Převrácená hodnota $\frac{1}{a}$ bez dělení:

1) $f(x) = a\cdot x - 1$, $f\left(\frac{1}{a}\right) = 0$

$f'(x) = a$

Newt. m.: $x_{k+1} = x_k - \frac{a\cdot x_k - 1}{a} = x_k - \frac{a x_k}{a} + \frac{1}{a} = \frac{1}{a}$

2) $f(x) = \frac{1}{x\cdot a} - 1$, $f'(x) = \frac{-1}{a}\frac{1}{x^2}$

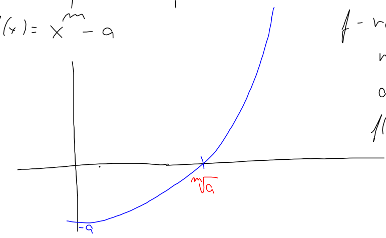
Newt. m.: $x_{k+1} = x_k + \frac{\frac{1}{x_k\cdot a} - 1}{\frac{1}{a x_k^2}} = x_k + \frac{\frac{1 - a x_k}{a\cdot x_k}}{\frac{1}{a x_k^2}} = x_k + x_k(1 - a x_k)$

$= x_k(2 - a x_k)$

bře 8-9:59

Four. podm. pro $\sqrt[m]{a}$:

$f(x) = x^m - a$



f - rostoucí, konvexní na $[0,\infty)$

$a > 0$, $b < \infty$

$f(a) < 0$, $\boxed{f(b) > 0}$

$x_0 = b$ $\boxed{b > \sqrt[m]{a}}$

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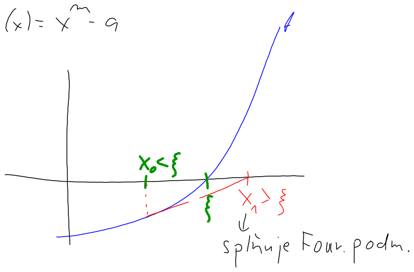
$f(x) = x^m - a$



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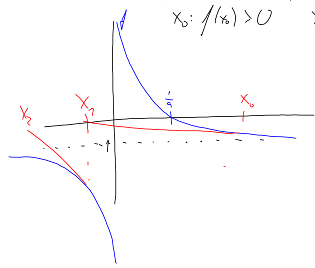
Four. podmínky pro výpočet $\frac{1}{a}$

$f(x) = \frac{1}{ax} - 1$ | f - konvexní, klesající

x_0: $f(x_0) > 0$ $x_0 \in (0, \frac{1}{a})$



Pro $x_0 > \frac{1}{a}$ nemusí N. m. konvergovat.

bře 8-10:44

Řád metody sečen ξ - řešení

$f(\xi) = 0$

$x_{k+1} = x_k - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}\cdot f(x_k)$

$x_{k+1} - \xi = x_k - \xi - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}\cdot\left[f(x_k) - f(\xi)\right]\frac{x_k - \xi}{x_k - \xi}$

$x_{k+1} - \xi = (x_k - \xi)\left[1 - \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}\cdot\frac{f(x_k) - f(\xi)}{x_k - \xi}\right] = (x_k - \xi)\left[1 - \frac{\frac{f(x_k) - f(\xi)}{x_k - \xi}}{\frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}}\right]$

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$x_{k+1} - \xi = (x_k - \xi)\frac{\frac{f(x_{k-1}) - f(x_k)}{x_{k-1} - x_k} - \frac{f(x_k) - f(\xi)}{x_k - \xi}}{\frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}}$

$x_{k+1} - \xi = (x_k - \xi)(x_{k-1} - \xi)\frac{\frac{\frac{f(x_{k-1}) - f(x_k)}{x_{k-1} - x_k} - \frac{f(x_k) - f(\xi)}{x_k - \xi}}{x_{k-1} - \xi}}{\frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}}$

$x_{k+1} - \xi = (x_k - \xi)(x_{k-1} - \xi)\frac{\frac{1}{2}f''(\alpha_k)}{f'(\beta_k)}$

$\alpha_k \to \xi$ $k \to \infty$

$\beta_k \to \xi$ — " —

$e_{k+1} = e_k\cdot e_{k-1}\cdot\left[\frac{f''(\xi)}{2f'(\xi)} + o(1)\right]$

bře 8-11:17

$$|e_{k+1}| \approx |e_k| \cdot |e_{k-1}| \cdot C$$

Nechť metoda sečen je řádu p:

$$\frac{|e_{k+1}|}{|e_k|^p} \approx L \quad , \quad \frac{|e_k|}{|e_{k-1}|^p} \approx L \qquad |e_k| \approx L \cdot |e_{k-1}|^p$$

$$\frac{|e_{k+1}|}{|e_k|^p} \approx \frac{|e_k|}{|e_k|^p} \cdot |e_{k-1}| \cdot C = \frac{|e_{k-1}|}{|e_k|^{p-1}} \cdot C \approx L \approx \frac{|e_k|}{|e_{k-1}|^p}$$

$$\approx L$$

$$\frac{|e_{k-1}|}{|e_k|^{p-1}} \cdot C \approx \frac{|e_{k-1}|}{L \cdot (|e_{k-1}|^p)^{p-1}} \cdot C = \frac{|e_{k-1}|}{|e_{k-1}|^{p^2-p}} \cdot \frac{C}{L} \quad \text{- ex. lim}$$

$$p^2 - p = 1$$

$$\boxed{\text{řeš: } p = \frac{1+\sqrt{5}}{2}} \leftarrow \boxed{p^2 = p+1}$$

bře 8-11:26