate. There are numerous approaches to tackle this problem, including but not limited to systems of differential equations and modified Leslie matrix model.

> *Task: Use a population model to describe population dynamics of a fishery of your choice. Estimate the model parameters based on historical data. How well does the model correspond to the historical data? Try to find a sustainable fish yield and make predictions about the population size in 5, 10, 20, 50, and 100 years from now. Interpret the results.*

**Useful links:**

Population dynamics of fisheries overview:

http://en.wikipedia.org/wiki/Population_dynamics_of_fisheries

Harvesting in matrix population models (accessible through discovery.muni.cz):

http://www.jstor.org/discover/10.2307/2529719?sid=21105982729843&uid=4&uid=3737856&uid=2129&uid=2&uid=70

Commercial fisheries statistics:

http://www.st.nmfs.noaa.gov/commercial-fisheries/

## Epidemic Modeling

Even though epidemic models are not strictly population models, they describe dynamics of a population over time under certain conditions (disease). Also, both models approach their goals from a very similar angle. An epidemic model is a compartment model with population separated into different groups (compartments). The simplest epidemic models differentiate between susceptible and infectious individuals. More sophisticated models then extends this concept by another compartment, the group of recovered individuals.

> *Task: Use the SIR (or SIRS) epidemic model to simulate an epidemic outbreak in given population. For example, you can model the potential outbreak of the 2009 influenza H1N1 in Brno. Estimate the model parameters based on historical data. Given your analysis of the model, present a few policies that you think will decrease both the total number infected and the time that it takes for the disease to run its course through our population. Interpret the results.*

**Useful links:**

Epidemic models overview:

http://en.wikipedia.org/wiki/Epidemic_model

Simple epidemic models:

http://mysite.science.uottawa.ca/rsmith43/MAT4996/Epidemic.pdf

WHO influenza statistics:

http://apps.who.int/globalatlas/dataQuery/default.asp


**The Limits of Growth**

The Limits of Growth refer to the now almost legendary study in the mathematical modeling circles. It is also the name of a book that presents the result of a mathematical model called World3 created by Donella Meadows, Denis Meadows, and Jorgen Randers. The authors of the model asked themselves as how the growing human population interacts with expanding global economy, and how they can both adapt to carrying capacity of the planet Earth. They created a complex model which is composed from five interrelated systems, the food system, the industrial system, the population system, the non-renewable resources system, and the pollution system.

> *Task: Use the ideas of World3 model to simulate the interaction between the economics and the limits of the planet Earth. Estimate the model parameters based on historical data. Given your analysis of the model, present several scenarios of the population growth and interpret the results. Especially, make predictions about the population size and its age distribution in 5, 10, 20, 50, and 100 years from now, and explain how the population is influenced by different parts of the model.*

**Useful links:**

The Limits of Growth overview:

http://en.wikipedia.org/wiki/The_Limits_to_Growth

Simple World model:

www.mdpi.com/2071-1050/5/3/896/pdf


## 3. Modeling tips

STELLA Modeling & Simulation software is an easy to understand software designed to model system dynamics. It's very simple to use and doesn't require and prior programing knowledge. Practically all of the tasks above can be easily modeled using STELLA (the software should be installed in some of the universities' computers).

Free trial version is available here:

http://www.iseesystems.com/softwares/Education/StellaSoftware.aspx