Homework 2

March 1, 2013

RULES: To pass the homework you either need to make 2 exercises and hand me the result by the next friday or you have to make all three exercises – in that case there is no deadline.

1. Finish the proof of the 5–lemma i.e prove that in the diagram of Abelian groups

$$\begin{tikzcd}
{} \arrow[r] & A \arrow[r] \arrow[d, "a"] & B \arrow[r] \arrow[d, "b"] & C \arrow[r] \arrow[d, "c"] & D \arrow[r] \arrow[d, "d"] & E \arrow[r] \arrow[d, "e"] & {} \\
{} \arrow[r] & A' \arrow[r] & B' \arrow[r] & C' \arrow[r] & D' \arrow[r] & E' \arrow[r] & {}
\end{tikzcd}$$

   where alll squares commute, the rows are exact sequences and a, b, d, e are isos, the morphism c is mono.

2. Let us have the following short exact sequence

$$0 \longrightarrow A \xrightarrow{i} B \xrightarrow{j} C \longrightarrow 0$$

   then the following are equivalent:

   (a) There exists $p : B \to A$ such that $p \circ i = id_A$.

   (b) There exists $q : C \to B$ such that $j \circ q = id_C$.

   (c) There are p, q as above such that $(i \circ p + q \circ j) = id_B$.

   We have shown that the first two conditions are equivalent. Finish the proof.

3. Decide and prove whether the hawaian earring space

$$H = \bigcup_{n \in \mathbb{N}} \{x \in \mathbb{R}^2 | \, \|x - (\tfrac{1}{n}, 0)\| = \frac{1}{n}\}$$

   is a CW complex