Krazin. metoda

Dk. řádu.

It. fce: $g(x) = x + \dfrac{f^2(x)}{f(x) - f(x+f(x))}$

$\xi : f(\xi) = 0$

$\xi$ : – pevný bod $g$: $\lim\limits_{x \to \xi} \dfrac{f(x)}{f(x) - f(x+f(x))} \overset{\text{l'Hosp. pr.}}{=} \lim\limits_{x \to \xi} \dfrac{f'(x)}{f'(x) - f'(x+f(x))[1+f'(x)]}$

$= \lim\limits_{x \to \xi} \dfrac{f'(x)}{f'(x) - f'(x+f(x)) - f'(x+f(x)) \cdot f'(x)} = \dfrac{f'(\xi)}{f'(\xi) - f'(\xi) - f'(\xi)^2}$

$= -\dfrac{1}{f'(\xi)}$

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$g(\xi) = \xi + \lim\limits_{x \to \xi} f(x) \cdot \lim\limits_{x \to \xi} \dfrac{f(x)}{f(x) - f(x+f(x))} = \xi + 0 \cdot \dfrac{1}{-f'(\xi)} = \xi$

$\xi$ – pevný bod $g$

Řád $\geq 2 \Rightarrow g'(\xi) = 0$:

$g'(\xi) = \lim\limits_{x \to \xi} \dfrac{g(x) - g(\xi)}{x - \xi} = \lim\limits_{x \to \xi} \dfrac{x + \frac{f^2(x)}{f(x) - f(x+f(x))} - \xi}{x - \xi} = 1 - 1 = 0$

$\lim\limits_{x \to \xi} \dfrac{\frac{f^2(x)}{f(x) - f(x+f(x))}}{x - \xi} = \lim\limits_{x \to \xi} \dfrac{f(x) - f(\xi)}{x - \xi} \cdot \lim\limits_{x \to \xi} \dfrac{f(x)}{f(x) - f(x+f(x))} = f'(\xi)\left(-\dfrac{1}{f'(\xi)}\right) = -1$

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Př:

$f(x) = x^2 - 10$

$x_0 = 3$, $f(x_0) = -1$

$x_0 + f(x_0) = 3 - 1 = 2$

$x_0 - f(x_0) = 3 + 1 = 4$

$x_0 = 4$, $f(x_0) = 6$

$x_0 + f(x_0) = 10$

$x_0 - f(x_0) = -2$

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Řád modifikované N. metody:

$\lim\limits_{x \to \infty} \dfrac{|x_{k+1} - \xi|}{|x_k - \xi|^2} = C < \infty$ :

$x_{k+1} = x_k - M \dfrac{f(x_k)}{f'(x_k)}$

$x_{k+1} - \xi = x_k - \xi - M \dfrac{f(x_k)}{f'(x_k)}$

Tayl. rozvoj:

$f(x_k) = f(\xi) + f'(\xi)(x_k - \xi) + \frac{1}{2} f''(\xi)(x_k - \xi)^2 + \dots + \dfrac{f^{(M)}(\xi)}{M!}(x_k - \xi)^M + \dfrac{f^{(M+1)}(\alpha)}{(M+1)!}(x_k - \xi)^{M+1} =$

$= \dfrac{f^{(M)}(\xi)}{M!}(x_k - \xi)^M + \dfrac{f^{(M+1)}(\alpha)}{(M+1)!}(x_k - \xi)^{M+1} = \dfrac{(x_k - \xi)^M}{M!}\left[f^{(M)}(\xi) + \dfrac{f^{(M+1)}(\alpha)}{M+1} \cdot (x_k - \xi)\right]$

$f'(x_k) = f'(\xi) + f''(\xi)(x_k - \xi) + \frac{1}{2} f'''(\xi)(x_k - \xi)^2 + \dots + \dfrac{f^{(M)}(\xi)}{(M-1)!}(x_k - \xi)^{M-1} + \dfrac{f^{(M+1)}(\beta)}{M!} \cdot (x_k - \xi)^M =$

$= \dfrac{(x_k - \xi)^{M-1}}{(M-1)!}\left[f^{(M)}(\xi) + \dfrac{f^{(M+1)}(\beta)}{M}(x_k - \xi)\right]$

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$x_{k+1} - \xi = x_k - \xi - M \dfrac{\frac{(x_k - \xi)^M}{M!}\left[f^{(M)}(\xi) + \frac{f^{(M+1)}(\alpha)(x_k - \xi)}{M+1}\right]}{\frac{(x_k - \xi)^{M-1}}{(M-1)!}\left[f^{(M)}(\xi) + \frac{f^{(M+1)}(\beta)(x_k - \xi)}{M}\right]} =$

$= (x_k - \xi)\left[1 - \dfrac{f^{(M)}(\xi) + \frac{f^{(M+1)}(\alpha)(x_k - \xi)}{M+1}}{f^{(M)}(\xi) + \frac{f^{(M+1)}(\beta) \cdot (x_k - \xi)}{M}}\right] =$

$= (x_k - \xi) \dfrac{\frac{f^{(M+1)}(\beta)}{M}(x_k - \xi) - \frac{f^{(M+1)}(\alpha)(x_k - \xi)}{M+1}}{f^{(M)}(\xi) + \frac{f^{(M+1)}(\beta)(x_k - \xi)}{M}} = (x_k - \xi)^2 \dfrac{\frac{f^{(M+1)}(\beta)}{M} - \frac{f^{(M+1)}(\alpha)}{M+1}}{f^{(M)}(\xi) + \frac{f^{(M+1)}(\beta)(x_k - \xi)}{M}}$

$\lim\limits_{k \to \infty} \dfrac{x_{k+1} - \xi}{(x_k - \xi)^2} = \lim\limits_{k \to \infty} \dfrac{\frac{f^{(M+1)}(\beta)}{M} - \frac{f^{(M+1)}(\alpha)}{M+1}}{0/0} = \dfrac{\frac{f^{(M+1)}(\xi)}{M} - \frac{f^{(M+1)}(\xi)}{M+1}}{f^{(M)}(\xi)} = \dfrac{1}{M \cdot (M+1)} \dfrac{f^{(M+1)}(\xi)}{f^{(M)}(\xi)} < \infty$

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Aitkenova metoda

$x_{k+1} - \xi = (C + o(1))(x_k - \xi)$ $\quad 0 < |C| < 1$

$\dfrac{x_{k+1} - \xi}{x_k - \xi} = C + o(1) \Rightarrow \lim\limits_{k \to \infty} \dfrac{|x_{k+1} - \xi|}{|x_k - \xi|^1} = |C| \Rightarrow$ řád konv. posl. je 1

Dk věty: jmenovatel $\neq 0$

$x_{k+1} - x_k = (x_{k+1} - \xi) - (x_k - \xi) = (C + o(1))(x_k - \xi) - (x_k - \xi) = (C - 1 + o(1))(x_k - \xi)$

$x_{k+2} - x_{k+1} = (C - 1 + o(1))(x_{k+1} - \xi) = (C - 1 + o(1))(C + o(1))(x_k - \xi)$

$x_{k+2} - 2x_{k+1} + x_k = (x_{k+2} - x_{k+1}) - (x_{k+1} - x_k) = (C - 1 + o(1))(C + o(1))(x_k - \xi) - (C - 1 + o(1))(x_k - \xi)$

$= (x_k - \xi)\left[(C^2 - C + o(1)) - (C - 1 + o(1))\right] = (x_k - \xi)\left[(C - 1)^2 + o(1)\right] \neq$ pro velká $k$

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$$\lim_{k\to\infty} \frac{\hat{x}_k - \xi}{x_k - \xi} = \lim_{k\to\infty} \frac{x_k - \xi - \frac{(x_{k+1} - x_k)^2}{x_{k+2} - 2x_{k+1} + x_k}}{x_k - \xi} =$$

$$= \lim_{k\to\infty} \frac{x_k - \xi - \frac{(C - 1 + o(1))^2 (x_k - \xi)^2}{((C-1)^2 + o(1))(x_k - \xi)}}{x_k - \xi} = \lim_{k\to\infty} \frac{x_k - \xi - \frac{((C-1)^2 + o(1)) \cdot (x_k - \xi)}{((C-1)^2 + o(1))}}{x_k - \xi} =$$

$$= \lim_{k\to\infty} \left(1 - \frac{(C-1)^2 + o(1)}{(C-1)^2 + o(1)}\right) = 1 - 1 = 0$$

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