Věta - ekvivalence pevných bodů:

Dk: 1. $\varphi(x) = x - \dfrac{(g(x)-x)^2}{g(g(x)) - 2g(x) + x}$, $\varphi(\xi) = \xi \Rightarrow$

$$\Rightarrow \frac{(g(\xi)-\xi)^2}{g(g(\xi)) - 2g(\xi) + \xi} = 0 \Rightarrow g(\xi) = \xi$$

2. $g(\xi) = \xi \Rightarrow \varphi(\xi) = \xi - \lim\limits_{x\to\xi} \dfrac{(g(x)-x)^2}{g(g(x)) - 2g(x) + x} = \xi - \lim\limits_{x\to\xi} \dfrac{2(g(x)-x)(g'(x)-1)}{g'(g(x))g'(x) - 2g'(x) + 1} =$

$$= \xi - \frac{2(g(\xi)-\xi)\cdot(g'(\xi)-1)}{g'(\xi)^2 - 2g'(\xi) + 1} = \xi - \frac{2(\xi-\xi)\cdot(g'(\xi)-1)}{(g'(\xi)-1)^2} = \xi$$

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Řád konvergence Steff. metody:

$p=1$, $g(\xi) = \xi$, $g'(\xi) \neq 1 \Rightarrow \varphi(\xi) = \xi$

Chceme: $\varphi'(\xi) = 0$

$$\varphi'(\xi) = \lim_{x\to\xi} \frac{\varphi(x) - \varphi(\xi)}{x-\xi} = \lim_{x\to\xi} \frac{\varphi(x)-\xi}{x-\xi} = \lim_{x\to\xi} \frac{x - \frac{(g(x)-x)^2}{g(g(x))-2g(x)+x} - \xi}{x-\xi} =$$

$$= 1 - \lim_{x\to\xi} \frac{g(x)-x}{x-\xi} \cdot \frac{g(x)-x}{g(g(x))-2g(x)+x} = 1 - \lim_{x\to\xi} \frac{g(x)-x}{x-\xi} \cdot \lim_{x\to\xi} \frac{g(x)-x}{g(g(x))-2g(x)+x} =$$

$$= 1 - \lim_{x\to\xi} \frac{g'(x)-1}{1} \cdot \lim_{x\to\xi} \frac{g'(x)-1}{g'(g(x))\cdot g'(x) - 2g'(x) + 1} = 1 - (g'(\xi)-1)\cdot \frac{g'(\xi)-1}{(g'(\xi)-1)^2} = 1 - 1 = 0$$

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Věta o hranicích kořenů $\quad |a \pm b| \le |a| + |b|$

$\quad |a \pm b| \ge |a| - |b|$

Dk: $A = \max\{|a_{m-1}|, \dots, |a_1|, |a_0|\}$

$P(x) = a_m x^m + \dots + a_1 x + a_0$

$|x| > 1 \quad |P(x)| = |a_m x^m + \dots + a_1 x + a_0| \ge |a_m x^m| - (|a_{m-1} x^{m-1} + \dots + a_1 x + a_0|) \ge$

$$\ge |a_m||x|^m - (|a_{m-1}||x|^{m-1} + \dots + |a_1|\cdot|x| + |a_0|) \ge$$

$$\ge |a_m||x|^m - A(|x|^{m-1} + \dots + |x| + 1) = |a_m||x|^m - A\frac{|x|^m - 1}{|x|-1} >$$

$$> |a_m||x|^m - A\frac{|x|^m}{|x|-1} = |x|^m\left(|a_m| - \frac{A}{|x|-1}\right)$$

Pro $|a_m| - \frac{A}{|x|-1} > 0$ je $|P(x)| > 0$, tj. $|a_m| > \frac{A}{|x|-1} \Rightarrow |x| - 1 > \frac{A}{|a_m|} \Rightarrow |x| > 1 + \frac{A}{|a_m|}$

pro x: $|x| > 1 + \frac{A}{|a_m|}$ je $|P(x)| > 0 \Rightarrow$ je-li ξ - kořen P, pak

$$|\xi| \le 1 + \frac{A}{|a_m|}$$

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Subst $y = \frac{1}{x}$

$P(x) = a_m x^m + \dots + a_1 x + a_0 = 0 \quad / : x^m$

$$a_m + a_{m-1} y + \dots + a_1 y^{m-1} + a_0 y^m = 0$$

$Q(y) = a_0 y^m + \dots + a_{m-1} y + a_m$

Pro kořen α pol. Q platí: $|\alpha| \le 1 + \frac{B}{|a_0|}$

$\xi = \frac{1}{\alpha}$ - kořen $P \qquad \left|\frac{1}{\xi}\right| \le 1 + \frac{B}{|a_0|}$

$\alpha = \frac{1}{\xi} \qquad |\xi| \ge \dfrac{1}{1 + \frac{B}{|a_0|}}$

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