Důkaz řádu pro modif. N. metodu

$$f(x) = f(\xi) + (x-\xi)f'(\xi) + \frac{1}{2}(x-\xi)^2 f''(\xi) + \dots + \frac{1}{(M-1)!}(x-\xi)^{M-1} f^{(M-1)}(\xi) + \frac{1}{M!}(x-\xi)^M f^{(M)}(\xi+\alpha(x-\xi)),\ \alpha \in (0,1) \Rightarrow f(x) = \frac{1}{M!}(x-\xi)^M f^{(M)}(\xi+\alpha(x-\xi))$$

$$f'(x) = f'(\xi) + (x-\xi)f''(\xi) + \dots + \frac{1}{(M-1)!}(x-\xi)^{M-1} f^{(M)}(\xi+\beta(x-\xi)),\ \beta \in (0,1)$$

$$f'(x) = \frac{1}{(M-1)!}(x-\xi)^{M-1} f^{(M)}(\xi+\beta(x-\xi))$$

$$x_{k+1} = x_k - M\frac{\frac{1}{M!}(x_k-\xi)^M f^{(M)}(\xi+\alpha(x_k-\xi))}{\frac{1}{(M-1)!}(x_k-\xi)^{M-1} f^{(M)}(\xi+\beta(x_k-\xi))} = x_k - \frac{f^{(M)}(\xi+\alpha(x_k-\xi))\cdot(x_k-\xi)}{f^{(M)}(\xi+\beta(x_k-\xi))}$$

$-\xi$

$$\frac{x_{k+1}-\xi}{(x_k-\xi)^2} = \frac{1}{x_k-\xi} - \frac{f^{(M)}(\xi+\alpha(x_k-\xi))}{(x_k-\xi)f^{(M)}(\xi+\beta(x_k-\xi))} = \frac{f^{(M)}(\xi+\beta(x_k-\xi)) - f^{(M)}(\xi+\alpha(x_k-\xi))}{(x_k-\xi)f^{(M)}(\xi+\beta(x_k-\xi))}$$

bře 22-7:58

$$\lim_{k\to\infty}\frac{x_{k+1}-\xi}{(x_k-\xi)^2} = \lim_{x\to\xi}\frac{f^{(M)}(\xi+\beta(x-\xi)) - f^{(M)}(\xi+\alpha(x-\xi))}{(x-\xi)f^{(M)}(\xi+\beta(x-\xi))} =$$

$$= \frac{f^{(M+1)}(\xi+\beta(x-\xi))\cdot\beta - f^{(M+1)}(\xi+\alpha(x-\xi))\cdot\alpha}{f^{(M)}(\xi+\beta(x-\xi)) + (x-\xi)f^{(M+1)}(\xi+\beta(x-\xi))\cdot\beta} = \frac{f^{(M+1)}(\xi)(\beta-\alpha)}{f^{(M)}(\xi)} < \infty$$

bře 22-8:17

ξ - pevný bod $\varphi \Rightarrow \xi$ je pevný bod g :

$$\varphi(\xi) = \xi \Rightarrow \frac{(g(\xi)-\xi)^2}{g(g(\xi)) - 2g(\xi) + \xi} = 0 \Rightarrow g(\xi) = \xi$$

ξ - pevný bod $g \Rightarrow \xi$ - pevný bod φ

$$\varphi(\xi) = \lim_{x\to\xi}\left(x - \frac{(g(x)-x)^2}{g(g(x)) - 2g(x) + x}\right) = \lim_{x\to\xi}\frac{x\,g(g(x)) - g^2(x)}{g(g(x)) - 2g(x) + x} =$$

$$= \lim_{x\to\xi}\frac{g(g(x)) + x\,g'(g(x))g'(x) - 2g(x)\cdot g'(x)}{g'(g(x))\cdot g'(x) - 2g'(x) + 1} = \frac{\xi + \xi g'(\xi)^2 - 2\xi g'(\xi)}{g'(\xi)^2 - 2g'(\xi) + 1} =$$

$$= \frac{\xi(1 + g'(\xi)^2 - 2g'(\xi))}{(g'(\xi)-1)^2} = \xi$$

bře 22-8:32

Hranice kořenů - Dk: nechť $x_k : |x_k| > 1$

$$|P(x)| = |a_m x^m + a_{m-1}x^{m-1} + \dots + a_1 x + a_0| \geq |a_m x^m| - |a_{m-1}x^{m-1} + \dots + a_0| \geq$$

$$\geq |a_m x^m| - \left(|a_{m-1}x^{m-1}| + |a_{m-2}x^{m-2}| + \dots + |a_1 x| + |a_0|\right) \geq |a_m x^m| - A\left(|x^{m-1}| + \dots + |x| + 1\right) =$$

$$= |a_m x^m| - A\frac{|x|^m - 1}{|x|-1} = |a_m||x|^m - A\frac{|x|^m-1}{|x|-1} > |a_m||x|^m - A\frac{|x|^m}{|x|-1} = |x|^m\left(|a_m| - \frac{A}{|x|-1}\right)$$

pro $\left(|a_m| - \frac{A}{|x|-1}\right) > 0$ je i $|P(x)| > 0$

$$|a_m| - \frac{A}{|x|-1} > 0 \Rightarrow |a_m| > \frac{A}{|x|-1} \Rightarrow \frac{|x|-1}{A} > \frac{1}{|a_m|} \Rightarrow |x| > 1 + \frac{A}{|a_m|}$$

$\Leftarrow \quad \Leftarrow \quad \Leftarrow$

pro $x: |x| > 1 + \frac{A}{|a_m|}$ je $P(x) \neq 0 \Rightarrow$ kořeny jsou v abs. h. menší než $1 + \frac{A}{|a_m|}$

Druhá nerovnost: $y = \frac{1}{x}$ $\quad P(y) = a_0 y^m + \dots + a_{m-1}y + a_m = 0 \Rightarrow A \longleftrightarrow B$

$a_m \longleftrightarrow a_0$

bře 22-9:14