HOMEWORK 1 – 2019

Solve exercises 1 to 3 or 3 to 5.

Exercise 1. Show that $SS^k = S^{k+1}$. *Hint:* Consider the map $f\colon S^k \times I \to S^{k+1}$, where

$$f(x,t) = ((\sqrt{1-(2t-1)^2})x, 2t-1)$$

Exercise 2. Suppose that the inclusion $A \hookrightarrow X$ is a cofibration and consider a map $f : A \to Y$. Prove that the inclusion $Y \hookrightarrow X \cup_f Y$ is also a cofibratin.

Exercise 3. Prove: If X is a Hausdorff space, then its diagonal

$$\Delta = \{(x,x) \in X \times X\}$$

is a closed subspace of $X \times X$.

Exercise 4. Let X be a Hausdorff space and $A \subseteq X$ be its retract. Prove that A is closed. *Hint:* Use the previous exercise and the map $X \to X \times X\colon x \mapsto (x, r(x))$ where $r\colon X \to A$ is a retraction.

Exercise 5. The consequence of the previous exercise is: If X is Hausdorff and $A \hookrightarrow X$ is a cofibration, then A is closed. *Hint:* $X \times I$ is Hausdorff.