Economic Modelling 37 (2014) 340–349

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

The dynamics of a banking duopoly with capital regulations

Luciano Fanti *

Department of Economics and Management, University of Pisa, Via Cosimo Ridolfi, 10, I–56124 Pisa, Italy

ARTICLE INFO

Article history:
Accepted 1 November 2013

JEL Classification:
C62
G21
G28
D43
L13

Keywords:
Bifurcation
Chaos
Cournot
Oligopoly
Banking
Capital regulation

ABSTRACT

This paper analyses the dynamics of a banking duopoly game with heterogeneous and homogeneous players (as regards the type of expectations' formation), to investigate the effects of the capital requirements introduced by international accords (Basel-I in 1988 and more recently Basel-II and Basel-III), in the context of the Monti-Klein model. This analysis reveals that the policy of introducing a capital requirement tends to stabilise the market equilibrium (both with heterogeneous and homogeneous banks). Moreover, it is shown that 1) when the capital standard is reduced the market stability is lost through a flip bifurcation and subsequently a cascade of flip bifurcations may lead to periodic cycles and chaos; 2) when the expectations are heterogeneous even the case of multi-stability may be present.

Therefore, although on the one side the capital regulation is harmful for the equilibrium loans' volume and profit, on the other side it is effective in keeping or restoring the stability of the Cournot–Nash equilibrium in the banking duopoly.

1. Introduction

As noted by Vives (2010b, p. 1) "the recent history of the financial sector can be divided into two periods. The first, from the 1940s up to the 1970s, was characterised by tight regulation, intervention, and stability, while the second was marked by liberalisation and greater instability."

The recent financial turmoil 2008–2009 has made high in the current political agenda the importance of a regulation of the banking industry, having stressed the threat of a systemic risk due to a bank run and the inability of depositors to monitor banks.

In particular, the ongoing financial crisis has sparked a debate about the need for a profound shake-up of financial regulation. Admittedly, most of discussion grounds on well-established and sophisticated microeconomics of banking, which however is prevalently either in a static context or assumes banks' perfect foresight. Since the crisis represents "intrinsically" an out-of-equilibrium market behaviour as well as causes per se a more unpredictable environment for banks' decisions, we investigate the banking market stability under the assumption of bounded rationality rather than of perfect foresight.

The predominant instruments employed in the regulation of banking, aiming to prevent banks in investing in too risky projects and to render more safe the banking system for depositors, may be considered 1) a deposit insurance contract offered by the government (e.g. Chan et al. (1992); 2) a capital requirement (e.g. Kim and Santomero (1988), Rochet (1992)); 3) a joint use of deposit insurance and capital requirements (e.g. Giammarino et al. (1993)).

While each of these instruments has been largely studied in its pro and cons, we only focus on the second one, because the international accords of the last decades as regards the banking industry regulation (namely Basel I, II and very recently III) are substantially based on it.[1]

Another reason why the imposition of some capital standard is important concerns the problem of corporate bank governance. This is because the regulation through capital requirements may be optimally used to establish a threshold of corporate control between bank's owners and regulators (which represent the interests of depositors who are unable to monitor management) (e.g. Dewatripont and Tirole, 1993).

In a nutshell, the capital to asset ratio imposed under Basel-I Norms (1988) by the regulator was fixed at 8%, while the new banking capital regulation (Basel II) prescribes a similar capital adequacy ratio which is, however, risk weighted. The idea underlying Basel II is to calibrate the

[1] The evolution of political debate about the banks' regulation may be so resumed: "the general trend in banking regulation has been to control risk-taking through capital requirements and appropriate supervision. Both risk-based (deposit) insurance and disclosure requirements have been proposed to limit risk-taking behaviour. …Capital requirements, supervision, and market discipline are the three pillars on which the Basel II regulatory reform was based." Vives (2010b, p.12).

* Tel.: +39 050 22 16 369; fax: +39 050 22 16 384.
E-mail addresses: lfanti@ec.unipi.it, fanti.luciano@gmail.com.

0264-9993/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.econmod.2013.11.010

capital requirement so that it covers the Value at Risk (expected and unexpected) from the loan under some assumptions.[2] More theoretically, the risk calibration of the capital requirement is due to the fact that when banks are regulated by a flat-capital requirement, this may lead to an increase in the bank's probability of failure because the banker may choose to compensate the loss in utility caused by the reduction in leverage with the choice of a riskier portfolio. Therefore the regulator can eliminate this adverse effect by using a risk–based capital requirement approach (Kim and Santomero (1988)).

As regards Basel III, the main content of such an accord – only focusing on the issue of capital requirements (which is crucial in this paper) – is a further increase of the capital requirements: in particular it will require banks to hold 1) 4.5% of common equity (up from 2% in Basel II), 2) 6% of Tier I capital (up from 4% in Basel II) of risk-weighted assets, 3) a mandatory capital conservation buffer of 2.5% plus a discretionary countercyclical buffer (up to another 2.5% of capital during periods of high credit growth).

The literature on banking and regulation is fairly vast (see, for a review, Santos (2000) and very recently Vives (2010a,b), to which we refer to). Only to mention someone, Blum and Hellwig (1995) discuss the macroeconomic implications of bank capital regulation, while, as regards particularly emerging economies, Vives (2006) discusses the role of banking capital regulation and Nieto Parra (2005) analyses in particular the behaviour of regulated foreign banks. As regards, more specifically, the assumption of non-competitive banking market Matutes and Vives (2000), among many others, consider an imperfect competition model where banks are differentiated, have limited liability and there are social costs of failure, and Allen and Gale (2004) consider banks competing à la Cournot in the deposit market and choose a risk level on the asset side, showing that, as the number of banks grows and depositors are insured, banks have maximal incentives to take risk on the asset side.

Despite the progress in the theory of banking regulation in the last two decades, there are still many relevant issues that are not fully investigated: for example, the theoretical research on the effects of banks' capital regulations on the dynamics of an imperfect competition banking industry is still limited.

In order to model the banking duopoly, a simplified version of the models of Klein (1971) and Monti (1972) – which are the standard models of the neoclassical theory of firm applied to the banking industry – is used.[3] In particular the model is adapted for banks' capital regulation, with the assumption that banks are risk-neutral. For the sake of precision, we recall that this model abstracts from the uncertainty,[4] and thus from both default risk (both for borrowers and banks) and risk for depositors (with corresponding insurance deposit mechanisms).

As to the dynamical context, the banking duopoly is analysed in accord with the recent strand of oligopoly literature in which firms' decisions are based on expectations different from the simple naïve expectations formation implicit in the original model by Cournot (1838) (according to which in every step each firm assumes the last values taken by the competitors without estimation of their future reactions).

In fact, more recently, several works, in particular following Dixit (1986), have considered more realistic mechanisms through which bounded rational players form their expectations on the decisions of the competitors and have shown that the Cournot model may lead to complex behaviours such as periodic cycles and chaos (e.g. Bischi et al., 2010; Fanti and Gori, 2012a, 2012b; Tramontana, 2010).[5] However, at the best of our knowledge, the issue of the dynamical relationship between capital regulation and stability in a banking duopoly has not been so far explored. Since the above mentioned papers on dynamic duopoly have shown that when one or both firms competing à la Cournot have expectations different from the traditional Cournot (naïve) type, complex dynamics may occur, then we investigate the specific problem of the dynamical effects of a capital regulation in a fully micro-founded banking industry when such expectations do exist. This fills the gap in the literature on dynamic Cournot duopolies. Moreover we note that the issue of the effects of capital regulations on stability takes on a greater importance when the banking industry is in "peril" of instability as in the current European situation.

The main result of the paper is that the introduction of sufficiently high capital requirements is effective for the purpose of keeping or restoring the banking industry stability, with heterogeneous as well as homogeneous banks' expectations.

The policy implication is that while on the one hand a banks' capital regulation induces a reduction in equilibrium profits and in the volume of loans, on the other hand it may prevent undesirable and unpredictable fluctuations and even a shrinking of the loans market.

Moreover, from a mathematical point of view, it is shown that the loss of the market equilibrium stability may occur through a flip bifurcation and that a cascade of flip bifurcations may lead to periodic cycles and chaos. Furthermore, a numerical analysis of the global behaviour has revealed that when banks are heterogeneous two stable attractors may co-exist (i.e. multistability) with their complicated basins of attraction. In such a case the implication for the regulation policy is that for identifying the effects of the policy on the long run evolution of the banking market criteria based on local stability are no longer sufficient and the market dynamics become dependent on the initial conditions (i.e. path-dependent), making difficult to predict which one of the multiple equilibria will be observed.

The paper is organised as follows. In Section 2 the model with the capital regulation is developed and the dynamical system of a duopoly game with heterogeneous expectations (one bounded rational bank and one naïve bank) is presented. In Section 3 the steady-state and the dynamics of the model are studied, showing explicit parametric conditions of the existence, local stability and bifurcation of the market equilibrium. In Section 4 the results of the previous section are numerically illustrated and complex dynamic behaviours are shown to occur depending on the level of capital requirement through usual bifurcation diagrams; moreover, a numerical sketch of the global behaviour is also offered. Section 5 considers homogeneous expectations, comparing the results with those in Sections 3 and 4. Section 6 concludes.

2. The model

The model is a simplified duopolistic version of Klein's (1971) and Monti's (1972) models, which represent the standard models as regards

[2] More technically, in order to fix the capital requirement under Basel-II, banks can choose between a "standardised" approach in which external rating agencies set the risk weight for the different types of loans (say corporate, banks, and sovereign claims) or an internal-rating-based approach in which banks estimate the probability of default and also the loss given by default.

[3] Indeed, a part from the further differences arising with uncertainty, there is a significant difference between bank and ordinary firm. In fact, while the latter mainly interacts with the other competitors in the output market and have no or little interactions in the input market, the former *i*) interacts in both the deposit (input) market and the loan (output) market, and *ii*) lends (borrows from) to other banks.

[4] In the presence of uncertainty, another – and more important–difference between banks and ordinary firms arises. Indeed, in contrast with the ordinary firms, banks have to face the problem of loans default risk (i.e. credit risk) and the own possible default risk. An important model embodying uncertainty in the Monti–Klein framework is developed by Dermine (1986), who extends it with bankruptcy risk and deposit insurance, showing that the independence between deposit and credit rates (assumed, in line with the original Monti–Klein framework, in the present paper for simplicity) would be lost and the direction of causality between the two rates would depend on whether a deposit insurance mechanism is present or not.

[5] Note that we assumed an informative context of bounded rationality instead of perfect foresight also because in the latter case the dynamic analysis is less interesting (broadly speaking, any market adjustment dynamics would tend to be prevented "by construction"). However, we recall, for the sake of precision, that Dana and Montrucchio (1986) showed that a complex trajectory can be an admissible solution to discounted dynamic optimization problems in a dynamic duopoly game with complete information and rational agents.

the microeconomic view of the banking industry (see for more detailed comments the textbook of Freixas and Rochet, 1997). This model is extended to embody a capital requirement, in line with the Basel-I, II and III Accords.

Since it is assumed, for simplicity, that there are no open positions between banks in the interbank market, the balance sheet of each bank is composed only of loans L on the asset side and of capital K and deposits D on the liability side. Again for simplicity, it is also assumed the same constant marginal costs c for deposits and loans. By contrast with the standard Monti–Klein model, there is no remuneration for deposits (however the marginal cost for deposits could be interpreted as the interest on deposits).[6]

As usual, a linear demand function for loans is assumed:

$$r_L\left(L_i+L_j\right)=a-b\left(L_i+L_j\right) \quad (1)$$

where a, $b > 0$ and r_L is the inverse demand function for loans.

Consequently, the profit function is as follows:

$$\pi_i=[a-b(L_i+L_j)]L_i-r_K K-c(D_i+L_i) \quad (2)$$

where r_K is the capital remuneration determined exogenously by the equilibrium in the capital markets.

By matching assets and liabilities in the balance sheet, we have:

$$L=K+D \quad (3)$$

and by denoting the capital requirement per unit of loans by γ, we have: $K \geq \gamma L$ where γ is a fixed percentage determined by the regulator.

It is assumed, for simplicity, that the capital requirement is binding, i.e.

$$K=\gamma L \quad (4)$$

Therefore, by using (3) and (4), the profit function becomes[7]:

$$\pi_i=[a-b(L_i+L_j)]L_i-L_i[(2c+(r-c_K)\gamma] \quad (5)$$

We assume, as usual in literature, that $a > 2c$ and that capital remuneration is higher than marginal cost, i.e. $r_K > c$.

From the profit maximisation by firm $i = \{1,2\}$, the marginal profits are obtained as:

$$\frac{\partial\pi_1(L_1,L_2)}{\partial L_1}=a-b(2L_1+L_2)-2c-\gamma(r_K-c), \quad (6.1)$$

$$\frac{\partial\pi_1(L_1,L_2)}{\partial L_2}=a-b(L_1+2L_2)-2c-\gamma(r_K-c) \quad (6.2)$$

The reaction or best reply functions of banks 1 and 2 are computed as the unique solution of Eqs. (6.1) and (6.2) for q_1 and q_2, respectively, and they are given by:

$$\frac{\partial\pi_1(L_1,L_2)}{\partial L_1}=0\Leftrightarrow L_1(L_2)=\frac{1}{2b}[a-2c-bL_2-\gamma(r_K-c)], \quad (7.1)$$

$$\frac{\partial\pi_1(L_1,L_2)}{\partial L_2}=0\Leftrightarrow L_2(L_1)=\frac{1}{2b}[a-2c-bL_1-\gamma(r_K-c)] \quad (7.2)$$

Following a vast recent dynamic oligopoly literature, (e.g. Leonard and Nishimura (1999), Den Haan (2001), Agiza and Elsadany (2003), Zhang et al. (2007), Tramontana (2010), Fanti and Gori, 2012a,b), I begin assuming heterogeneous expectations: i.e., firm 1 is bounded rational and firm 2 is naïve.[8]

The bank which has bounded rational expectations about the level of loans that should be set in the future, uses, as a consequence, the information on the current profit in such a way to increase or decrease loans at time $t+1$ depending on whether marginal profits are either positive or negative, as suggested, for instance, by Dixit (1986). Therefore, the adjustment mechanism of loans over time of the ith bounded rational bank is described by:

$$L_{i,t+1}=L_{i,t}+\alpha_i L_{i,t}\frac{\partial\pi_i}{\partial L_{i,t}}, \quad (8)$$

where the speed of adjustment of bank i's loans with respect to a marginal change in profits when L_i varies is assumed to be linear (i.e. $\alpha_i L_{i,t}$) with the coefficient $\alpha_i > 0$, in line with the above mentioned literature.[9]

The second duopolist, instead, is – in line with the traditional Cournot's assumption – a naive player which expects a level of loans of the rival equal to the last period's one.

Therefore, given these types of expectations formation, the two-dimensional system that characterises the dynamics of a Cournot banking duopoly is the following:

$$\begin{cases} L_{1,t+1}=L_{1,t}+\alpha L_{1,t}\left[a-b\left(2L_{1,t}+L_{2,t}\right)-2c-\gamma(r_K-c)\right] \\ L_{2,t+1}=\dfrac{1}{2b}\left[a-2c-bL_{1,t}-\gamma(r_K-c)\right] \end{cases} \quad (9)$$

3. Local stability analysis of the unique positive Cournot–Nash equilibrium

For simplicity, we define $\beta=\alpha-2c-\gamma(r_K-c)$. Map (9) has two fixed points: $E^1=\left(0,\frac{\beta}{2b}\right)$, located on the invariant coordinate axis, and $E^*=\left(\frac{\beta}{3b},\frac{\beta}{3b}\right)$, which is the unique Cournot–Nash equilibrium (i.e. $L_1=L_2=L^*$).

The fixed point E^1 is called a boundary equilibrium where only firm 2 serves the market as a monopolist and firm 1 does not produce and, for ensuring non-negative output of firm 2, $a > 2c-\gamma(r_K-c)$ holds. I first consider the stability properties of the equilibria by linearising the system (9) around each equilibrium point. As known, in order to study the local stability of the equilibrium points, I consider the Jacobian matrix $J(L_1,L_2)=\begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22}\end{pmatrix}$ evaluated at each equilibrium point. Then the following Lemma 1 holds:

Lemma 1. The boundary equilibrium E^1 is a saddle point.

This Lemma is straightforwardly derived from the Jacobian matrix $J\left(E^1\right)=\begin{pmatrix} \frac{\alpha\beta+2}{2} & 0 \\ -\frac{1}{2} & 0\end{pmatrix}$, whose eigenvalues are given by the diagonal entries and are $\lambda_1=\frac{\alpha\beta+2}{2}>1$, $\lambda_2=0$ and since $|\lambda_1|>1$, $|\lambda_2|<1$, then E^1 is a saddle point. Now we focus on the unique positive (i.e. Cournot–Nash) output equilibrium, which is determined by setting $L_{1,t+1}=L_{1,t}=L_1$ and $L_{2,t+1}=L_{2,t}=L_2$ in (9) and solving for (non-negative solutions of) L_1 and L_2:

$$L^*_1=L^*_2=L^*=\frac{1}{3b}[a-2c-\gamma(r_K-c)], \quad (10)$$

[6] Since a capital regulation based on the supply of loans is assumed, then the minimum capital requirements do not depend on the level of deposits and thus the presence or the absence of deposit remuneration is not relevant for our purposes. The alternative view about capital requirements linked with deposits is left for further research.

[7] Since capital requirement is binding and deposits are not remunerated, banks compete by choosing only loans, while in the original Klein–Monti framework they simultaneously choose loans, deposits and capital.

[8] The case of both bounded rational banks is investigated in Section 5.

[9] The coefficient α represents, loosely speaking, the "relative" speed of adjustment.

where $a > 2c - \gamma(r_K - c)$ holds to ensure $L^* > 0$. The equilibrium profit corresponding to the positive output is

$$\pi^* = \frac{\beta^2}{9b} \tag{11}$$

It is easy to see that both equilibrium loans and profits are reducing with an increasing capital requirement, γ.

The Jacobian matrix evaluated at the equilibrium point (10) is the following:

$$J(E^*) = \begin{pmatrix} -\frac{2\alpha\beta - 3}{3} & -\frac{\alpha\beta}{3} \\ -\frac{1}{2} & 0 \end{pmatrix} \tag{12}$$

The trace and determinant of the Jacobian matrix (12) are respectively given by:

$$T := Tr(J) = J_{11} + J_{22} = -\frac{2\alpha\beta - 3}{3} \tag{13}$$

$$D := Det(J) = J_{11}J_{22} - J_{12}J_{21} = \frac{-\alpha\beta}{6}, \tag{14}$$

so that the characteristic polynomial of (12) is:

$$G(\lambda) = \lambda^2 - tr(J)\lambda + \det(J), \tag{15}$$

whose discriminant is $Q := [Tr(J)]^2 - 4Det(J)$.

We now study the local stability properties of the Cournot–Nash equilibrium (Eq. (10)) by means of well-known stability conditions for a system in two dimensions with discrete time (see, e.g., Medio, 1992; Gandolfo, 2010), which are given by:

$$\begin{cases} (i) & F := 1 + T + D > 0 \\ (ii) & TC := 1 - T + D > 0 \\ (iii) & H := 1 - D > 0 \end{cases} . \tag{16}$$

For the particular case of the Jacobian matrix 12), while it can easily be seen that conditions (*ii*) is always fulfilled, conditions (*i*) and (*iii*) define surfaces in the parameter space on which a Flip bifurcation (i.e. a real eigenvalue that passes through 1) when $F = 0$ and a Neimark–Sacker bifurcation (i.e. the modulus of a complex eigenvalue pair that passes through 1) when $H = 0$, namely $Det(J) = 1$ and $|Tr(J)| < 2$, occur, respectively, as it can be easily ascertained from the following (17):

$$\begin{cases} (i) & F = \frac{-5\alpha\beta + 12}{6} > 0 \\ (iii) & H = \frac{-\alpha\beta + 6}{6} > 0 \end{cases} . \tag{17}$$

Therefore, the Cournot–Nash equilibrium L^* can lose stability through either a flip or a Neimark-Sacker bifurcation. The stability condition (*i*) in 17) represents a region *F* in the (α, γ) plane (i.e. the "relative" speed of adjustment and the level of capital requirement), bounded by the economic model assumption $\alpha > 0, \gamma > 0$. Therefore, the following equation B(α, γ) (i.e. the numerator of *F* in (17)) represents a bifurcation curve at which the positive equilibrium point $L^*_1 = L^*_2 = L^*$ looses stability through a flip (or period-doubling) bifurcation, that is:

$$B(\alpha, \gamma) := -5\alpha\beta + 12 = 0. \tag{18}$$

A simple inspection of Eq. (18) leads to the following remark.

Result 1. *The bifurcation curve* B(α, γ) *intersects the horizontal axis* at

$$\alpha = \alpha^F := \frac{12}{5\beta} \text{ or, alternatively, at } \gamma = \gamma^F := \frac{5\alpha(a - 2c) + 12}{[5\alpha(r_K - c)]} \tag{19}$$

Furthermore, the market equilibrium L^* is stable ($B(\alpha,\gamma) > 0$) when $\alpha < \alpha^F$ or, alternatively, when $\gamma > \gamma^F$.

Moreover, the following equation $N(\alpha,\gamma)$, i.e. the numerator of *H* in (17), represents a bifurcation curve at which the positive equilibrium point L^* loses stability through a Neimark–Sacker bifurcation, that is:

$$N(\alpha, \gamma) := -\alpha\beta + 6 = 0, \tag{20}$$

A simple inspection of Eq. (20) leads to the following result.

Result 2. *The bifurcation curve* $N(\alpha,\gamma)$ *intersects the horizontal axis* at

$$\alpha = \alpha^H := \frac{6}{\beta} \text{ and the market equilibrium } L^* \text{ is stable } (N(\alpha,\gamma) > 0) \text{ when } \alpha < \alpha^H \tag{21}$$

[10]

Given that the Nash equilibrium may become unstable either via flip or via Neimark–Sacker bifurcation (as shown in Results 1 and 2), we have to check which one occurs before the other one, starting from a stability situation and increasing the value of α (decreasing the value of γ).

Then the following result holds:

Result 3. *the equilibrium market L^* may lose stability only through a flip bifurcation.* This remark straightforwardly follows from the simple observation that $\alpha^F < \alpha^H$.[11]

Once established that only a flip bifurcation may occur, we focus on our parameter of interest, i.e. the capital requirement γ. In particular, we must investigate whether the solution for $\gamma = \gamma^F$ is feasible from an economic point of view. Therefore the following holds:

Result 4. *A flip bifurcation value of the capital requirement does exist, provided that the following threshold values of the speed of adjustment hold:*

$$0 < \gamma^F < 1 \Leftrightarrow \frac{12}{5(a - 2c)} < \alpha < \frac{12}{5(a - c - r_K)}. \tag{18}$$

Therefore, provided that the "relative" speed of adjustment is not too small (too high), in which case the market is always stable (unstable) independently of the level of the capital standard, the regulation through the choice of an appropriate level of capital requirement is feasible and effective in stabilising the banking duopoly.

Moreover, from the simple observation of the effects of parameters *c* and r_K on the flip bifurcation value of γ (i.e. $\frac{\partial\gamma^F}{\partial c} < 0$, $\frac{\partial\gamma^F}{\partial r_K} < 0$) we may see that both higher marginal costs and higher exogenous capital remuneration (a higher opportunity-cost of capital) favour the stabilising effect of the capital requirement.

4. A numerical illustration

The main purpose of this section is to show that the dynamic system (9) can generate, in addition to the local flip bifurcation and the resulting emergence of a two-period cycle analytically shown in Section 3, even complex behaviours. In accordance with the aim of the paper, the capital requirement parameter γ is taken as the bifurcation

[10] Alternatively, it can be easily shown, by solving N(α,γ) N 0 for γ, that the market is stable when γ N γH.

[11] Of course, the proof of Result 3 could be alternatively formulated in terms of γ, showing that $\gamma^H > \gamma^F$.

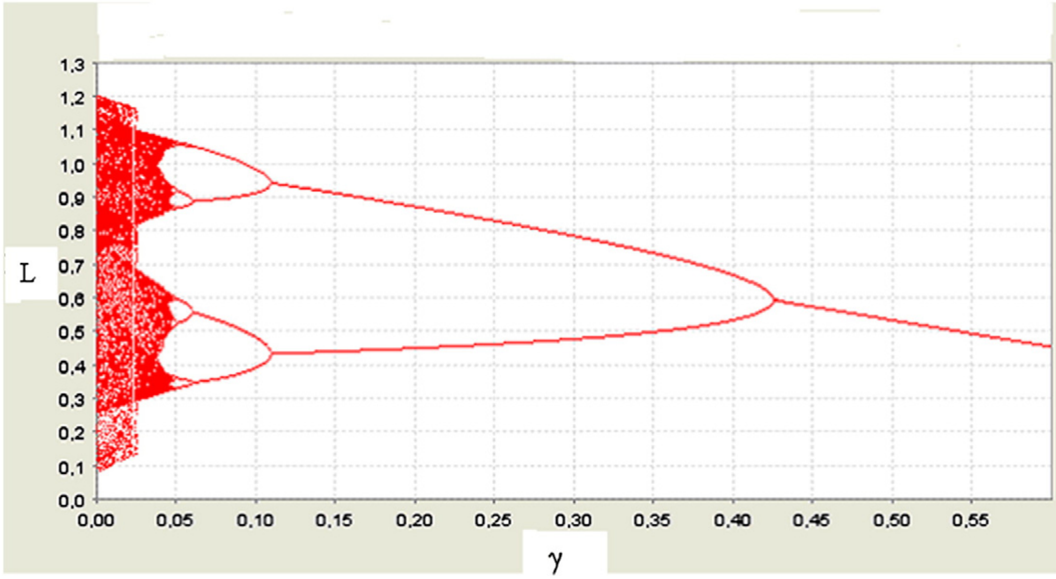

Fig. 1. Bifurcation diagram for γ. Initial conditions: $L_{1,0} = 0.30$ and $L_{2,0} = 0.31$.

parameter, and the following parameter set is chosen (only for illustrative purposes): $\alpha = 1.35$, $a = 3$, $r_K = 2.5$ and $c = 0.1$. To provide some numerical evidence for the dynamical chaotic behaviour of system (9), several numerical results may be resumed in a bifurcation diagram.

Fig. 1 depicts the bifurcation diagram for γ. The figure clearly shows that a decrease in the capital requirement implies that the map (9) converges to a fixed point for $1 > \gamma > 0.4259$. Starting from this interval, in which the positive fixed point (10) of system (9) is stable, Fig. 1 shows that the equilibrium volume of loans undergoes a flip bifurcation at $\gamma^F = 0.4259$. Then, a further decrease implies that a stable two-period cycle emerges for $0.4259 > \gamma > 0.11$. As long as the parameter γ reduces a four-period cycle, cycles of highly periodicity and a cascade of flip bifurcations that ultimately lead to unpredictable (chaotic) motions are observed.

Moreover Fig. 2 shows the shape of the chaotic attractor of system (9) [12] and its basin of attraction (which appears connected and regular)[13] for $\gamma = 0.01$.

To sum up, it is clearly illustrated that the banking industry is stable (unstable) for sufficiently high (low) levels of capital requirement.

However, although the local stability analysis above presented has driven clearcut results, we should consider that discrete time dynamical systems such as (9), which are also characterised by asymmetric expectations, may show other interesting dynamical events which cannot be studied by local methods (i.e. based on linear approximations around the attractors) but through a global study – often through numerical methods – of the dynamical system, as recently argued by, among others, Brock and Hommes, 1997; Bischi et al., 2000; Agliari et al., 2002. Among these global phenomena, those that may have important economic implications, especially for policy, are the presence of coexisting attractors and the structure of the boundaries that separate their basins of attraction. Then in the next section we proceed with a numerical analysis of the global behaviour of system (9) in order to characterise, although in a sketched way,[14] the presence of other phenomena, additive to those evidenced by the local analysis above presented.

4.1. Sketch of the global behaviour

We use the same parametric set, but $\alpha = 0.415$, a $= 10$ (meaning simply that the size of the loan market is larger and the "relative" speed of adjustment is smaller).

It is easy to observe that the bifurcation diagram (Fig. 3) shows a different shape with respect to Fig. 1, especially in the range $0.54 < \gamma < 0.59$ (Fig. 4), with evident symptoms of a multiplicity of attractors. For instance, the presence of discontinuous bifurcations when $\gamma = 0.58$ highlights the appearance of the co-existence of two attractors (namely, a period-4 cycle with a period-6 cycle) which persists, while the attractors' shape evolves, for further reductions of γ. In fact Fig. 5 depicts the shape of the chaotic attractor of system (9) for $\gamma = 0.555$: the period-4 cycle co-exists with a six-band chaotic attractor (and their basins of attraction appear rather irregular).

This means that for the given capital requirement, because two locally stable attractors coexist, policy-makers cannot know to which one a generic trajectory converges, even when starting very close to one of them.

5. Homogenous expectations

In this section I develop the dynamic model in the case in which both banks have bounded rational expectations.

Therefore, also recalling Eqs. (6.2) and (8) in Section 3, the two-dimensional system that characterises the dynamics of a Cournot banking duopoly under this type of expectations formation is the following:

$$\begin{cases} L_{1,t+1} = L_{1,t} + \alpha L_{1,t}\left[a - b\left(2L_{1,t} + L_{2,t}\right) - 2c - \gamma(r_K - c)\right] \\ L_{2,t+1} = L_{2,t} + \alpha L_{2,t}\left[a - b\left(2L_{2,t} + L_{,1,t}\right) - 2c - \gamma(r_K - c)\right] \end{cases}. \qquad (23)$$

[12] This figure shows the shape of the attractor, which is obtained, as usual, by representing many points $(L_1(t), L_2(t))$ of a trajectory after having discarded the early iterates, which constitute the so-called transient. The red-coloured region represents the basin of attraction of the bounded attractor, whereas the white region represents the set of points that generate unbounded trajectories, i.e. the basin of infinity.

[13] As known, the basins of attractions have either a simple or complicated structure: they can be connected (i.e. formed by a compact subset of the phase space containing the attractor itself) or disconnected (i.e. by the union of the subset of initial conditions around the attractor, called immediate basin, and by its disconnected preimages). Interestingly, even connected basins can have a complicated structure.

[14] Obviously, a thorough analysis of the global behaviour would require a more technical paper. I thank an anonymous referee for having pointed out the importance of the global analysis.

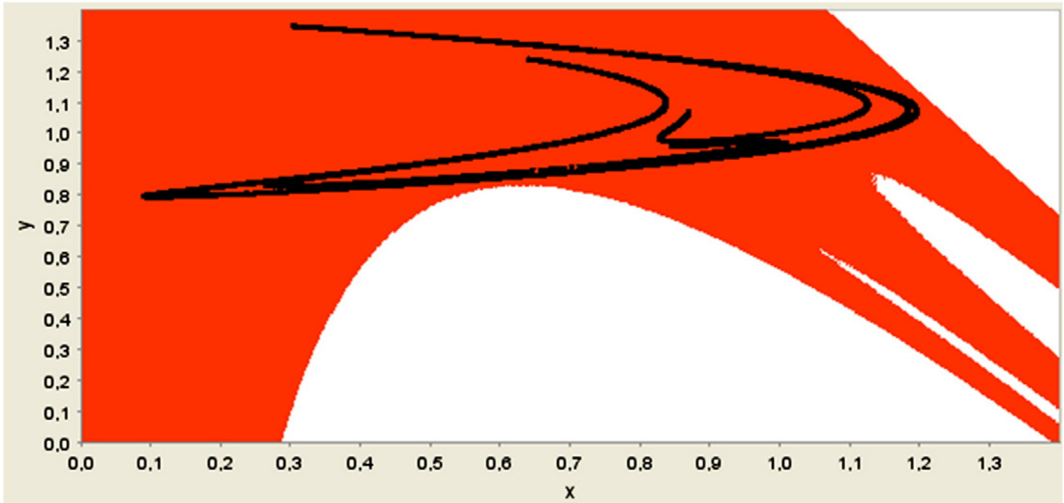

Fig. 2. Attractor of system (9) and its basin of attraction for the parameter value $\gamma = 0.01.(x = L_1(0), y = L_2(0))$.

Map (23) has four fixed points: $E^0 = (0,0)$, $E^1 = \left(0, \frac{\beta}{2b}\right)$, $E^2 = \left(\frac{\beta}{2b}, 0\right)$, located on the invariant coordinate axes, and

$$E^* = \left(\frac{\beta}{3b}, \frac{\beta}{3b}\right) \tag{24}$$

which is the unique Cournot–Nash equilibrium (i.e. $L_1 = L_2 = L^*$). The Cournot–Nash equilibrium is the same with the case of heterogeneous expectations. While the analysis of the local stability of the zero and boundaries equilibria easily shows that Lemma 1 also holds for such equilibria, I focus on the Cournot–Nash equilibrium, which in the case of heterogeneous expectations, as was shown in Section 3, could be destabilised through a flip bifurcation.

The Jacobian matrix evaluated at the Cournot–Nash equilibrium point (24) is the following:

$$J = \begin{pmatrix} -\frac{2\alpha\beta-3}{3} & -\frac{\alpha\beta}{3} \\ -\frac{\alpha\beta}{3} & -\frac{2\alpha\beta-3}{3} \end{pmatrix} \tag{25}$$

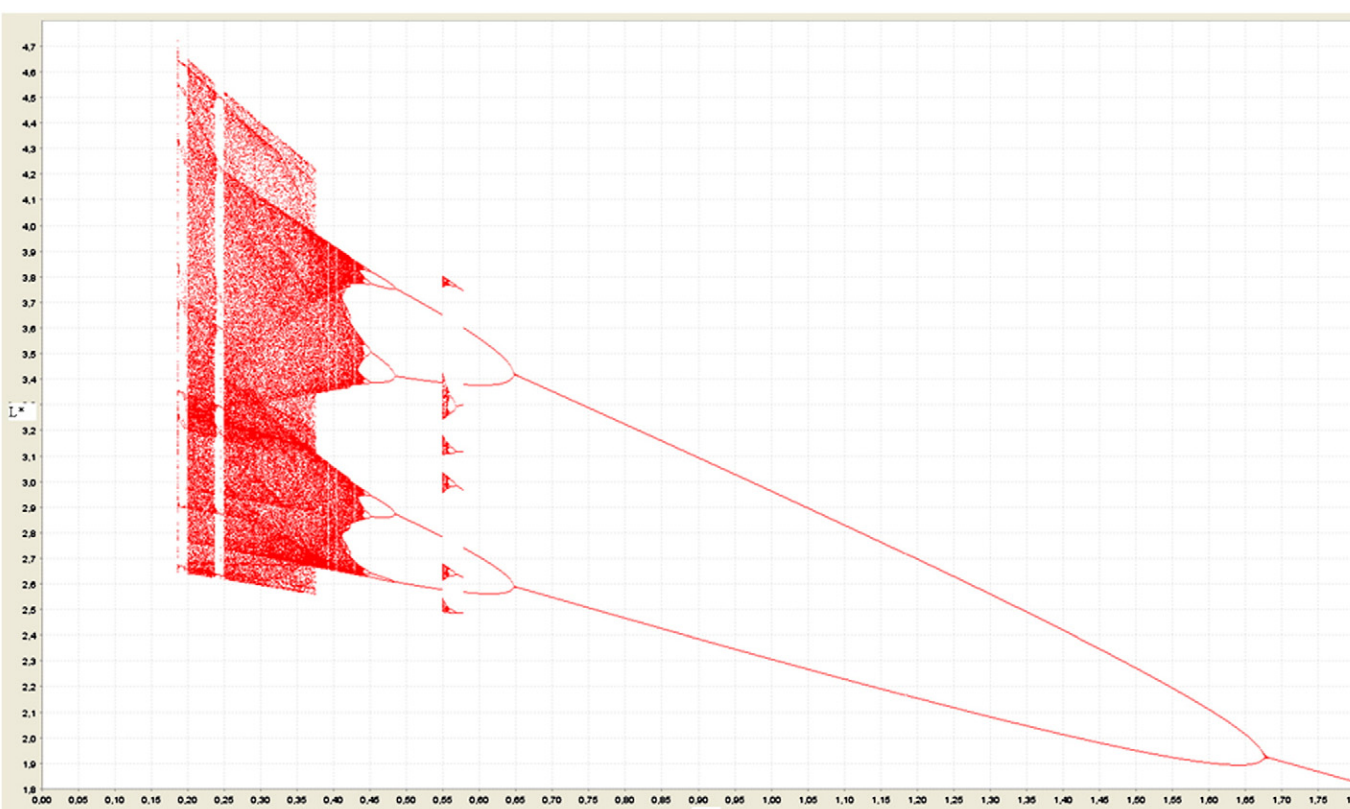

Fig. 3. Bifurcation diagram for γ. Initial conditions: $L_{1,0} = 0.90$ and $L_{2,0} = 0.50$.

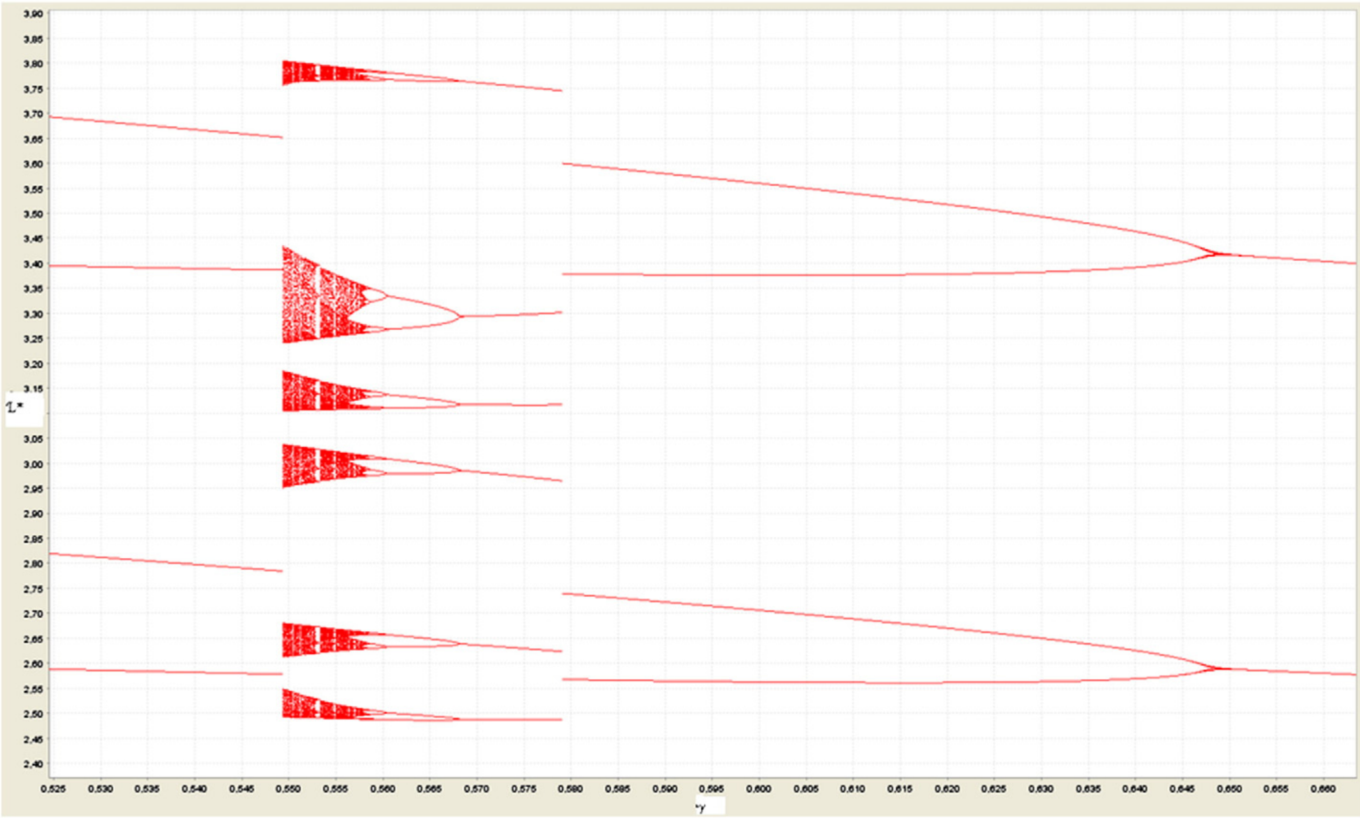

Fig. 4. Enlarged window of the bifurcation diagram in Fig. 3 for $0.525 < \gamma < 0.665$.

The trace, determinant and stability conditions of the Jacobian matrix (25) are respectively given by:

$$T := Tr(J) = -\frac{2\alpha\beta-3}{3}. \tag{26}$$

$$D := Det(J) = \frac{(\alpha\beta-3)(\alpha\beta-1)}{6}, \tag{27}$$

$$\begin{cases} (i) \quad F := 1+T+D = \dfrac{(\alpha\beta-6)(\alpha\beta-2)}{3} > 0 \\ (ii) \quad TC := 1-T+D = \dfrac{\alpha^3\beta^2}{3} > 0 \\ (iii) \quad H := 1-D = -\dfrac{\alpha\beta(\alpha\beta-4)}{3} > 0 \end{cases}. \tag{28}$$

Therefore, as in the case of heterogenous banks, the Cournot–Nash equilibrium L^* can lose stability through either a flip or Neimark–Sacker bifurcation.

Redefining opportunely, according to (28), the bifurcation curves $B(\alpha,\gamma)$ and $N(\alpha,\gamma)$, as in the case of heterogeneous banks in the previous section, the following result is obtained.

Result 5. *The bifurcation curve $B(\alpha,\gamma)$ intersects the horizontal axis at*

$$\gamma = \gamma_1{}^F := \frac{a\alpha-2(c\alpha+1)}{\alpha(r_K-c)} \text{ and } \gamma = \gamma_2{}^F := \frac{a\alpha-2(c\alpha+3)}{\alpha(r_K-c)} \tag{29}$$

Furthermore, the market equilibrium L^* is stable ($B(\alpha,\gamma) > 0$) when either $\gamma > \gamma_1{}^F$ or $\gamma < \gamma_2{}^F$.

The proof easily follows observing that $B(\gamma)$ is a U-shaped quadratic function.

Result 6. *The bifurcation curve $N(\alpha,\gamma)$ intersects the horizontal axis at* $\gamma = \gamma^H := \dfrac{a\alpha-2(c\alpha+2)}{\alpha(r_K-c)}$ *and the market equilibrium L^* is stable ($N(\alpha,\gamma) > 0$) when $\gamma > \gamma^H$.*

Then, again as in the case of heterogeneous banks, the following result holds:

Result 7. *The equilibrium market L^* may lose stability only through a flip bifurcation.* This Result straightforwardly follows from the simple observation that $\gamma_1{}^F > \gamma^H > \gamma_2{}^F$[15]

Finally also in the case of bounded rational banks the solution for $\gamma = \gamma_1{}^F$ may be feasible from an economic point of view, according to the following:

Result 8. *A flip bifurcation value of the capital requirement does exist, provided that the following threshold values of the speed of adjustment hold*:

$$0 < \gamma^F < 1 \iff \frac{2}{(a-2c)} < \alpha < \frac{2}{(a-c-r_K)}. \tag{30}$$

Therefore, also in this case the regulation through the choice of an appropriate level of capital requirement is feasible and effective in stabilising the banking duopoly when the "relative" speed of adjustment is neither too small nor too high, although the Cournot–Nash equilibrium appears to be slightly more prone to the triggering of cyclical instability than in the case of heterogeneous expectations (as easily seen by the algebraic comparison between (19) and (30) and by the graphical comparison between the bifurcation diagrams (Figs. 3 and 6)), independently of the level of the capital standard.

[15] Of course, the proof of Result 7 could be alternatively formulated in terms of α.

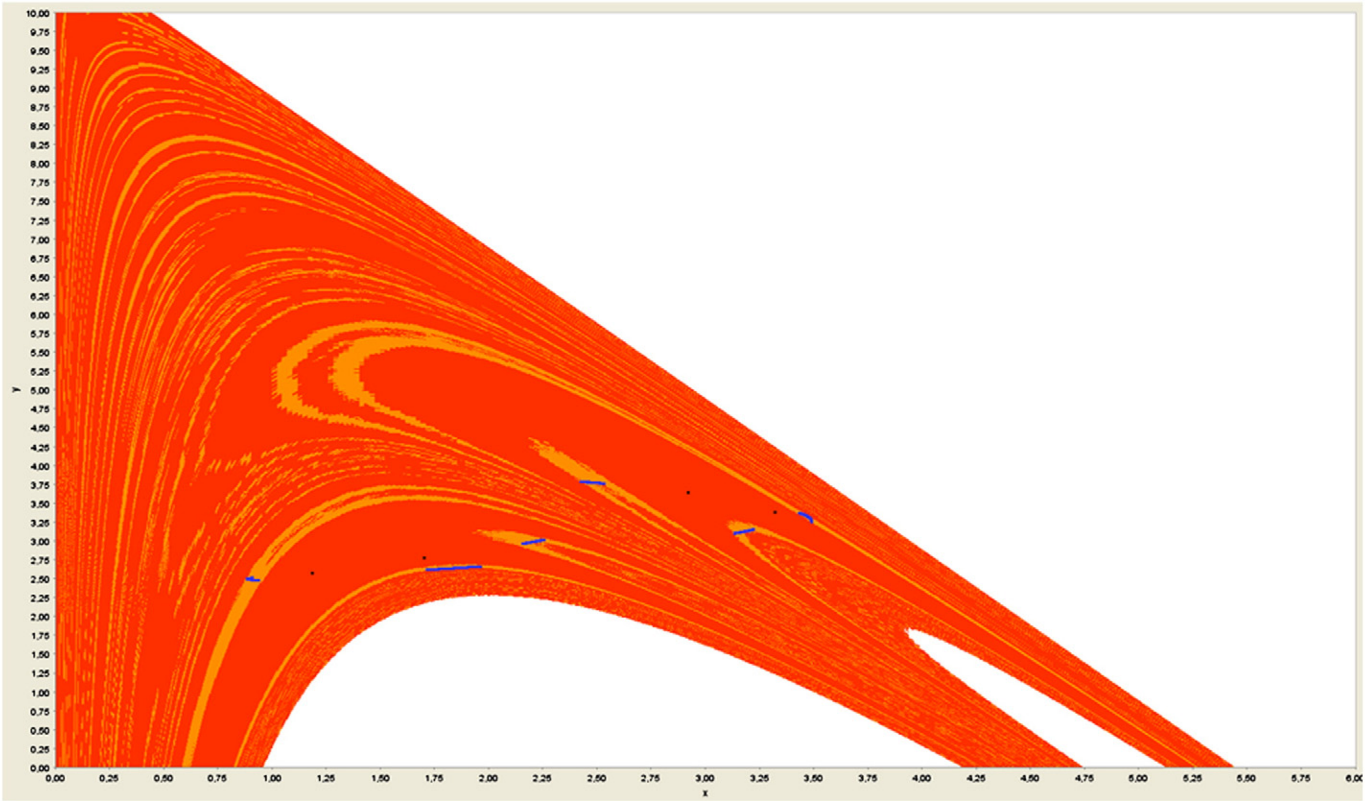

Fig. 5. Attractors of system (9) and their basins of attraction for the parameter value $\gamma = 0.555$.

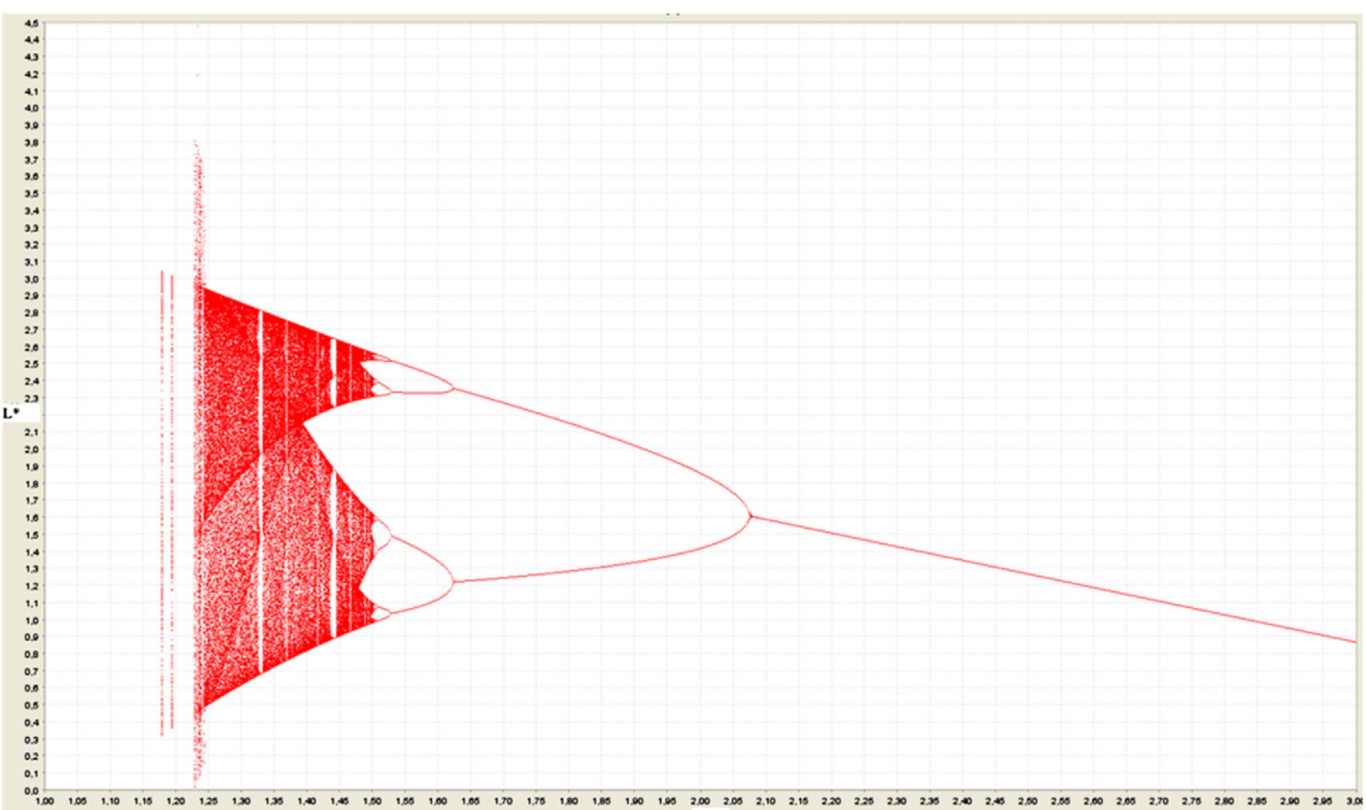

Fig. 6. Bifurcation diagram for γ. Initial conditions: $L_{1,0} = 0.50$ and $L_{2,0} = 0.55$.

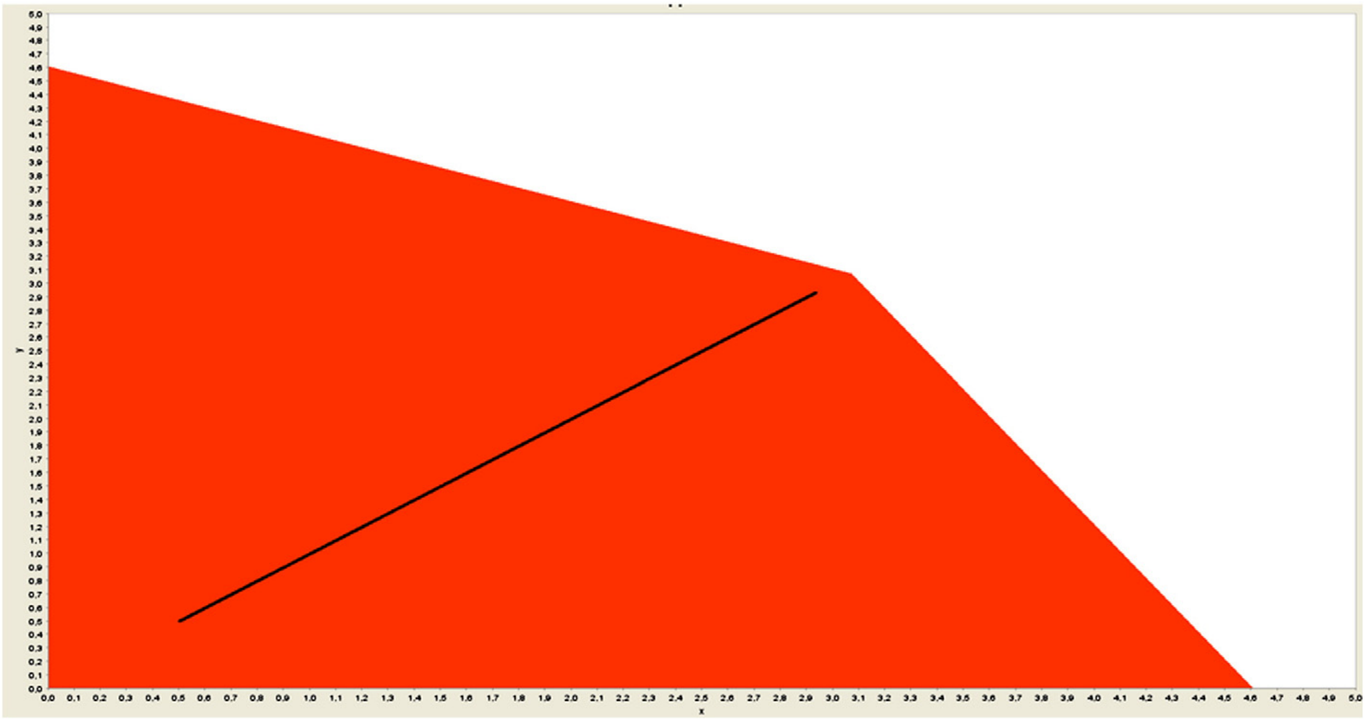

Fig. 7. Attractor of system (23) and its basin of attraction for the parameter value $\gamma = 1.25$.

We can now examine whether the global behaviour of system (23) is equal to that of system (9). Fig. 7 shows the uniqueness of the chaotic attractor and the regularity of its basin of attraction. In particular, it is noted that this is due to the fact that now banks are identical and system (23) becomes symmetric: this implies that synchronised dynamics is governed by a simpler one-dimensional model and the chaotic region is a small area along the diagonal, which is the line of equal volume of loans (see Bischi et al., 1999, for more technical details on the dynamical behaviour of the duopoly system with identical players). Then, the relatively more complicated global behaviour of system (9) may be ascribed to the asymmetry in the banks' expectations.[16]

6. Conclusions

Motivated by the important debate on banks' capital regulation, this paper analysed the dynamics of a Cournot banking duopoly game with both heterogeneous and homogeneous expectations with bounded rationality, and investigated the effects of the presence of capital requirements. For doing this, a simplified version of the Monti–Klein approach to the banks' behaviour is adopted, extended to embody a capital requirement, in line with the Basel-I, II and III Norms.

The main result is that such a capital regulation is effective for the purpose of stabilising the market equilibrium with both cases of expectations, and, under appropriate economic conditions, a reduction of the capital standard is responsible for the stability loss of such an equilibrium through a flip bifurcation, and for the consequent complex dynamic events. Moreover it is shown that when the expectations are heterogeneous even multi-stability may be present, with noteworthy policy implications.

In conclusion, the message is that, although, on the one hand, capital regulation reduces the equilibrium loans' volume and profit, on the other hand it may keep or restore the stability of the banking market equilibrium, and, furthermore, the latter result may constitute a warning for policy-makers as regards the possible effects of de-regulation policies.

[16] By passing, we note that this is another example that the heterogeneities of agents matter much.

Two remarks are appropriate to conclude, as a note of caution and as insights for future directions of research. Since both the aspect of defaults and the security for depositors are important at the light of the present debate about the banks' capital adequacy, the present model should be extended for embodying uncertainty and default risks.

Moreover, according to Matutes and Vives (2000), the capital requirement level should be – rather than an exogenous constant as in the present model (and in the Basel-I accord) — an increasing function of the intensity of competition (i.e. a decreasing function of the degree of product differentiation between banks, which would requires that the solvency requirement be tightened in a less products differentiated environment). A model's extension following the suggestions in Matutes and Vives (2000) is left for future research. Finally we note that dynamical analyses of the effects of banking regulations such as the present one may be of interest for the public policy responses to the recent banking industry crisis, especially in Europe.

References

Agiza, H.N., Elsadany, A.A., 2003. Nonlinear dynamics in the Cournot duopoly game with heterogeneous players. Physica A 320, 512–524.
Agliari, A., Bischi, G.I., Gardini, L., 2002. Some methods for the global analysis of dynamic games represented by noninvertible maps. In: Puu, T., Sushko, I. (Eds.), Oligopoly Dynamics: Models and Tools. Springer Verlag, pp. 31–83.
Allen, F., Gale, D., 2004. Competition and financial stability. J. Money Credit Bank. 3, 453–480.
Bischi, G.I., Gallegati, M., Naimzada, A., 1999. Symmetry-breaking bifurcations and representative firm in dynamic duopoly games. Ann. Oper. Res. 89, 253–272.
Bischi, G.I., Mammana, C., Gardini, L., 2000. Multistability and cyclic attractors in duopoly games. Chaos, Solitons Fractals 11, 543–564.
Bischi, G.I., Chiarella, C., Kopel, M., Szidarovszky, F., 2010. Nonlinear oligopolies. Stability and bifurcations. Springer-Verlag, Berlin.
Blum, J., Hellwig, M., 1995. The macroeconomic implications of capital adequacy requirements for banks. Eur. Econ. Rev. 39, 739–749.
Brock, W.A., Hommes, C., 1997. Rational route to randomness. Econometrica 65, 1059–1095.
Chan, Y.S., Greenbaum, S.I., Thakor, A.V., 1992. Is fairly priced deposit insurance possible? J. Financ. 47, 227–245.
Cournot, A., 1838. Recherches sur les Principes Mathématiques de la Théorie des Richessess. Hachette, Paris.
Dana, R.A., Montrucchio, L., 1986. Dynamic complexity in duopoly games. J. Econ. Theory 40, 40–56.
Den Haan, W.J., 2001. The importance of the number of different agents in a heterogeneous asset-pricing model. J. Econ. Dyn. Control. 25, 721–746.

Dermine, J., 1986. Deposit rates, credit rates and bank capital: the Klein-Monti model revisited. J. Bank. Financ. 10 (1), 99–114.

Dewatripont, M., Tirole, J., 1993. Efficient governance structure: implications for banking regulation. In: Mayer, C., Vives, X. (Eds.), Capital Markets and Financial Intermediation. Cambridge University Press, Cambridge, pp. 12–35.

Dixit, A.K., 1986. Comparative statics for oligopoly. Int. Econ. Rev. 27, 107–122.

Fanti, L., Gori, L., 2012a. The dynamics of a differentiated duopoly with quantity competition. Econ. Model. 29 (2), 421–427.

Fanti, L., Gori, L., 2012b. The codetermined firm in a Cournot duopoly: a stability analysis. Econ. Model. 29, 1242–1247.

Freixas, X., Rochet, J.C., 1997. Microeconomics of banking. MIT Press, Cambridge.

Gandolfo, G., 2010. Economic dynamics, Fourth Edition. Springer, Heidelberg.

Giammarino, R.M., Lewis, T.R., Sappington, D.E.M., 1993. An incentive approach to banking regulation. J. Financ. 48 (4), 1523–1542.

Kim, D., Santomero, A.M., 1988. Risk in banking and capital regulation. J. Financ. 43, 1219–1233.

Klein, M., 1971. A theory of the banking firm. J. Money Credit Bank. 3, 205–218.

Leonard, D., Nishimura, K., 1999. Nonlinear dynamics in the Cournot model without full information. Ann. Oper. Res. 89, 165–173.

Matutes, C., Vives, X., 2000. Imperfect competition, risk taking and regulation in banking. Eur. Econ. Rev. 44 (1), 1–34.

Medio, A., 1992. Chaotic Dynamics. Theory and applications to economics.Cambridge University Press, Cambridge (UK).

Monti, M., 1972. Deposit, credit, and interest rate determination under alternative bank objectives. In: Szego, G.P., Shell, K. (Eds.), Mathematical Methods in Investment and Finance, North-Holland, Amsterdam.

Nieto Parra, S., 2005. Macroeconomic implications of the new banking capital regulation in emerging markets: a duopoly model adapted to risk-averse banks. Rev. Econ. Rosario 8 (1), 59–83 (Bogotá (Colombia)).

Rochet, J.C., 1992. Capital requirements and the behaviour of commercial banks. Eur. Econ. Rev. 36, 1137–1178.

Santos, J., 2000. Bank capital regulation in contemporary banking theory: a review of the literature. BIS Working Paper 90.

Tramontana, F., 2010. Heterogeneous duopoly with isoelastic demand function. Econ. Model. 27, 350–357.

Vives, X., 2006. Banking and regulation in emerging markets. World Bank Res. Obs. 21 (2), 179–206.

Vives, X., 2010a. Competition and stability in banking. J. Econ. Chil. (The Chilean Economy), 13(2). Central Bank of Chile, pp. 85–112.

Vives, X., 2010b. Competition and stability in banking. CEPR, Policy, Insight n.50.

Zhang, J., Da, Q., Wang, Y., 2007. Analysis of nonlinear duopoly game with heterogeneous players. Econ. Model. 24, 138–148.