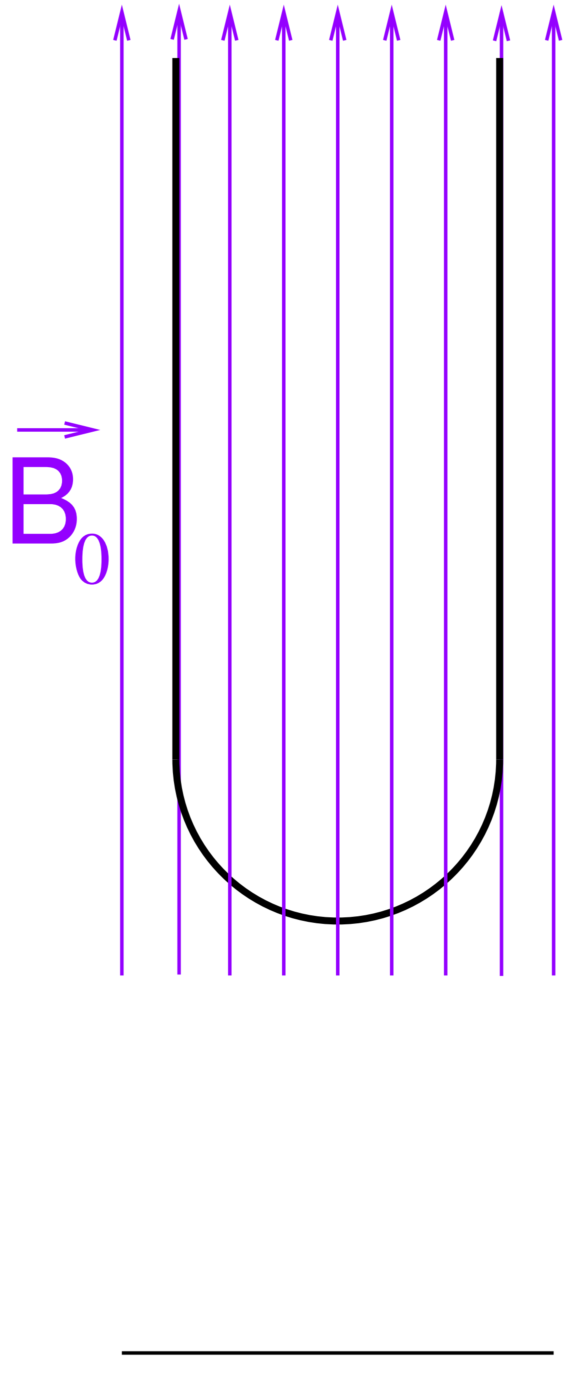
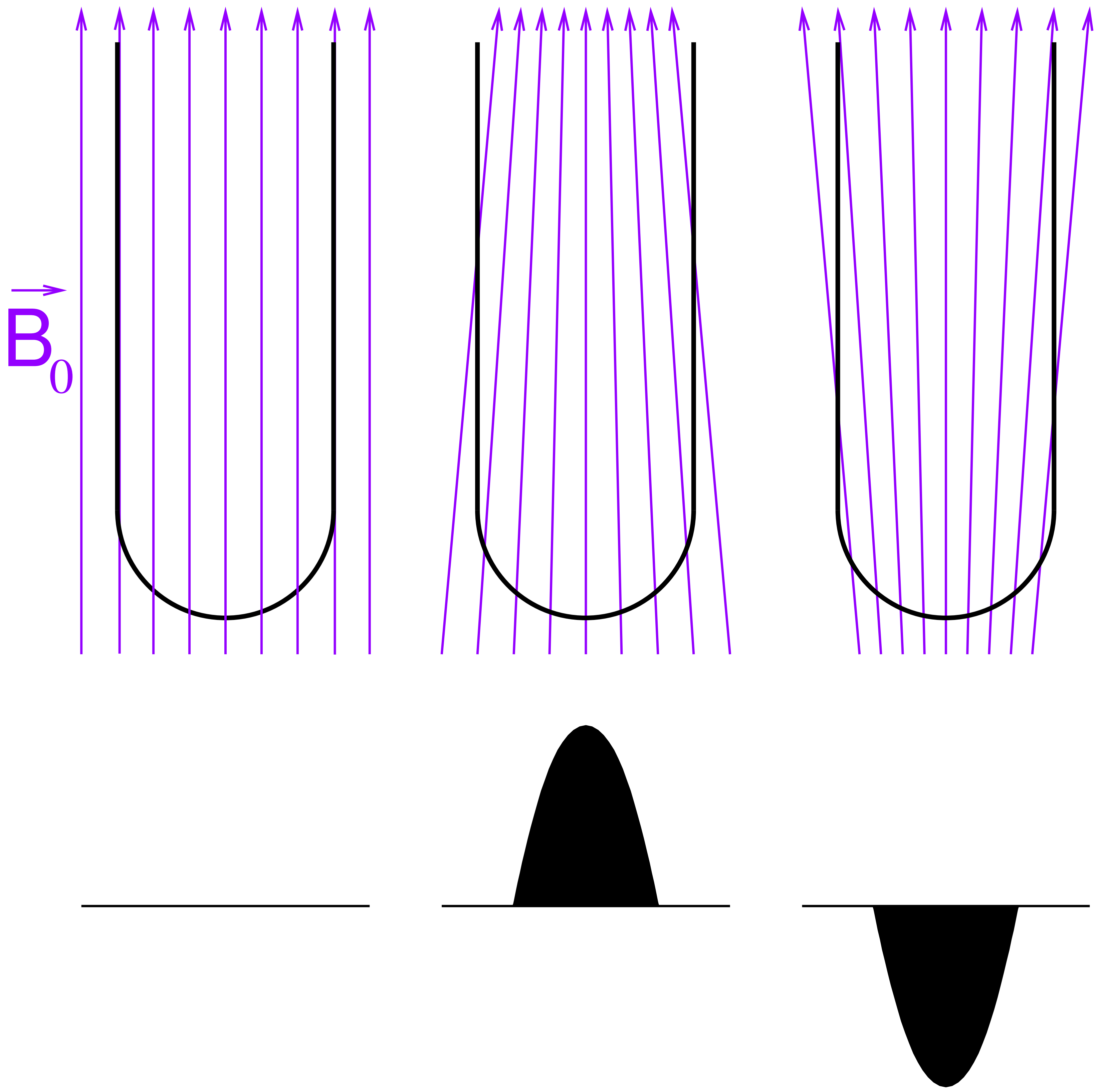
Lecture 13: Field gradients

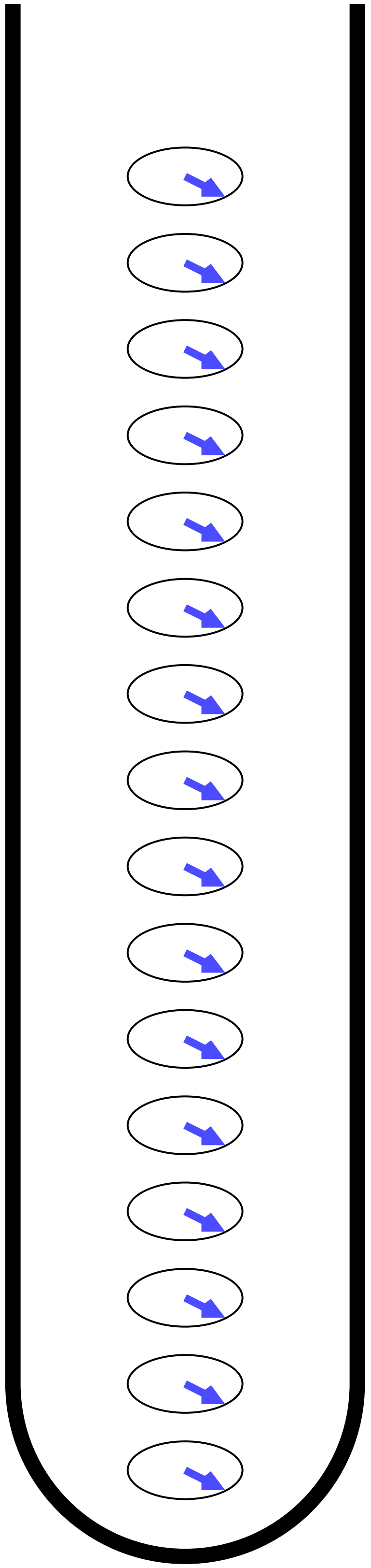
Homogeneous field



Pulsed field gradients (G_z)

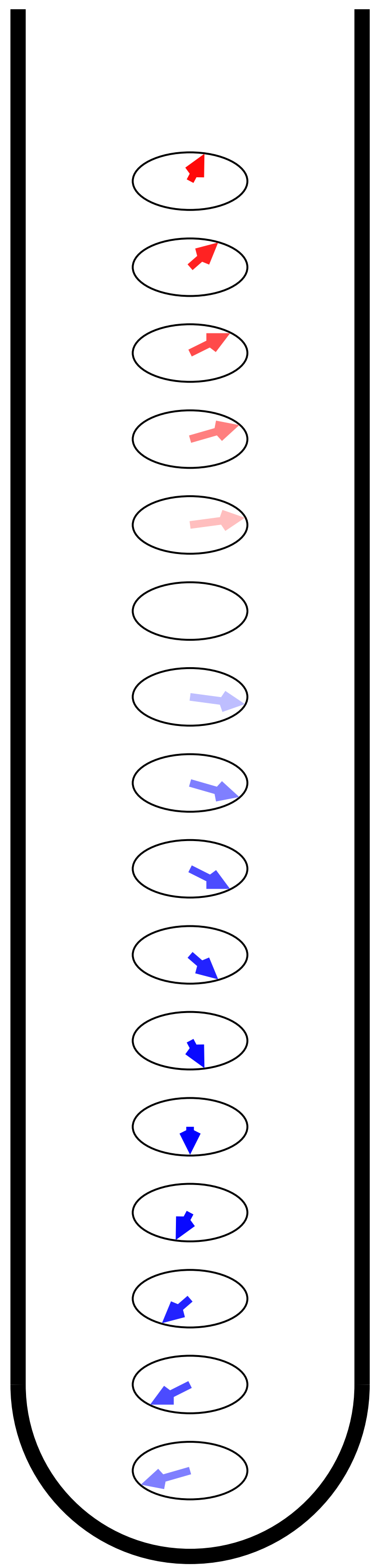


Gradients dephase transverse magnetization



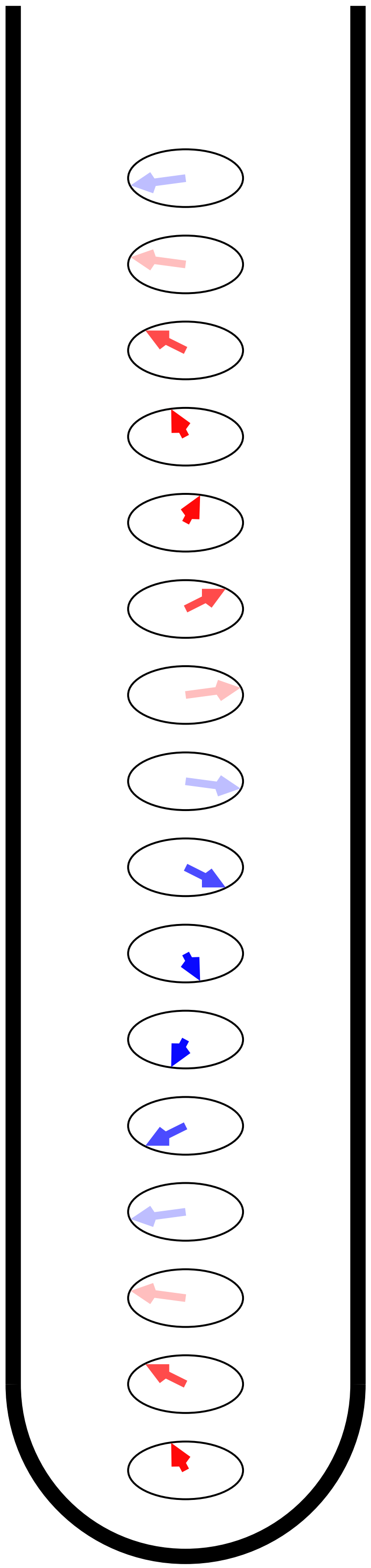
$G_z = 0$ units

Gradients dephase transverse magnetization



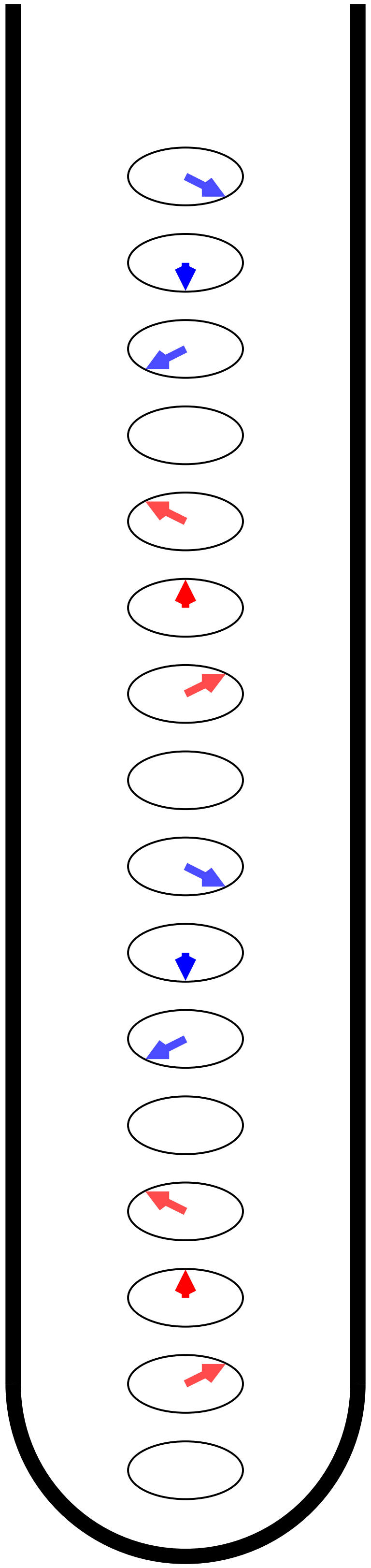
$G_z = 1$ units

Gradients dephase transverse magnetization



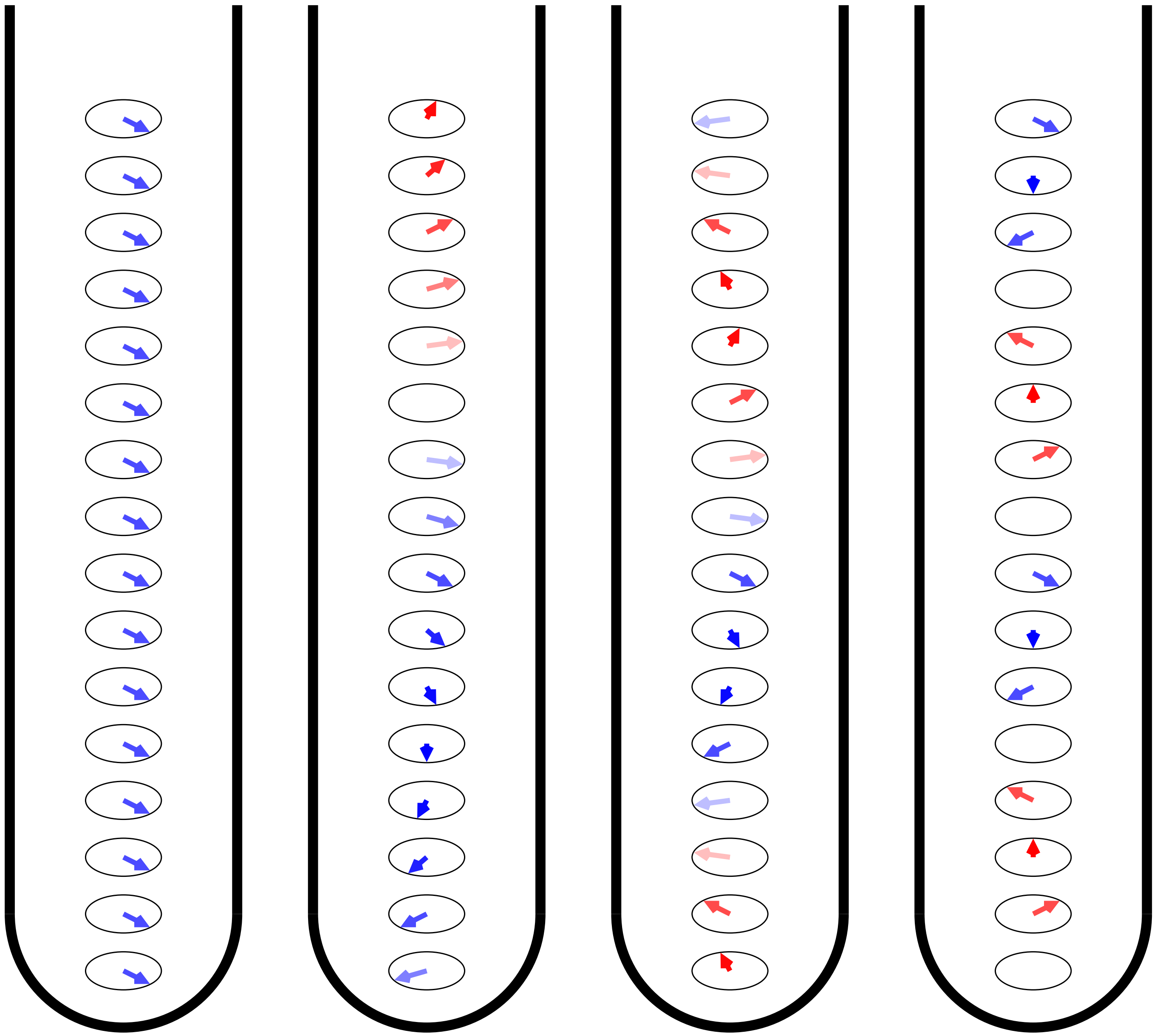
$G_z = 2$ units

Gradients dephase transverse magnetization



$G_z = 4$ units

Gradient-induced dependence of phase



$$G_z = \Delta B_0/\Delta z \quad \Rightarrow \quad \Omega'(z) = \Omega - \gamma G_z z$$

$$-\mathscr{I}_y \rightarrow -\mathscr{I}_y \cos(\Omega' t) + \mathscr{I}_x \sin(\Omega' t)$$

$$= -\mathscr{I}_y \cos(\Omega - \gamma G_z z t) + \mathscr{I}_x \sin(\Omega - \gamma G_z z t)$$

Gradient-induced dependence of phase

$$G_z = \Delta B_0/\Delta z \quad \Rightarrow \quad \Omega'(z) = \Omega - \gamma G_z z$$

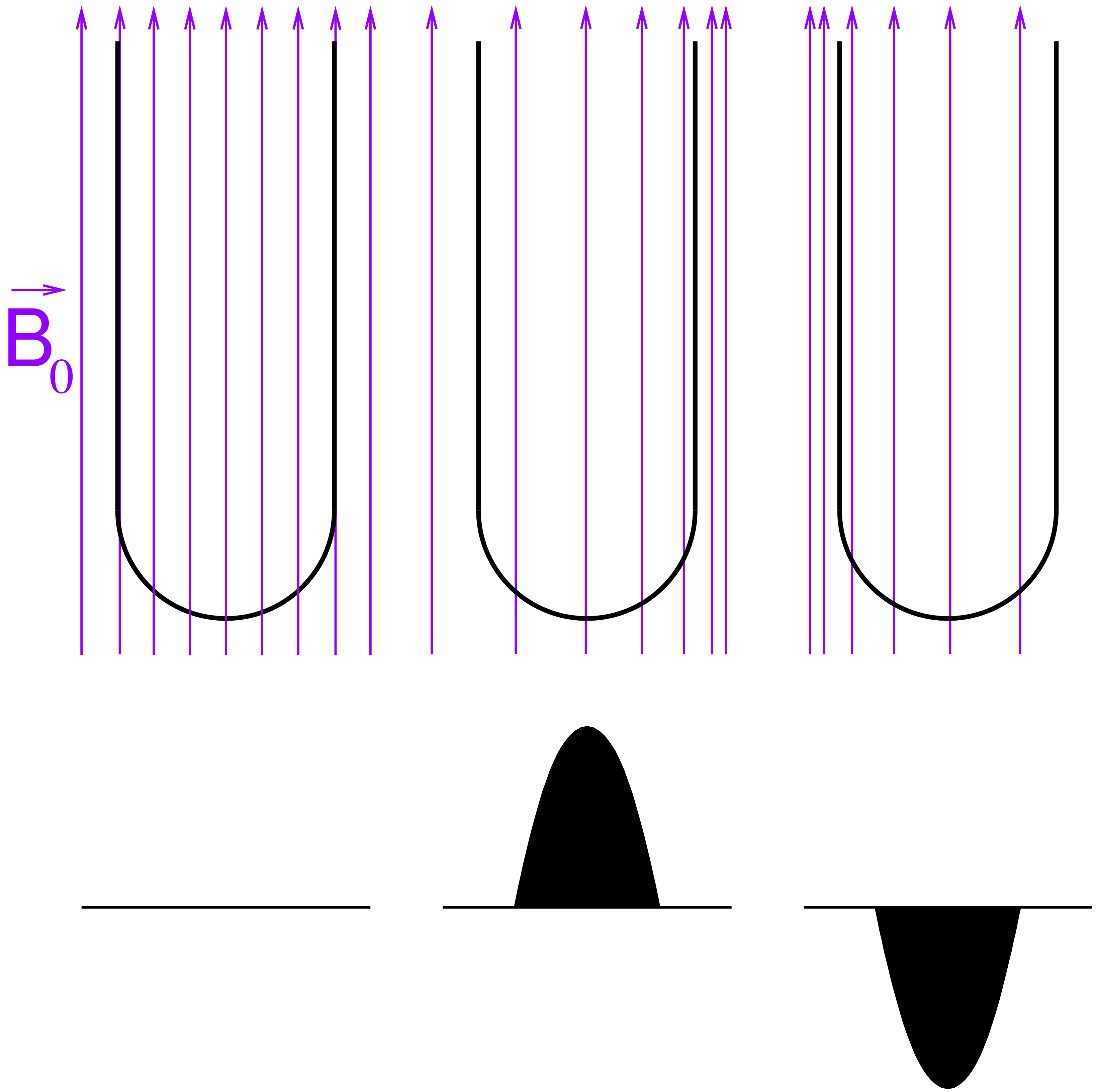
$$\begin{aligned} -\mathscr{I}_y &\to -\mathscr{I}_y \cos(\Omega' t) + \mathscr{I}_x \sin(\Omega' t) \\ &= -\mathscr{I}_y \cos(\Omega - \gamma G_z z t) + \mathscr{I}_x \sin(\Omega - \gamma G_z z t) \end{aligned}$$

$$\begin{aligned} \langle M_+ \rangle &= \mathrm{Tr}\left\{\widehat{\rho}(z,t)\mathscr{I}_+\right\} \\ &= \mathcal{N}\frac{\gamma^2\hbar^2 B_0}{4k_\mathrm{B}T}\mathrm{e}^{\mathrm{i}\frac{\pi}{2}}\mathrm{e}^{\mathrm{i}(\Omega-\gamma G_z z)t} \end{aligned}$$

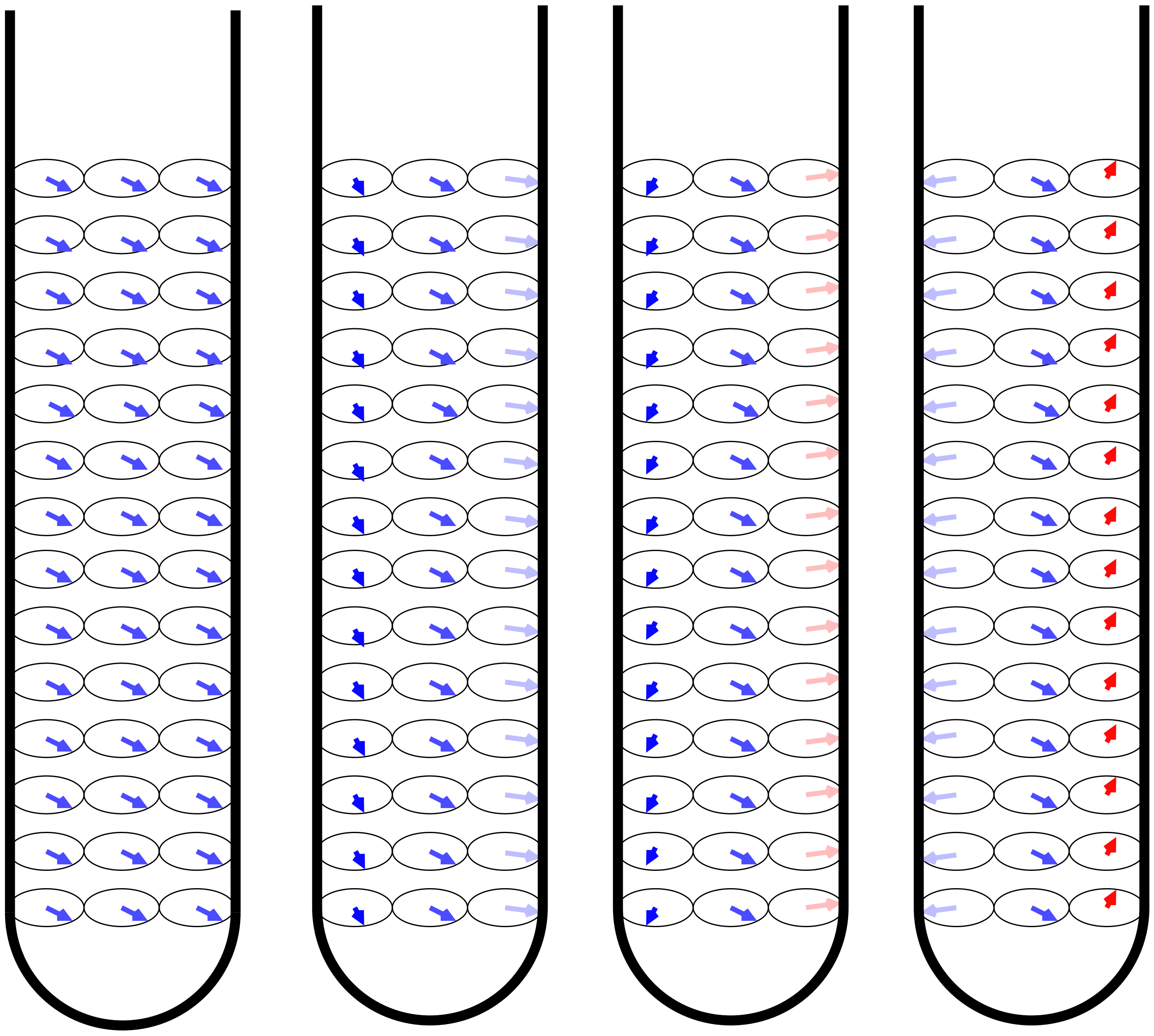
Performing phase correction and including relaxation:

$$\langle M_+ \rangle = \mathcal{N}\frac{\gamma^2\hbar^2 B_0}{4k_\mathrm{B}T}\mathrm{e}^{-R_2 t}\mathrm{e}^{\mathrm{i}(\Omega-\gamma G_z z)t}$$

Pulsed field gradients (G_y)



Gradient-induced dependence of phase

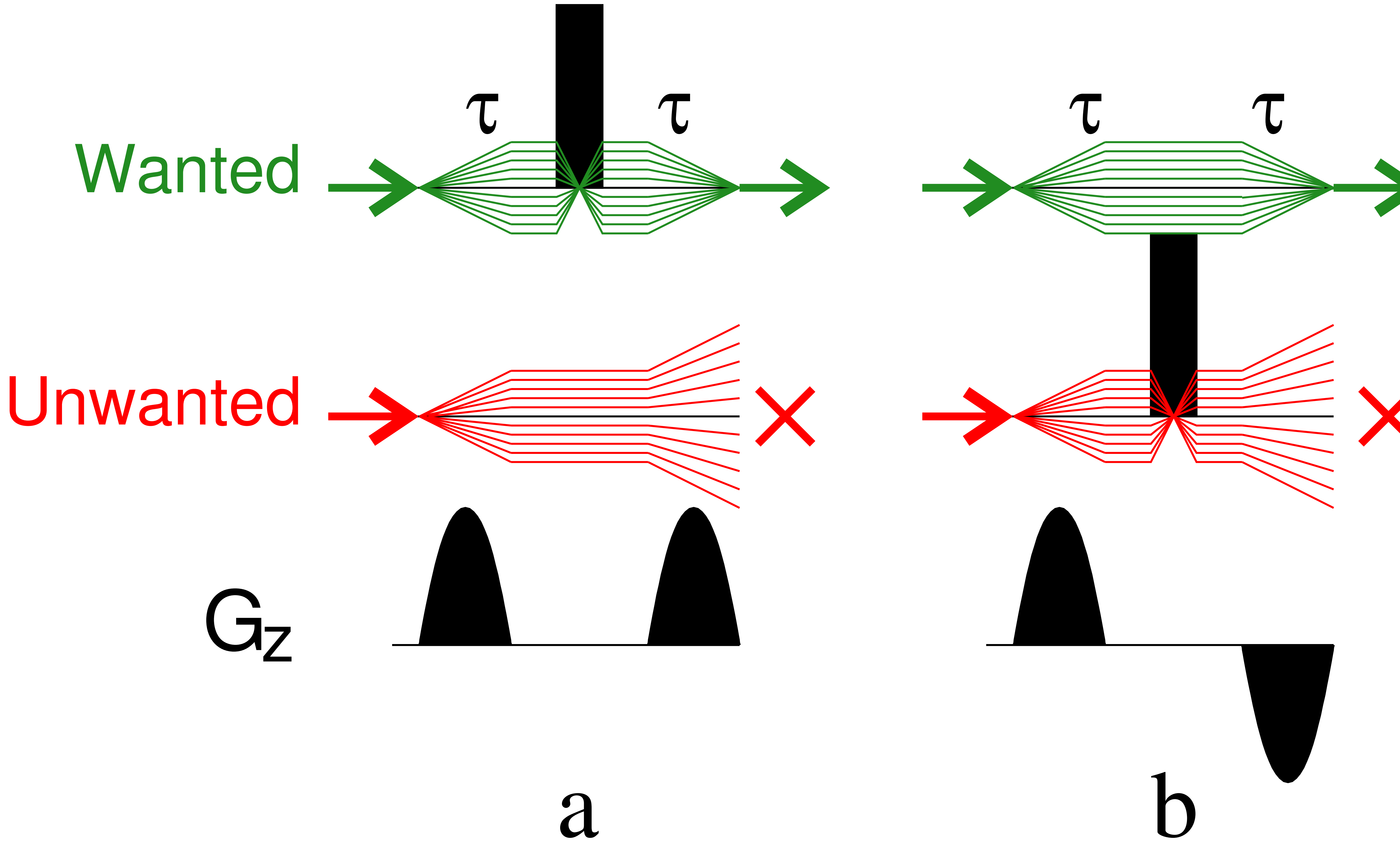


$$G_y = \Delta B_0/\Delta y \quad \Rightarrow \quad \Omega'(y) = \Omega - \gamma G_y y$$

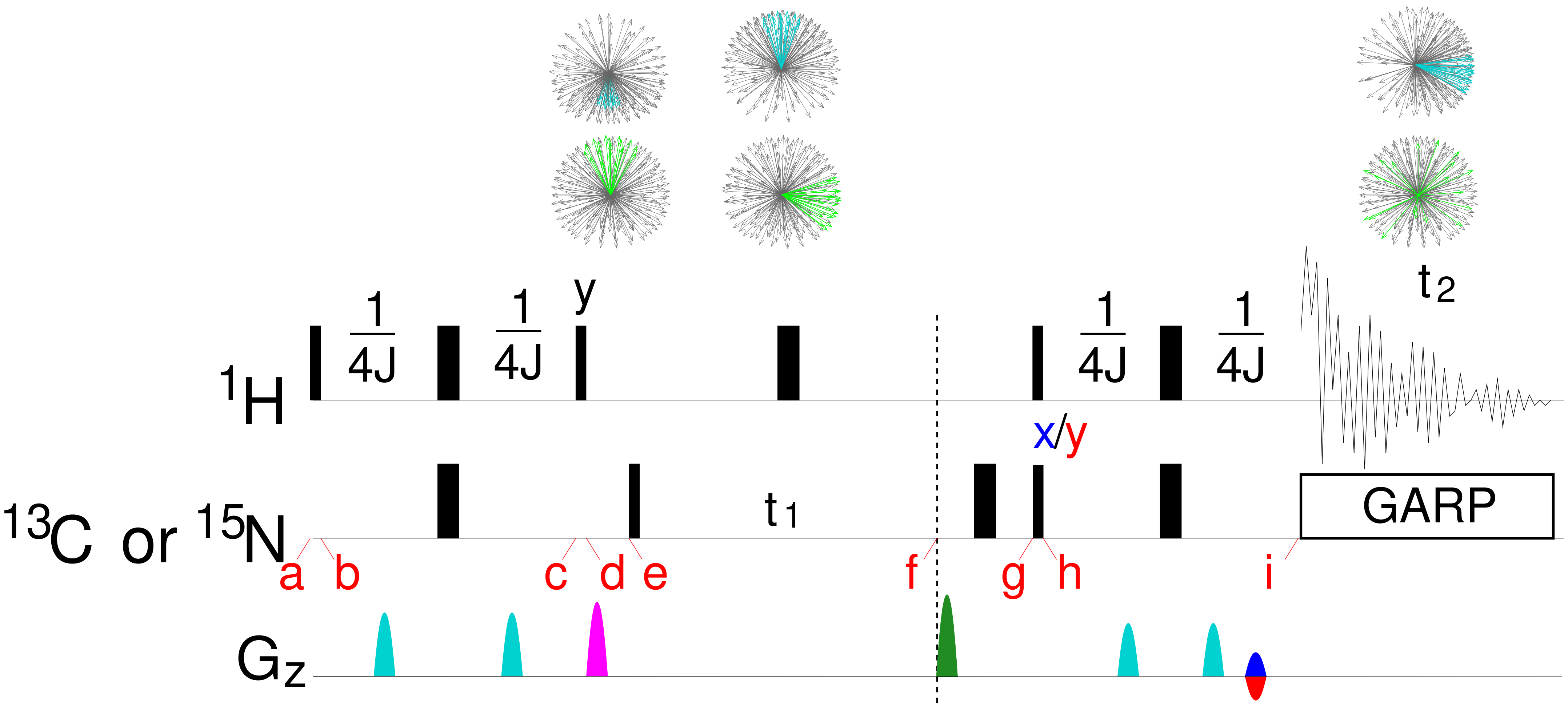
$$-\mathscr{I}_y \rightarrow -\mathscr{I}_y \cos(\Omega' t) + \mathscr{I}_x \sin(\Omega' t)$$

$$= -\mathscr{I}_y \cos(\Omega - \gamma G_y y t) + \mathscr{I}_x \sin(\Omega - \gamma G_y y t)$$

Gradient echoes



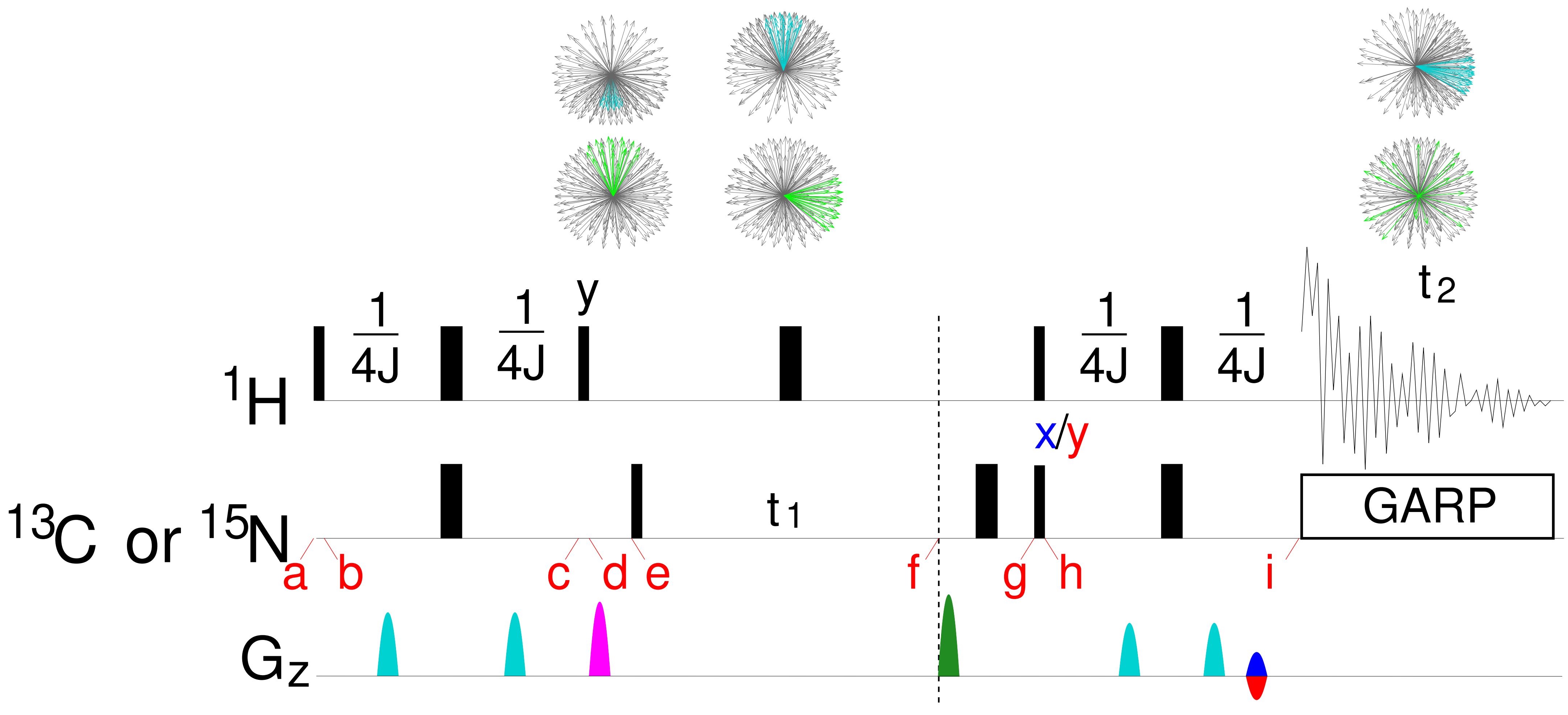
Gradient-enhanced HSQC



G_z Cleaning echo imperfections

G_z Cleaning INEPT imperfections

Gradient-enhanced HSQC



$$x: \quad G_z = \frac{\gamma_2}{\gamma_1} G_z$$

$$y: \quad G_z = -\frac{\gamma_2}{\gamma_1} G_z$$

Use of gradients

- Cleaning, filtering, selection
  similar use as phase cycling

- Translational diffusion measurement

- Imaging

GRADIENTS
AND MAGNETIC RESONANCE IMAGING

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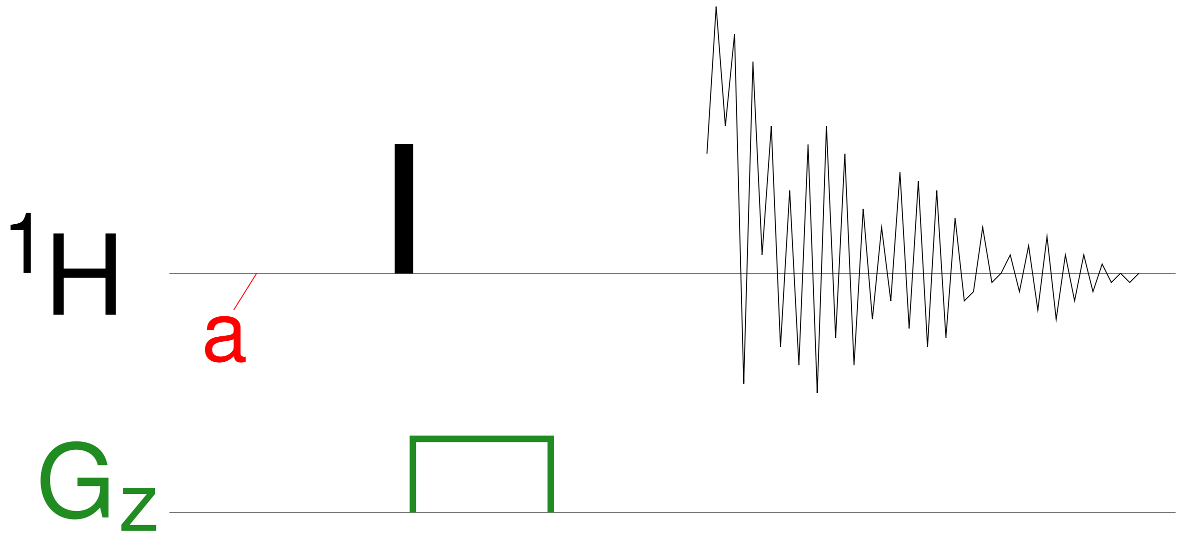
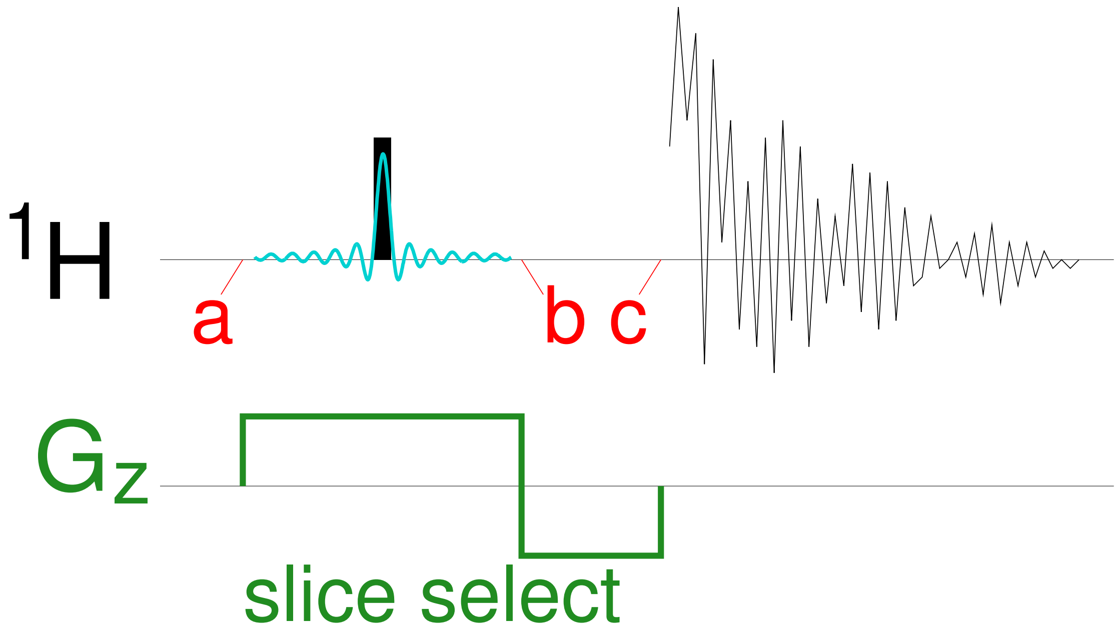
Copenhagen University Hospital Hvidovre

http://eprints.drcmr.dk/37/1/MRI_English_a4.pdf

Slice selection by G_z

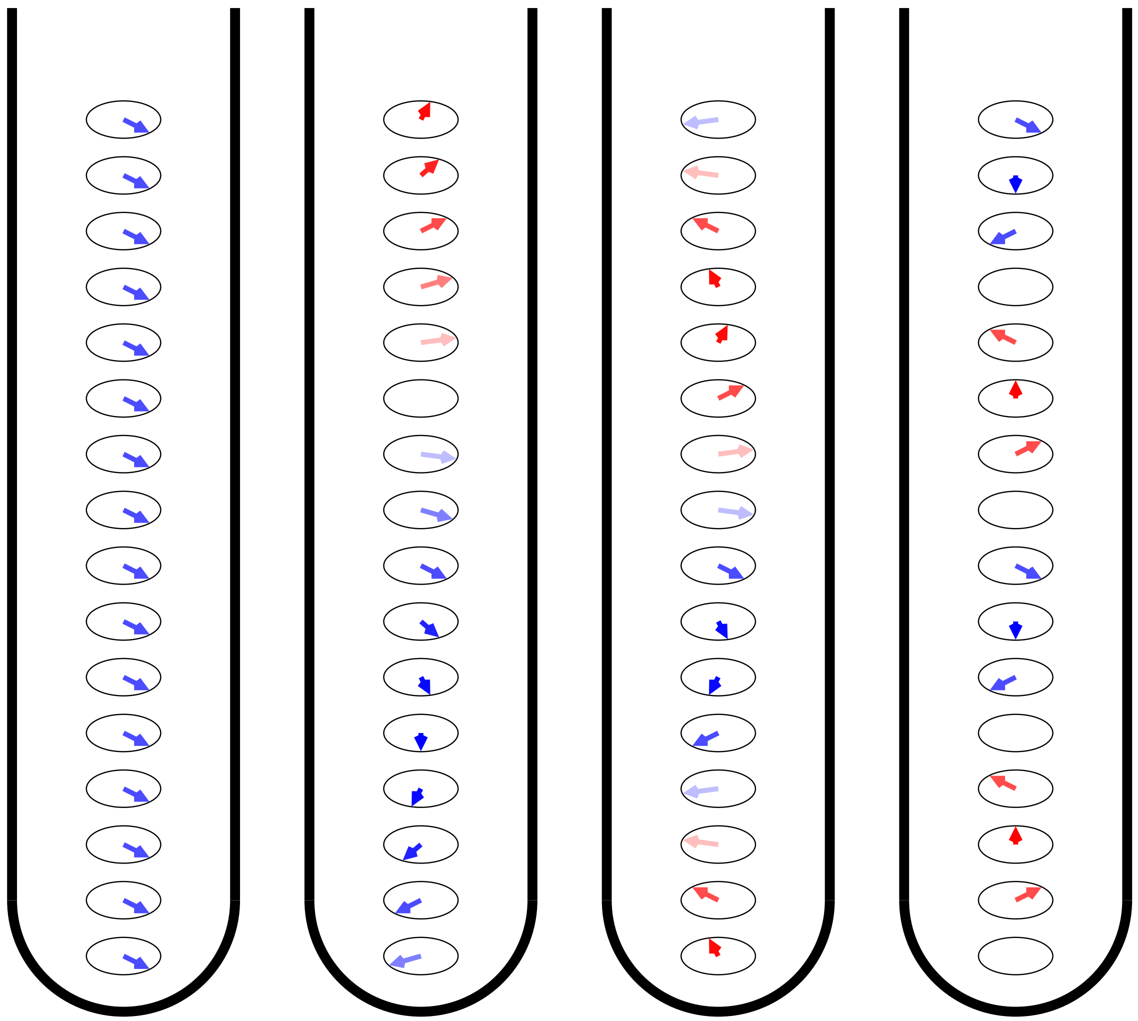


Slice selection by G_z



Selective pulse: **amplitude modulation**

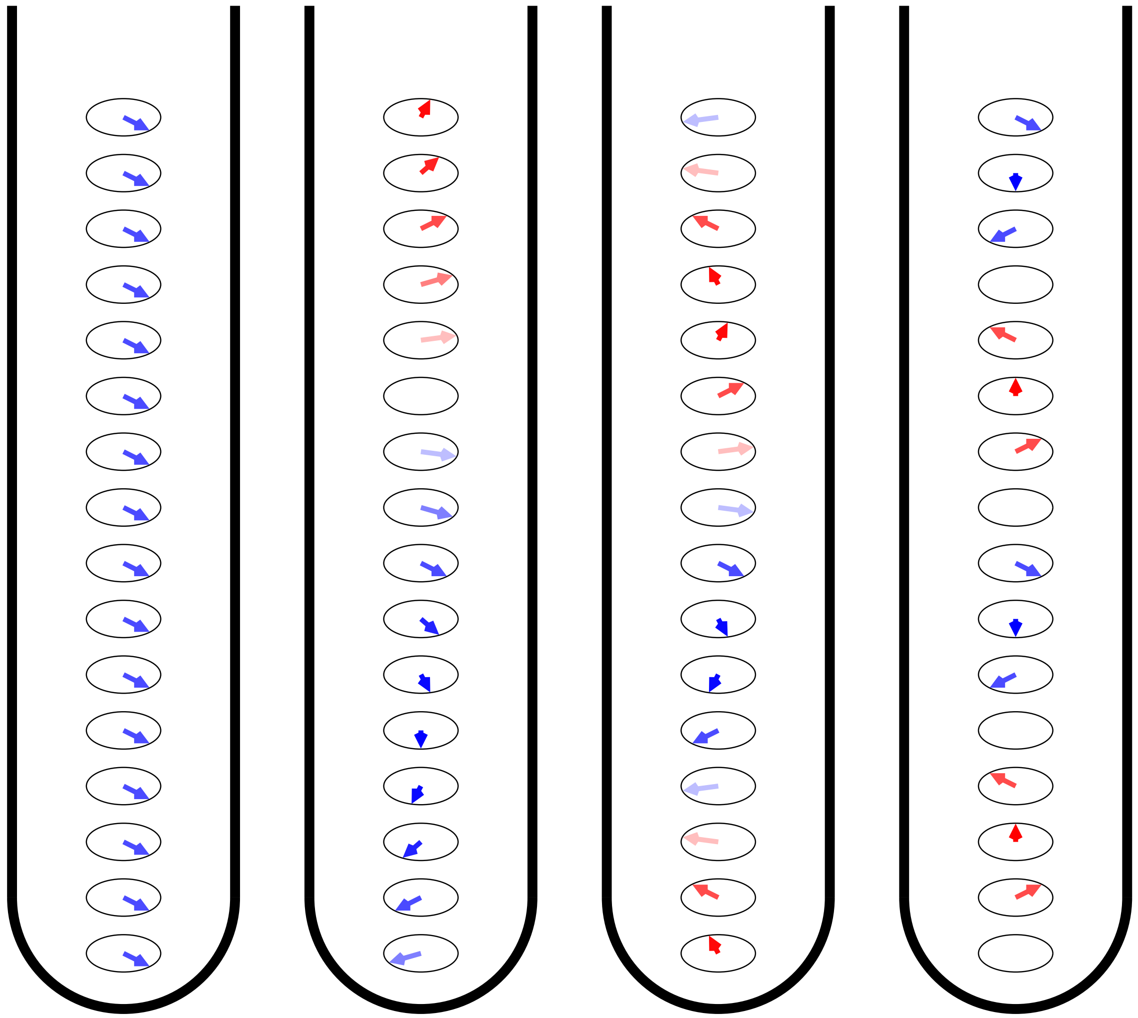
Slice selection



$$\langle M_+ \rangle = \frac{\overbrace{\gamma^2 \hbar^2 B_0}^{K}}{4 k_\mathrm{B} T} \mathrm{e}^{-R_2 t} \left\langle \mathcal{N}(z) \mathrm{e}^{\mathrm{i}\Omega t} \mathrm{e}^{-\mathrm{i}\,\overbrace{\underbrace{\gamma G_z t}_{k_z}\, z}^{\phi}} \right\rangle$$

$$= K \mathrm{e}^{\mathrm{i}\Omega t - R_2 t} \left\langle \mathcal{N}(z) \mathrm{e}^{-\mathrm{i} k_z z} \right\rangle$$

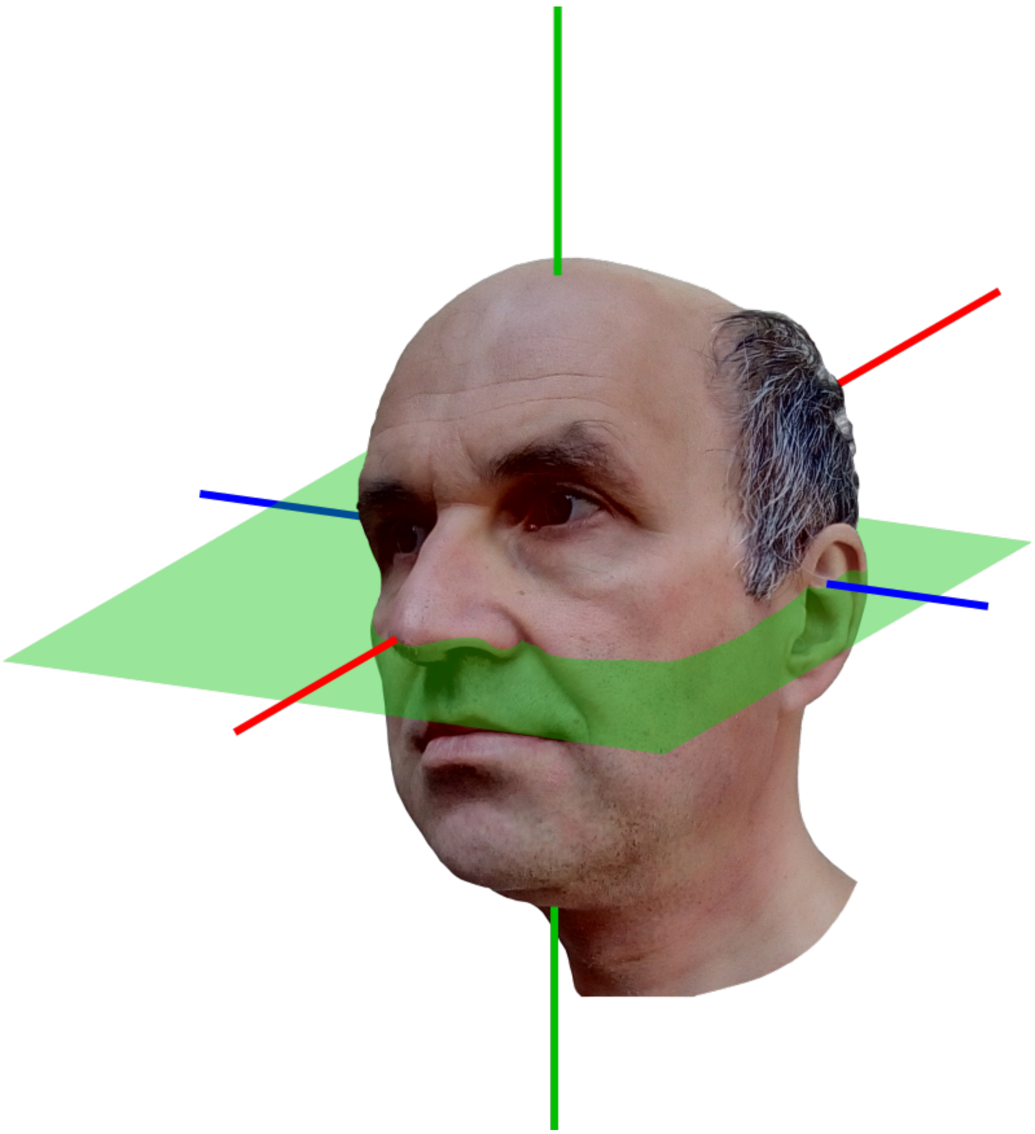
Slice selection



$$\gamma G_z z = \Omega : \qquad \langle M_+ \rangle = K \left\langle \mathrm{e}^{-R_2 t} \right\rangle \mathcal{N}(z) \overbrace{\left\langle \mathrm{e}^{\mathrm{i}(0)t} \right\rangle}^{1}$$

$$\gamma G_z z \neq \Omega : \qquad \langle M_+ \rangle = K \left\langle \mathrm{e}^{-R_2 t} \right\rangle \mathcal{N}(z) \underbrace{\left\langle \mathrm{e}^{\mathrm{i}(\Omega - \gamma G_z z)t} \right\rangle}_{0}$$

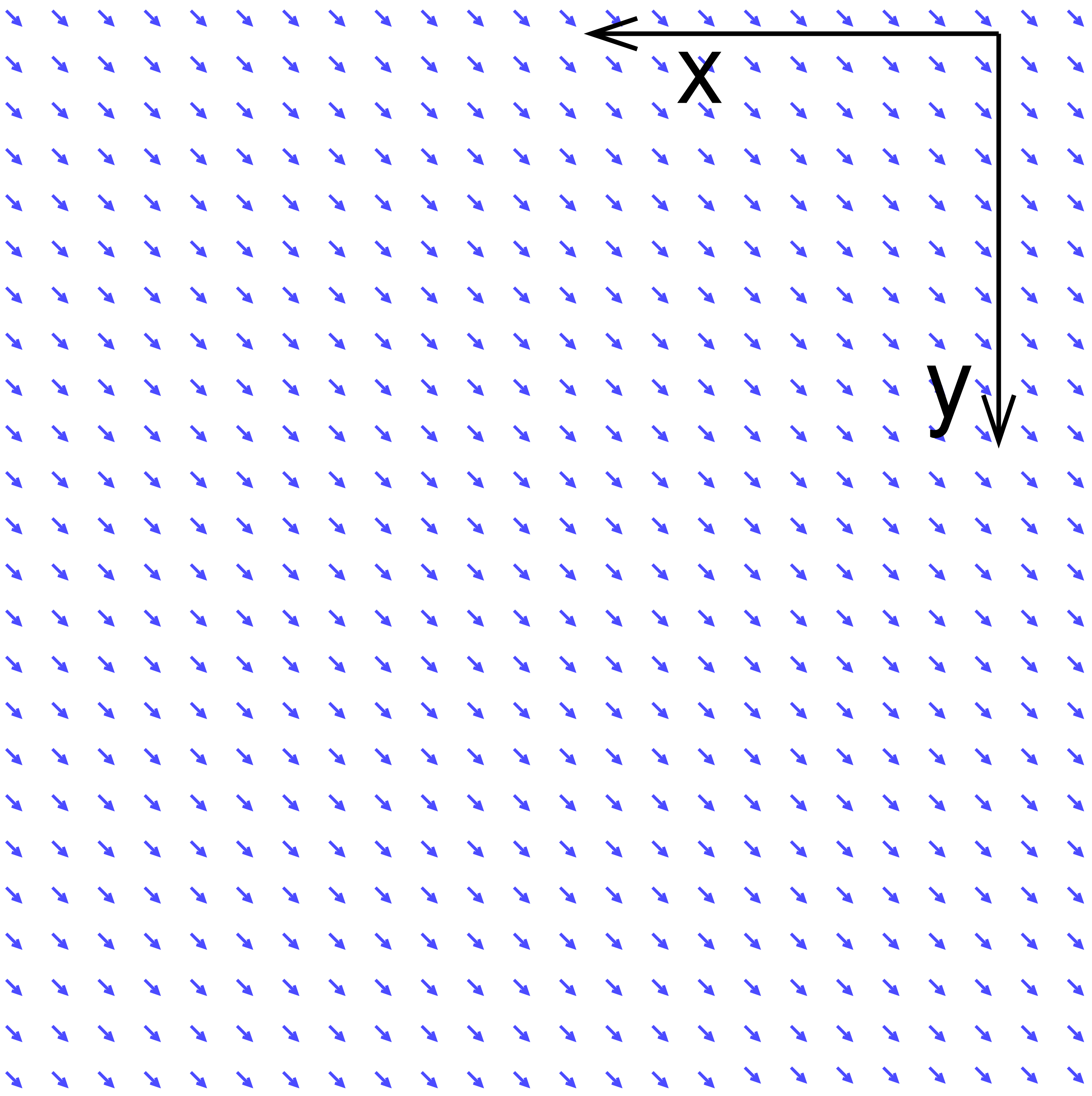
Axial slice selection by G_z



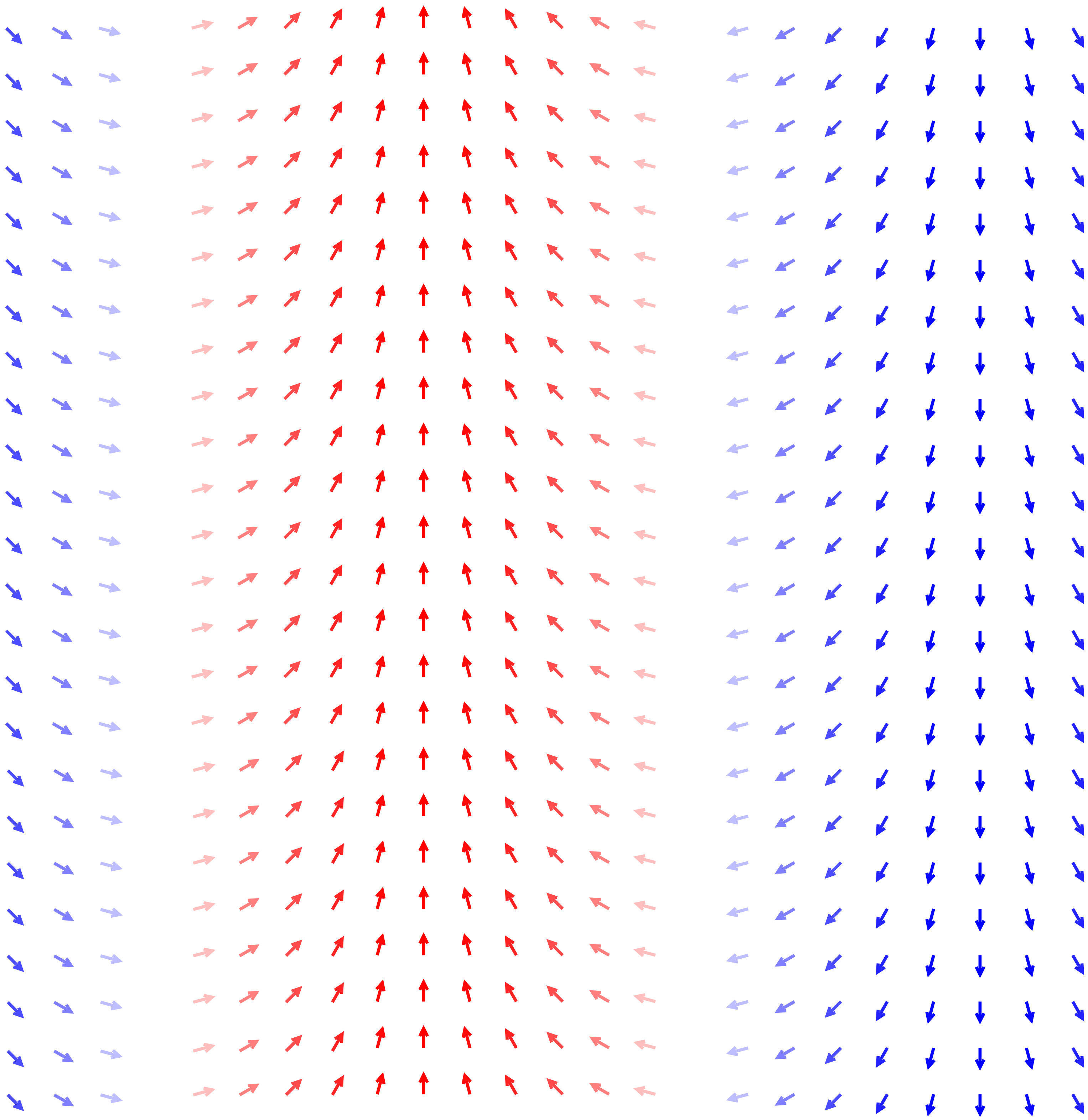
$$\gamma G_z z = \Omega : \qquad \langle M_+ \rangle = K \left\langle \mathrm{e}^{-R_2 t} \right\rangle \mathcal{N}(z) \overbrace{\left\langle \mathrm{e}^{\mathrm{i}(0)t} \right\rangle}^{1}$$

$$\gamma G_z z \neq \Omega : \qquad \langle M_+ \rangle = K \left\langle \mathrm{e}^{-R_2 t} \right\rangle \mathcal{N}(z) \underbrace{\left\langle \mathrm{e}^{\mathrm{i}(\Omega - \gamma G_z z)t} \right\rangle}_{0}$$

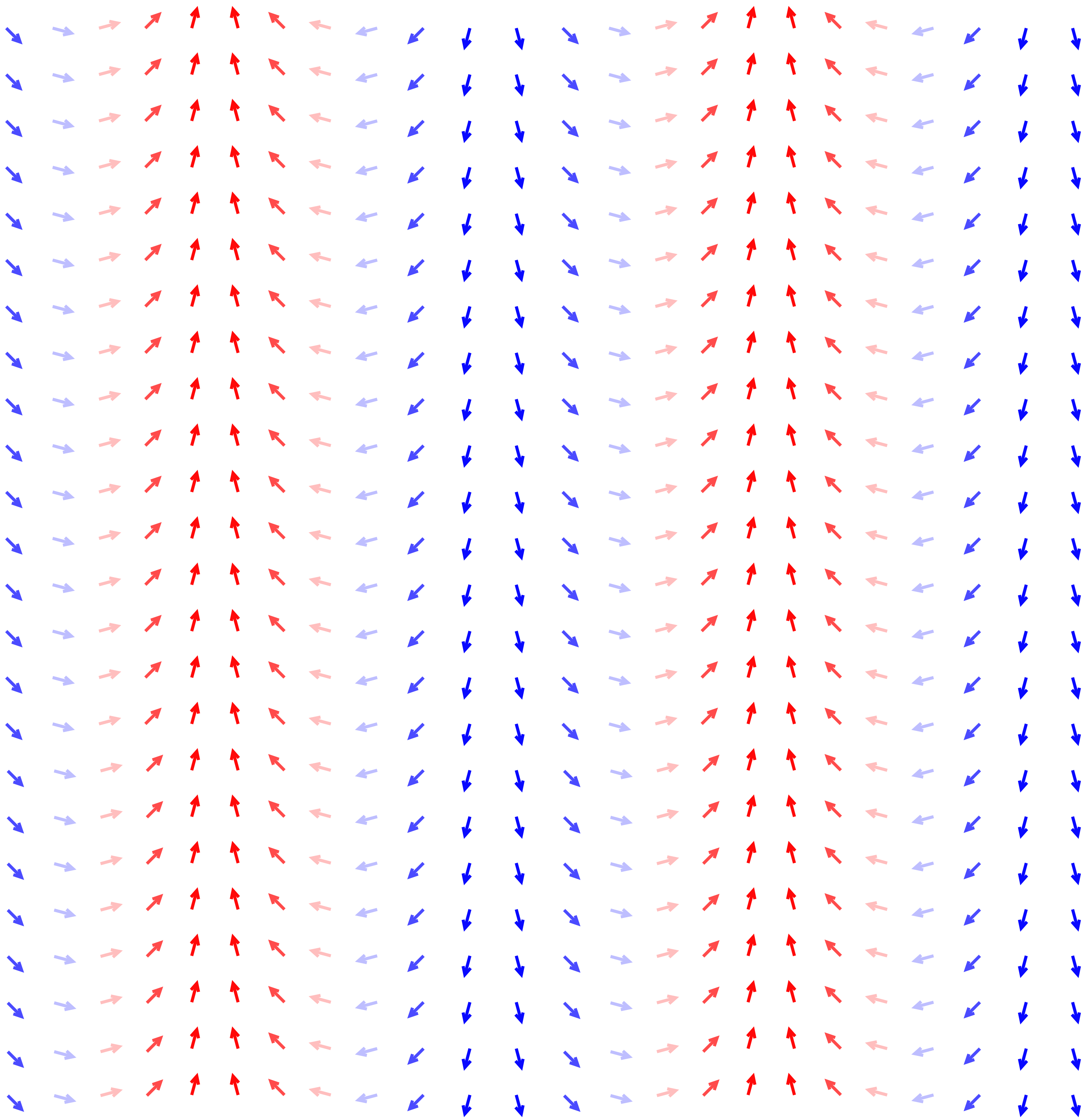
Magnetization in the slice



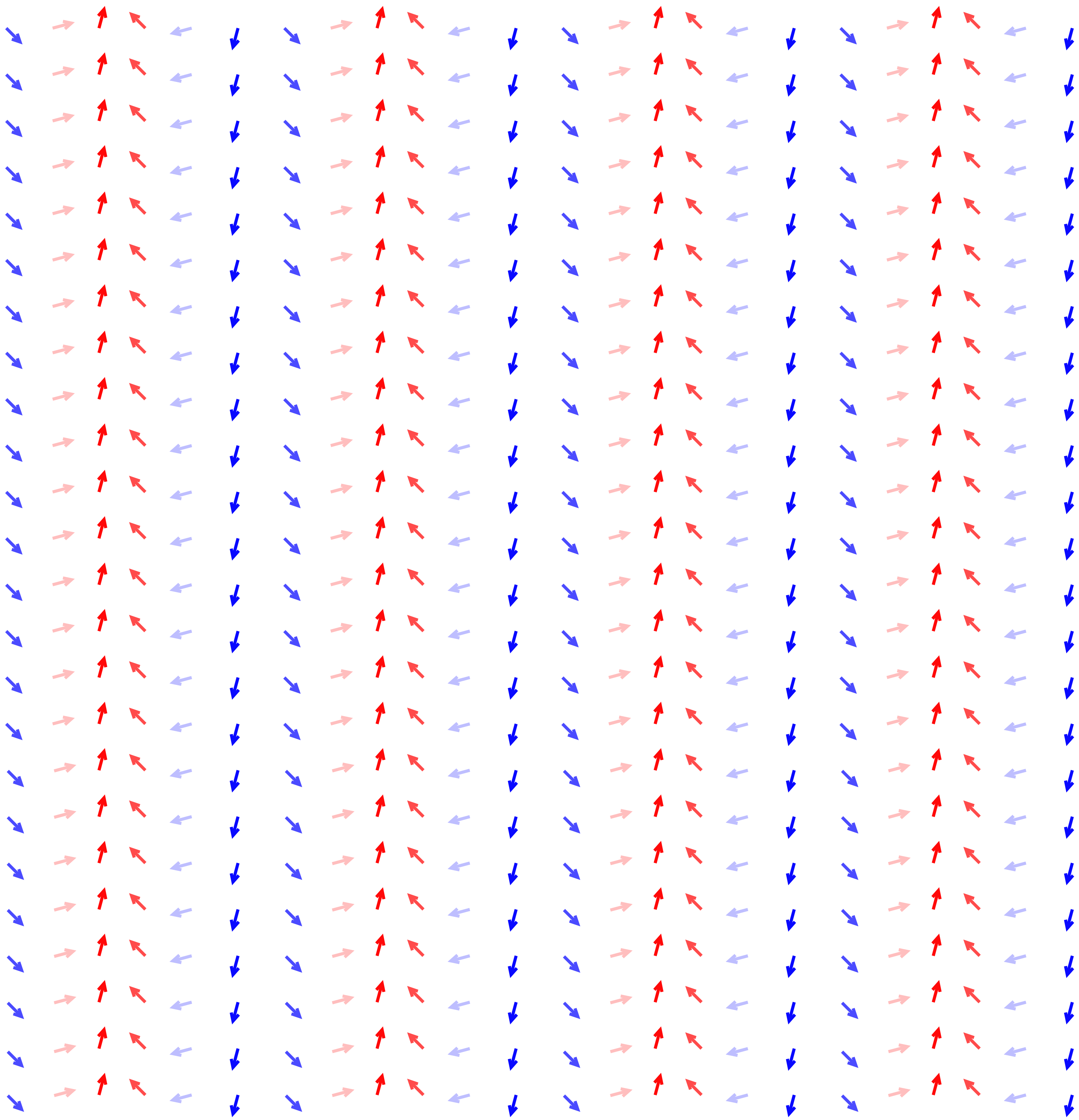
Magnetization in the slice with gradient G_x



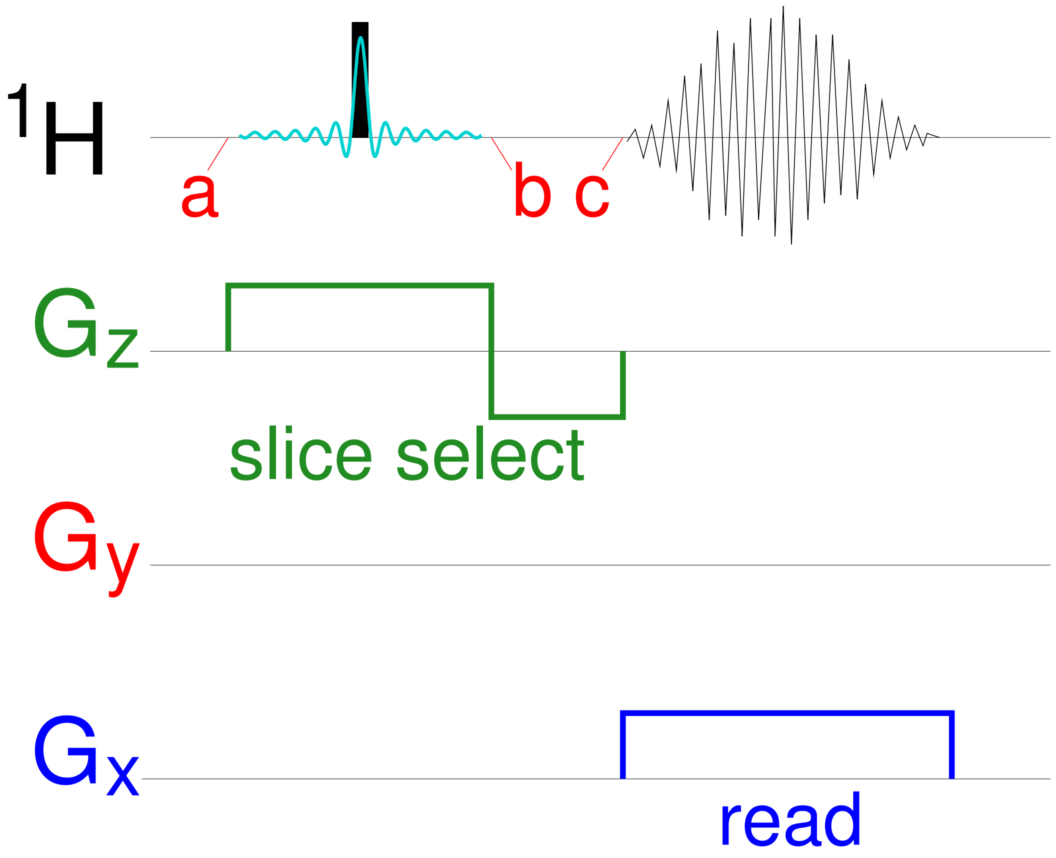
Magnetization in the slice with gradient G_x



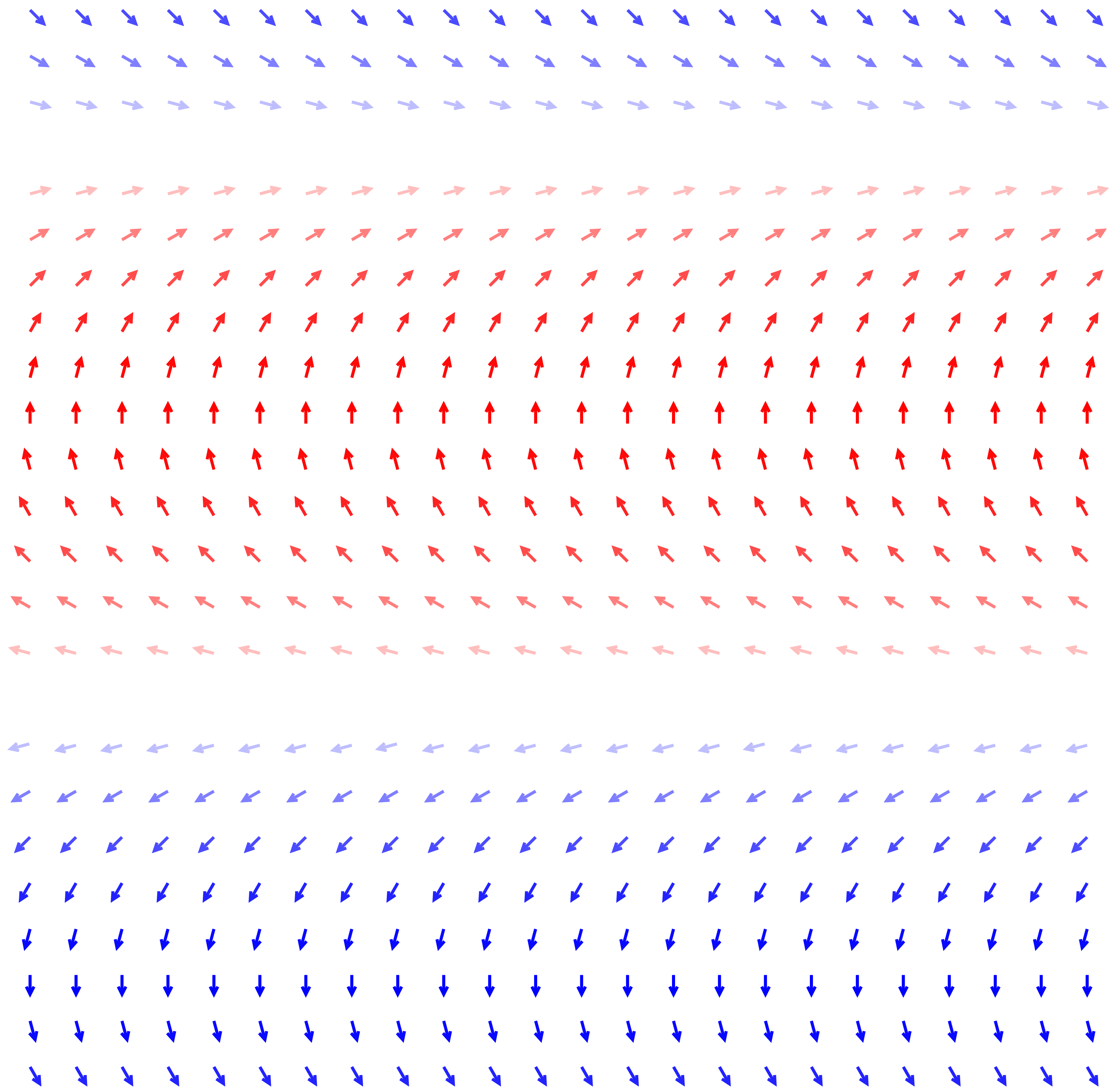
Magnetization in the slice with gradient G_x



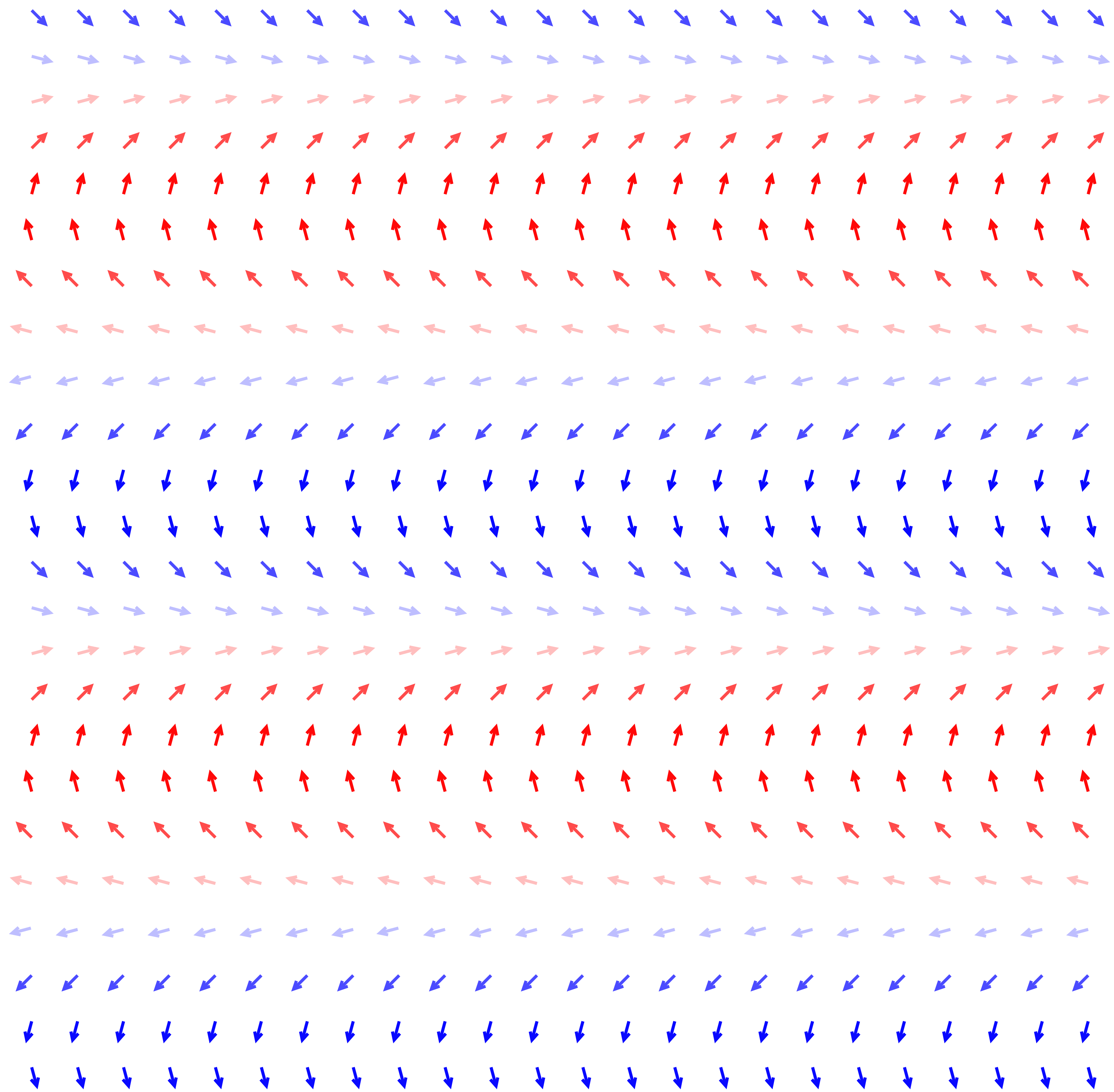
Magnetization in the slice with gradient G_x



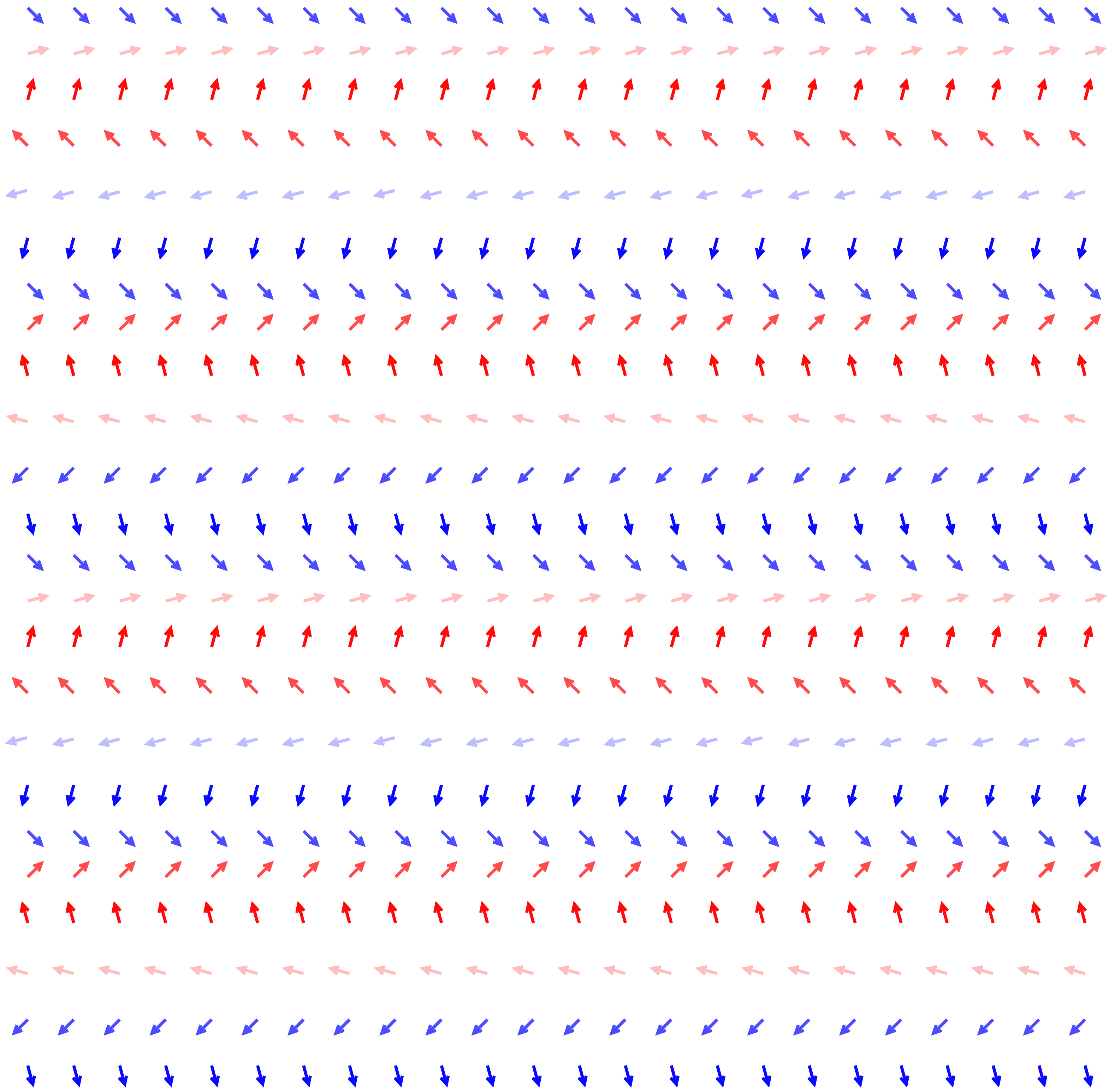
Magnetization in the slice with gradient G_y



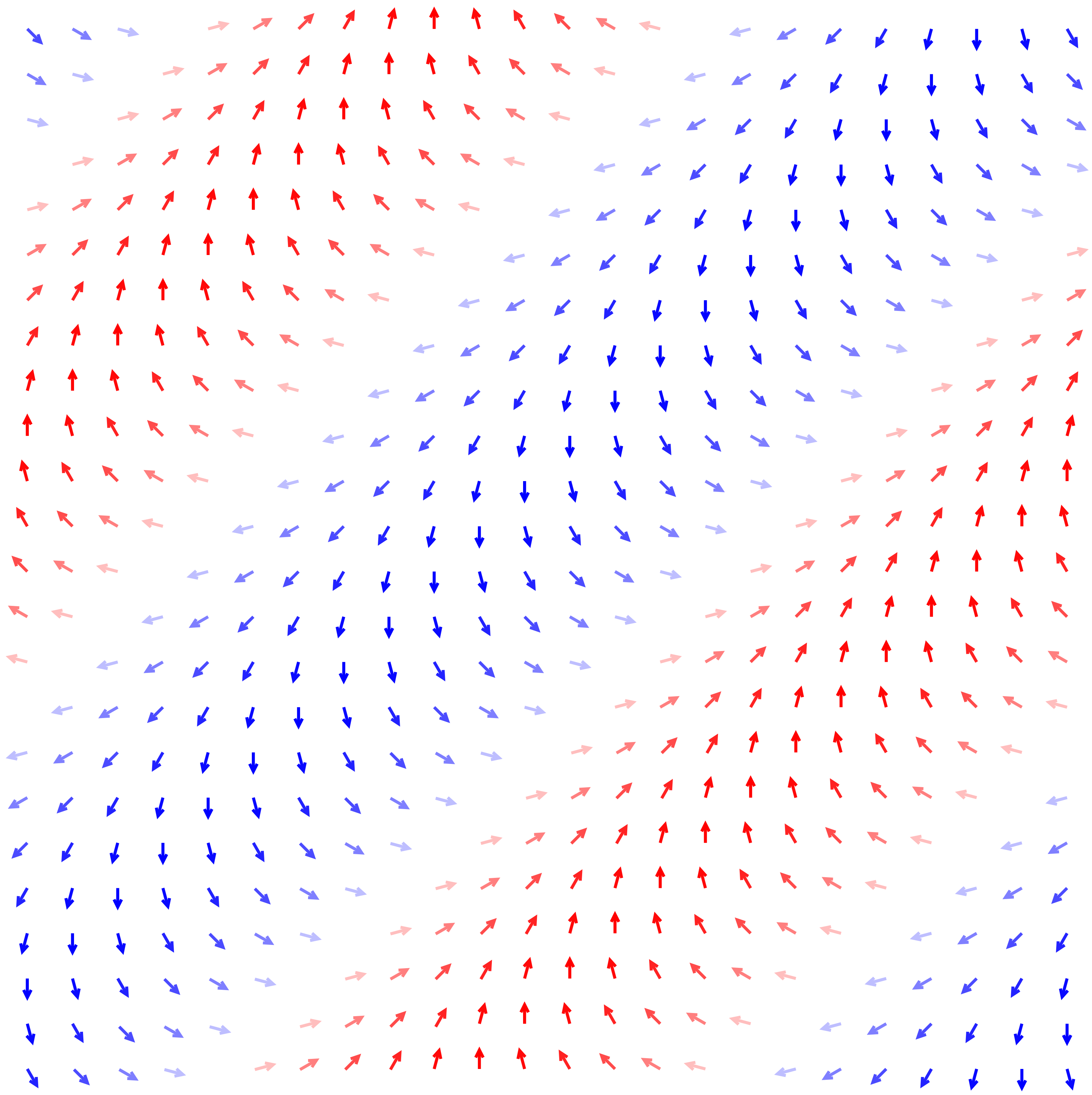
Magnetization in the slice with gradient G_y



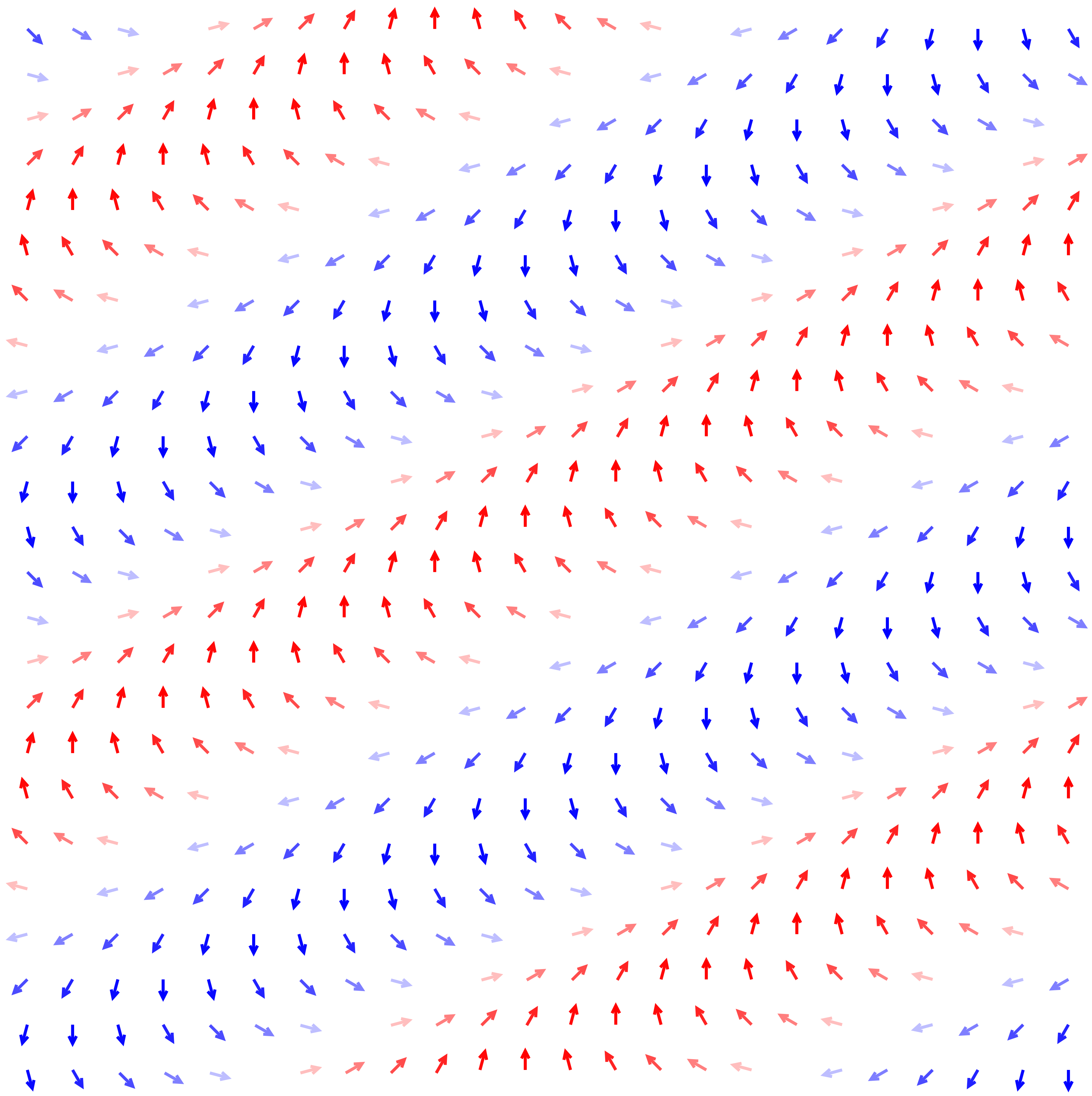
Magnetization in the slice with gradient G_y



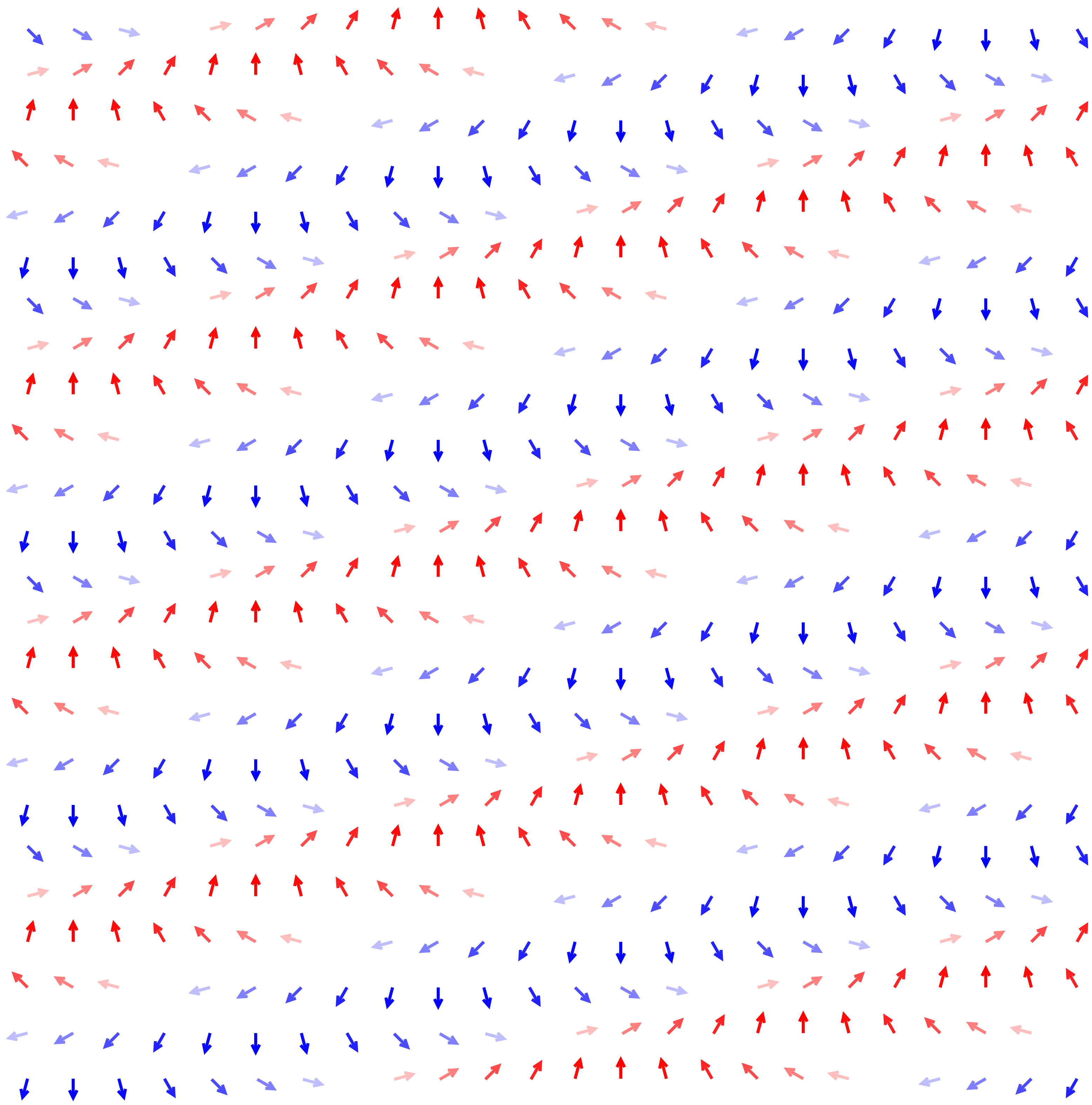
Combination of gradients G_x and G_y



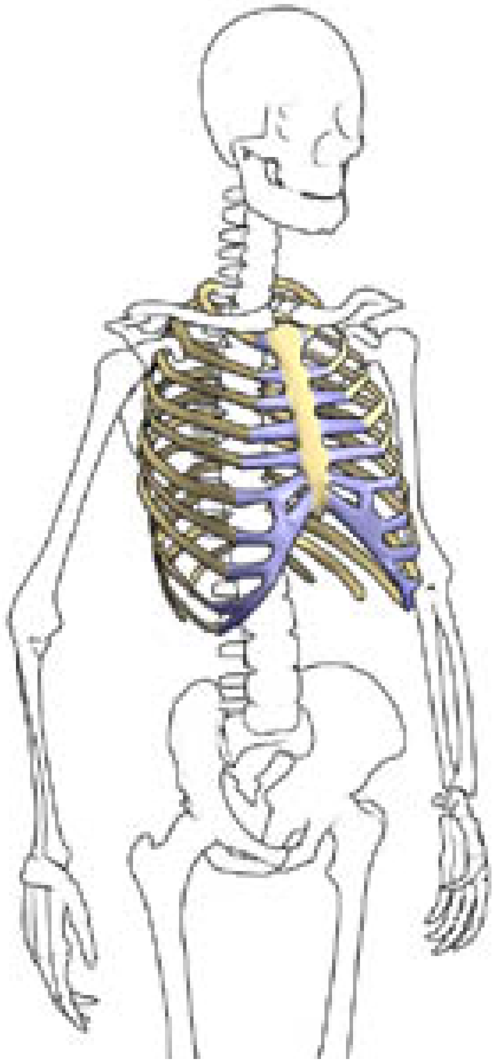
Combination of gradients G_x and G_y

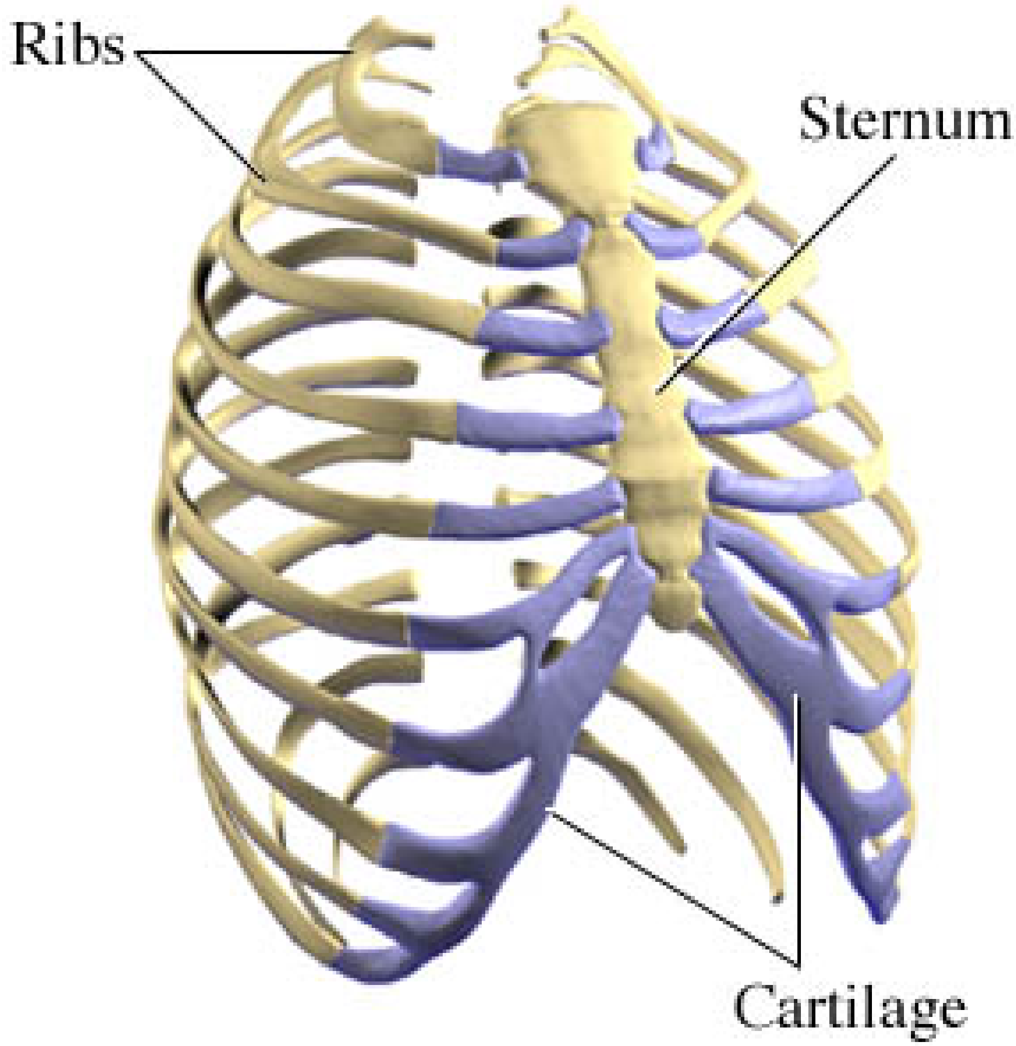


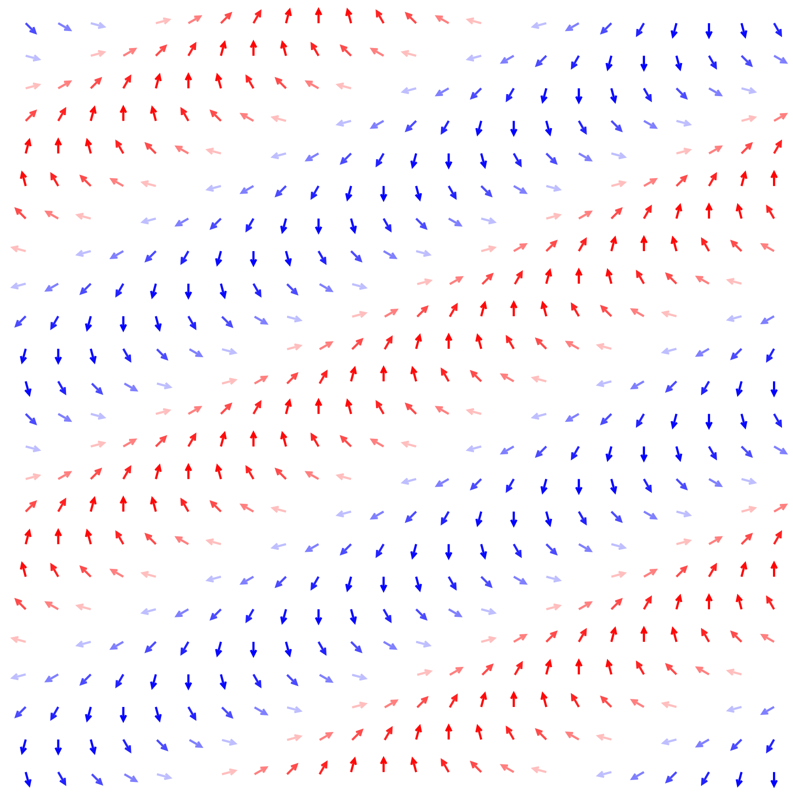
Combination of gradients G_x and G_y



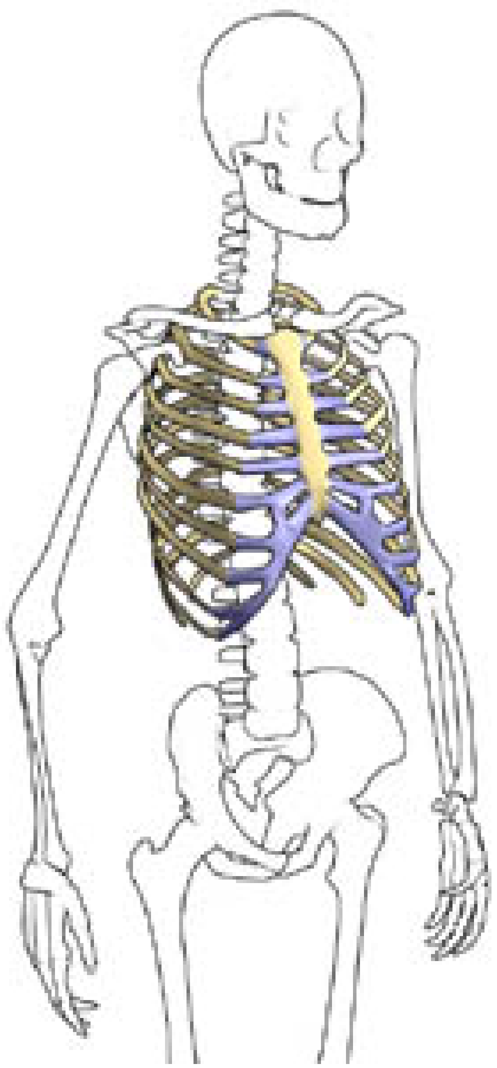
Regular patterns enhance signal

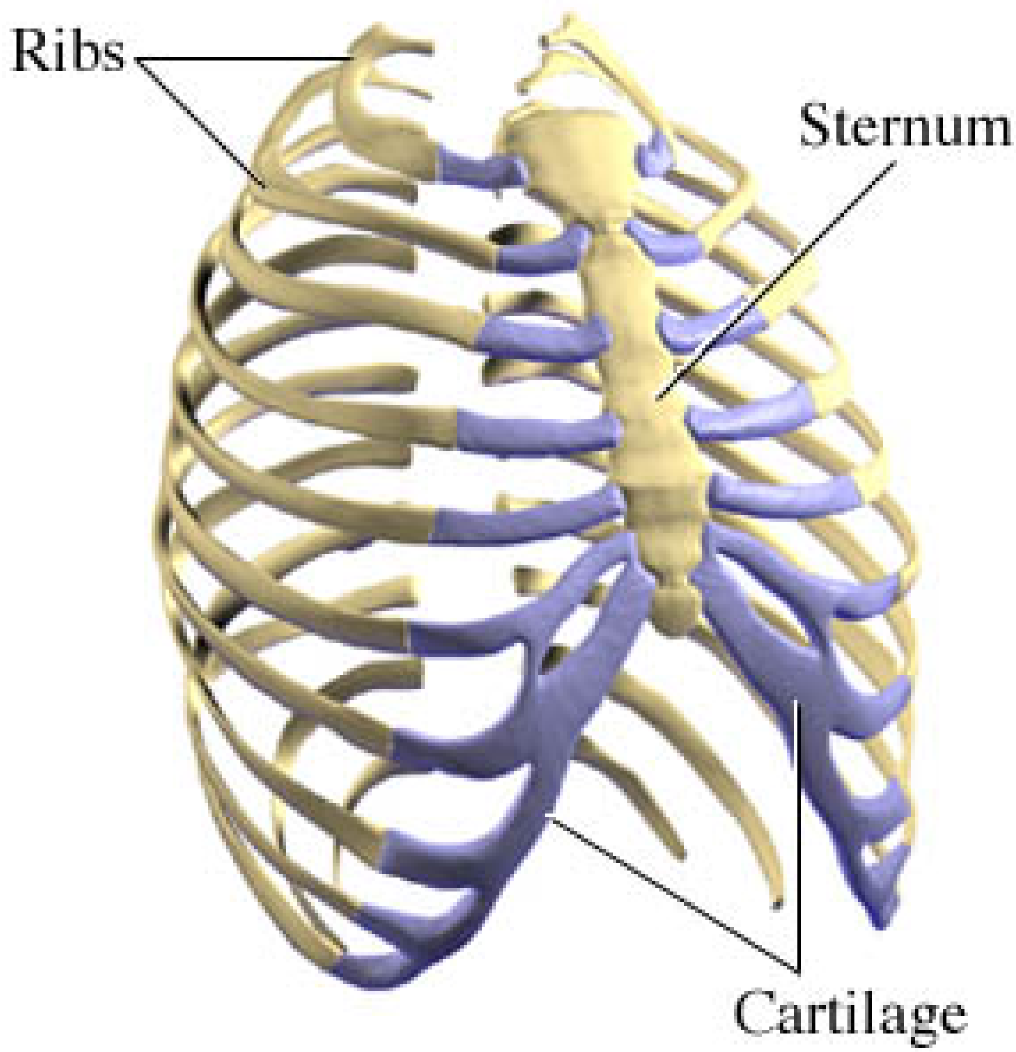


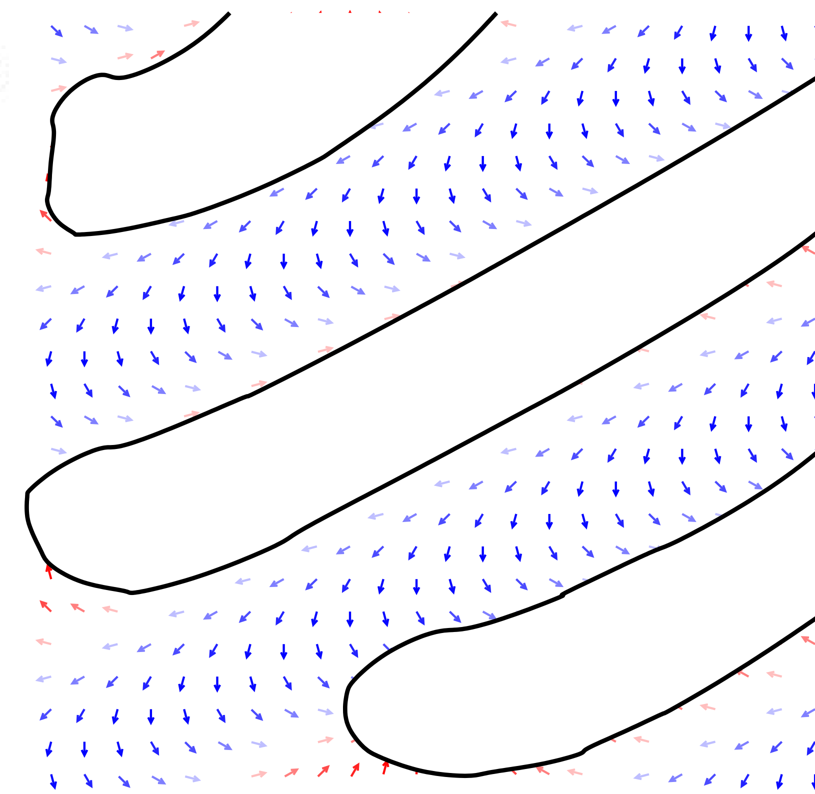




Regular patterns enhance signal







Unique shape as superposition of patterns

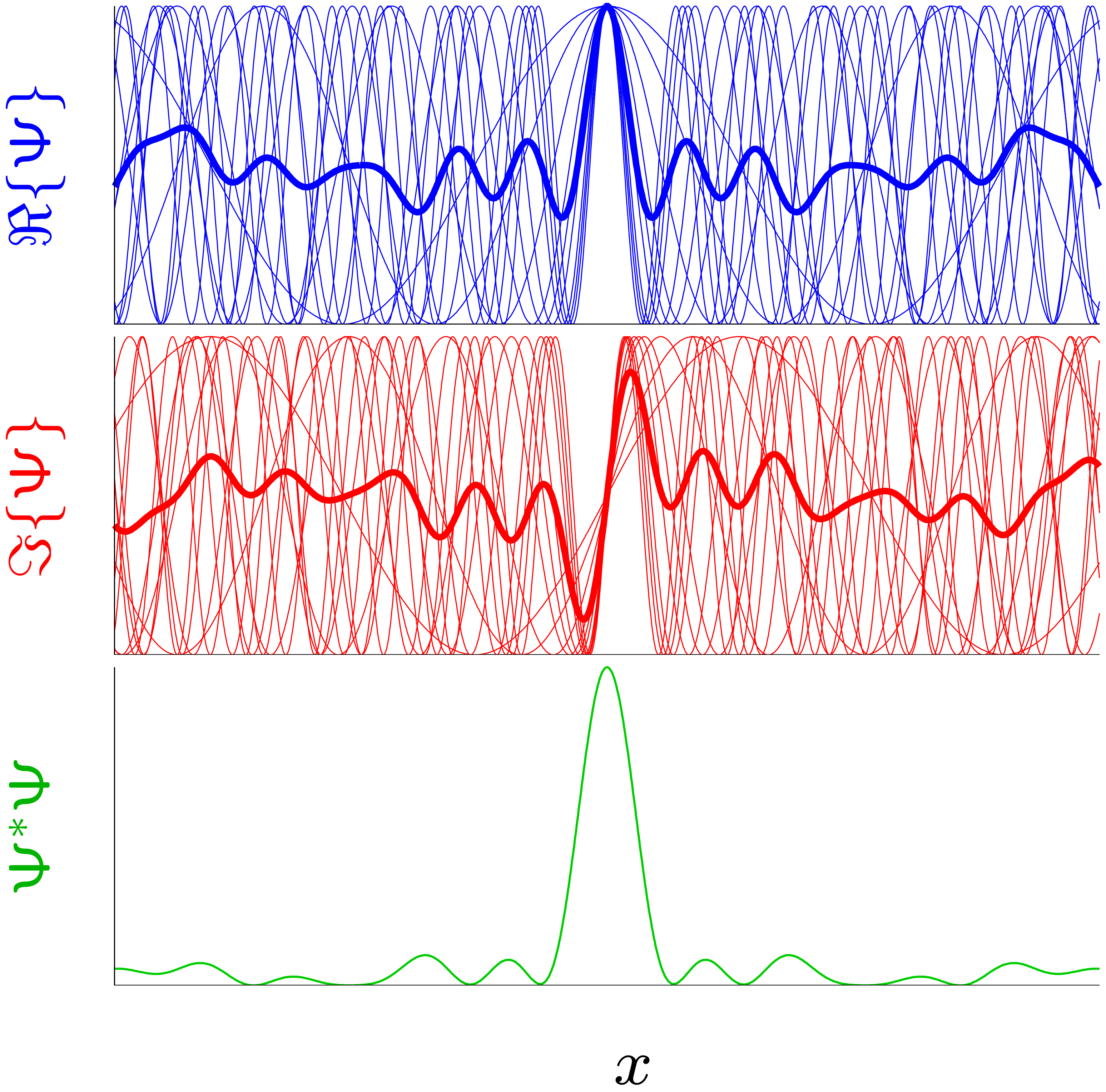


Image reconstruction

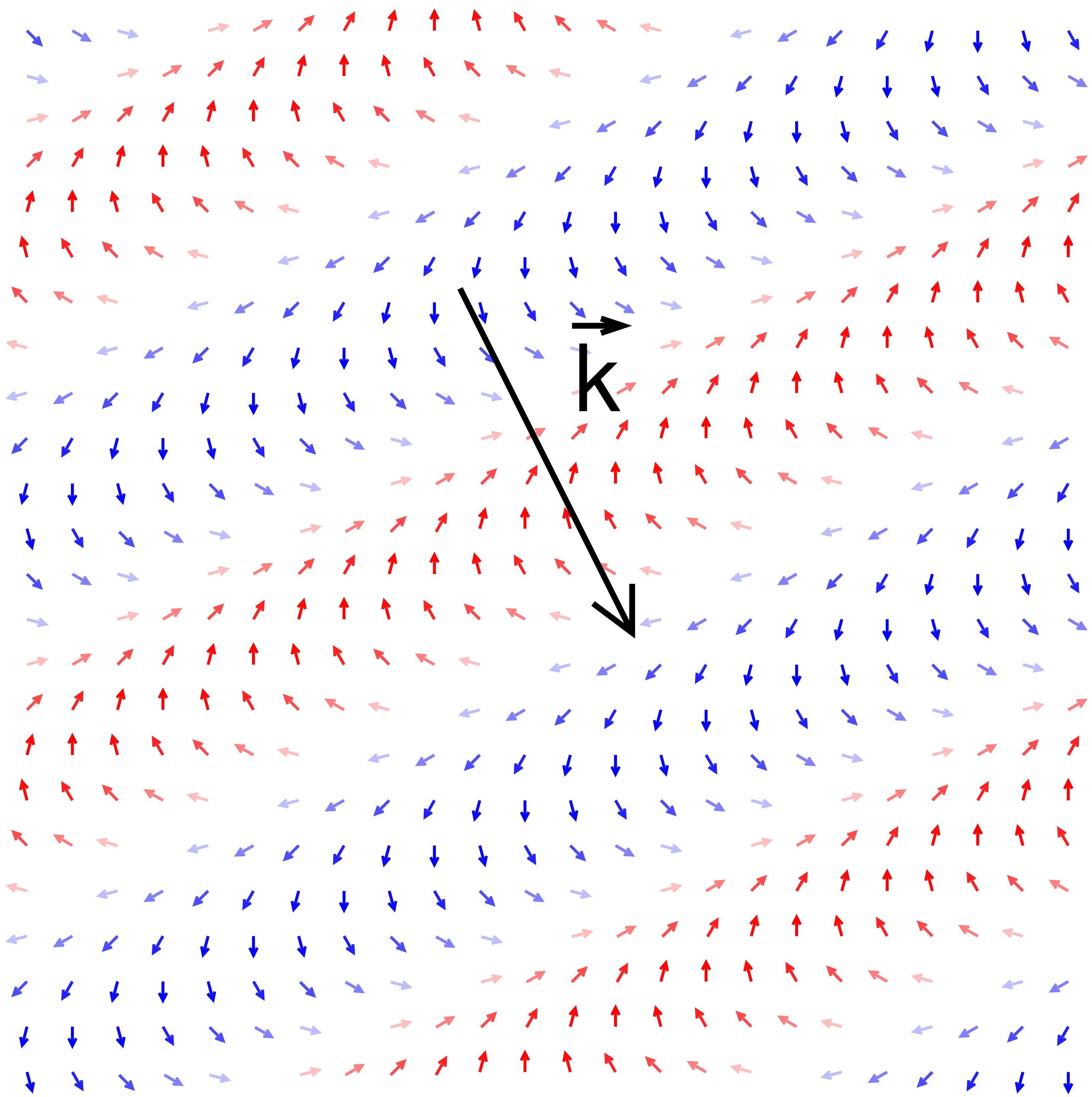
- resembles diffraction methods (crystallography)

- wavelength of the phase patterns generated by gradients

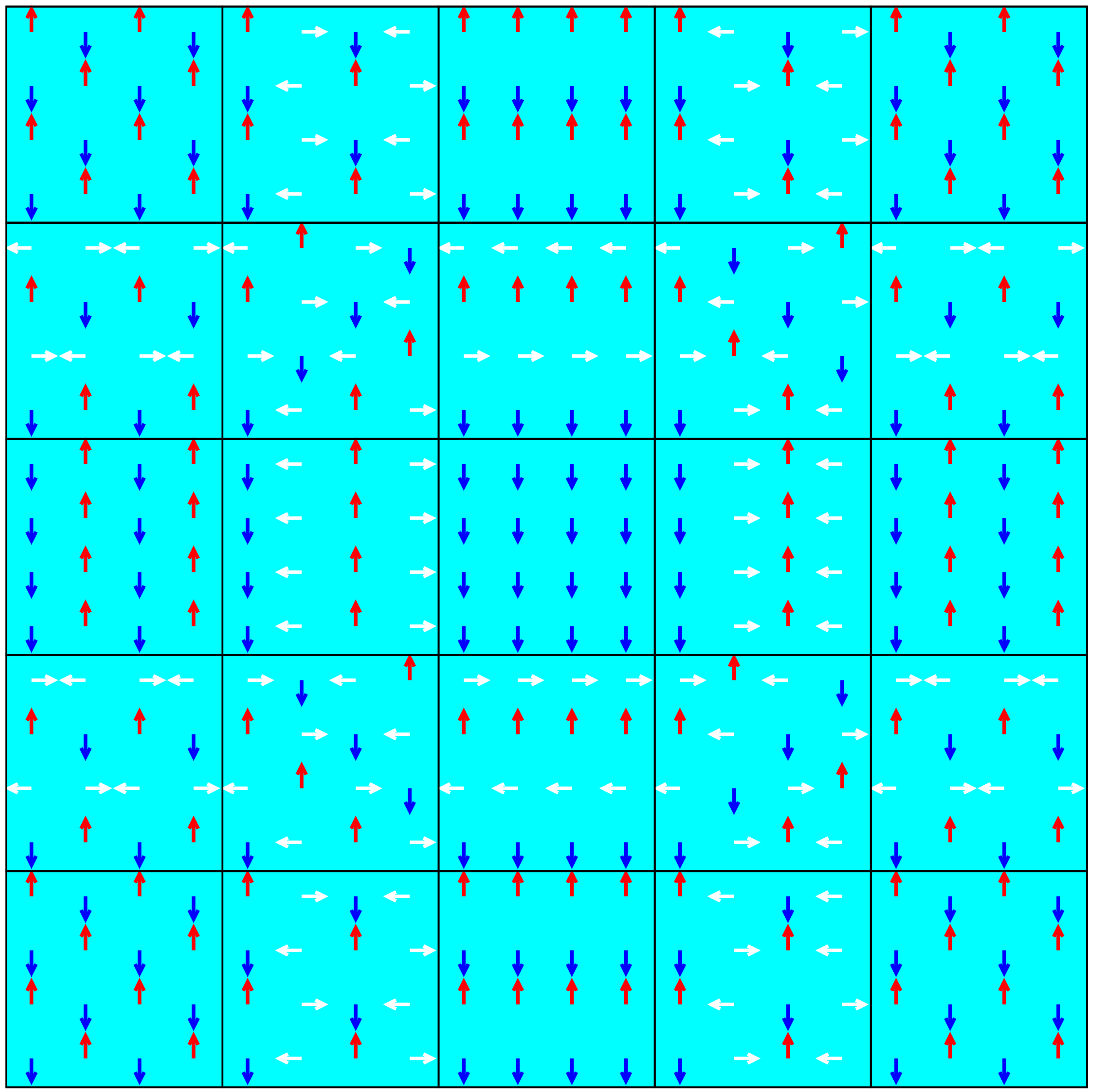
- wavelength of the radio waves is irrelevant
  (but starts to interfere at high field,
  where it approaches the body dimensions)

- Ω assumed to be identical
  differences must be corrected to avoid artifacts

k space

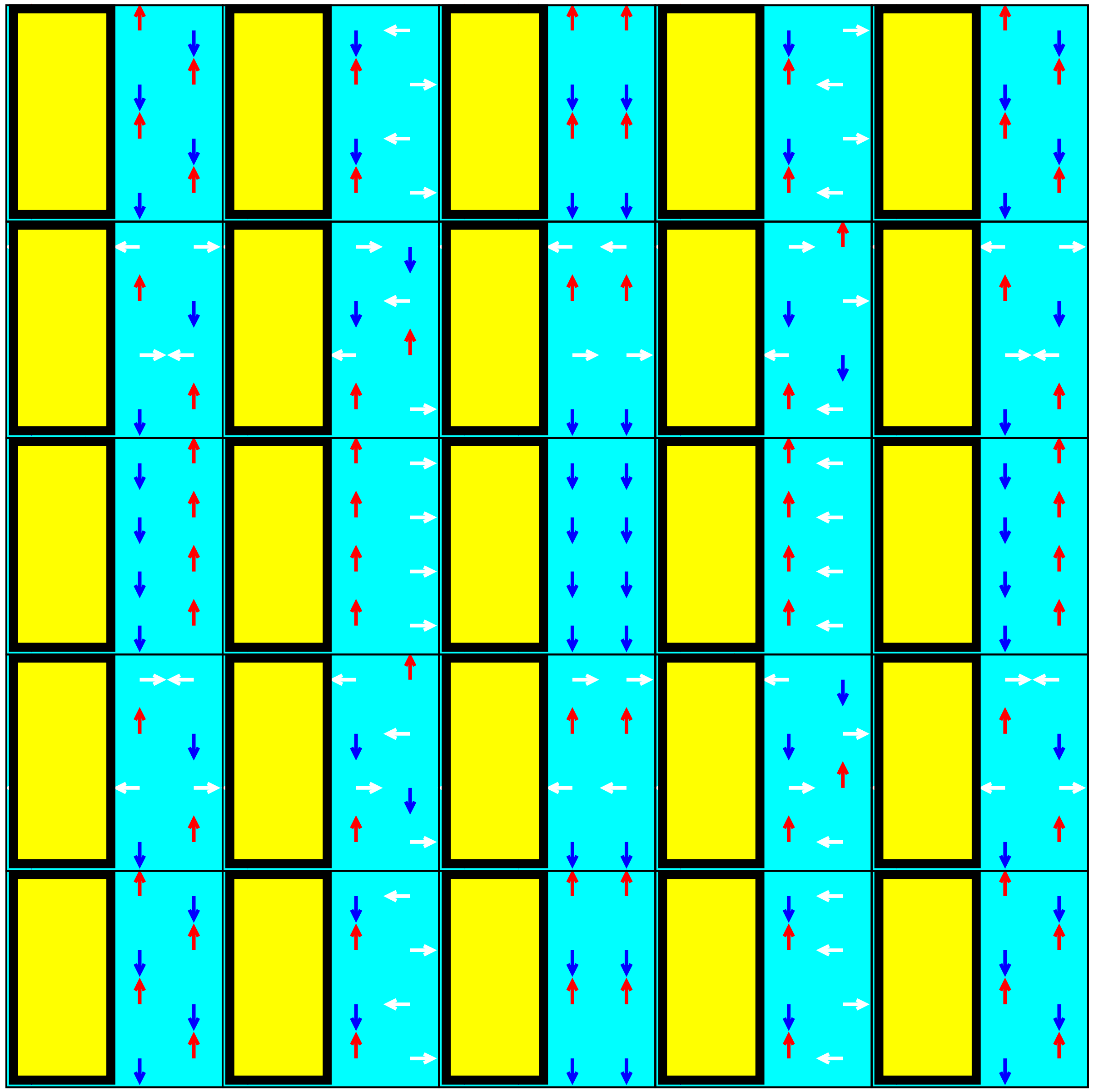


k space



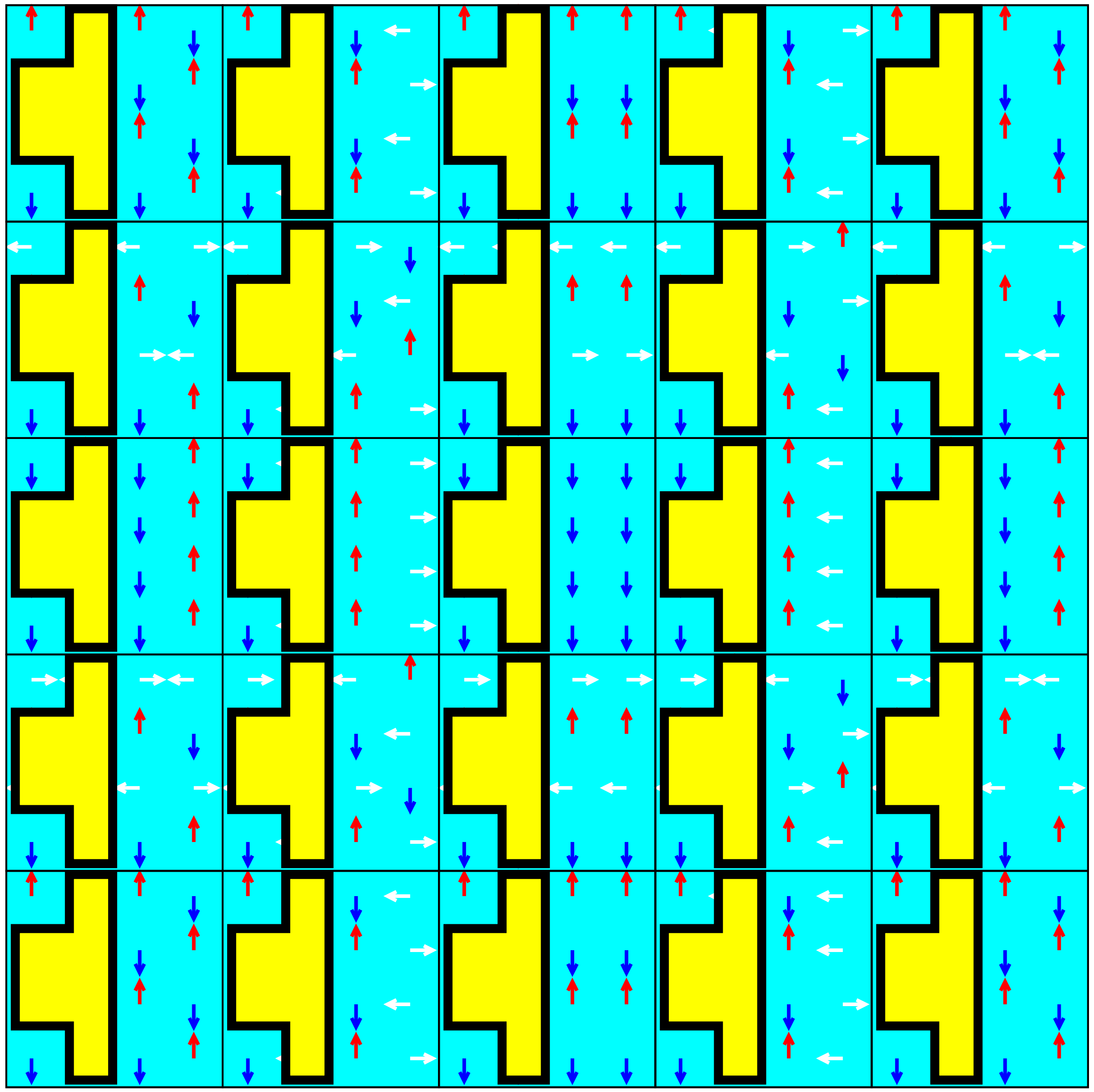
0	0	0	0	0
0	0	0	0	0
0	0	16	0	0
0	0	0	0	0
0	0	0	0	0

k space



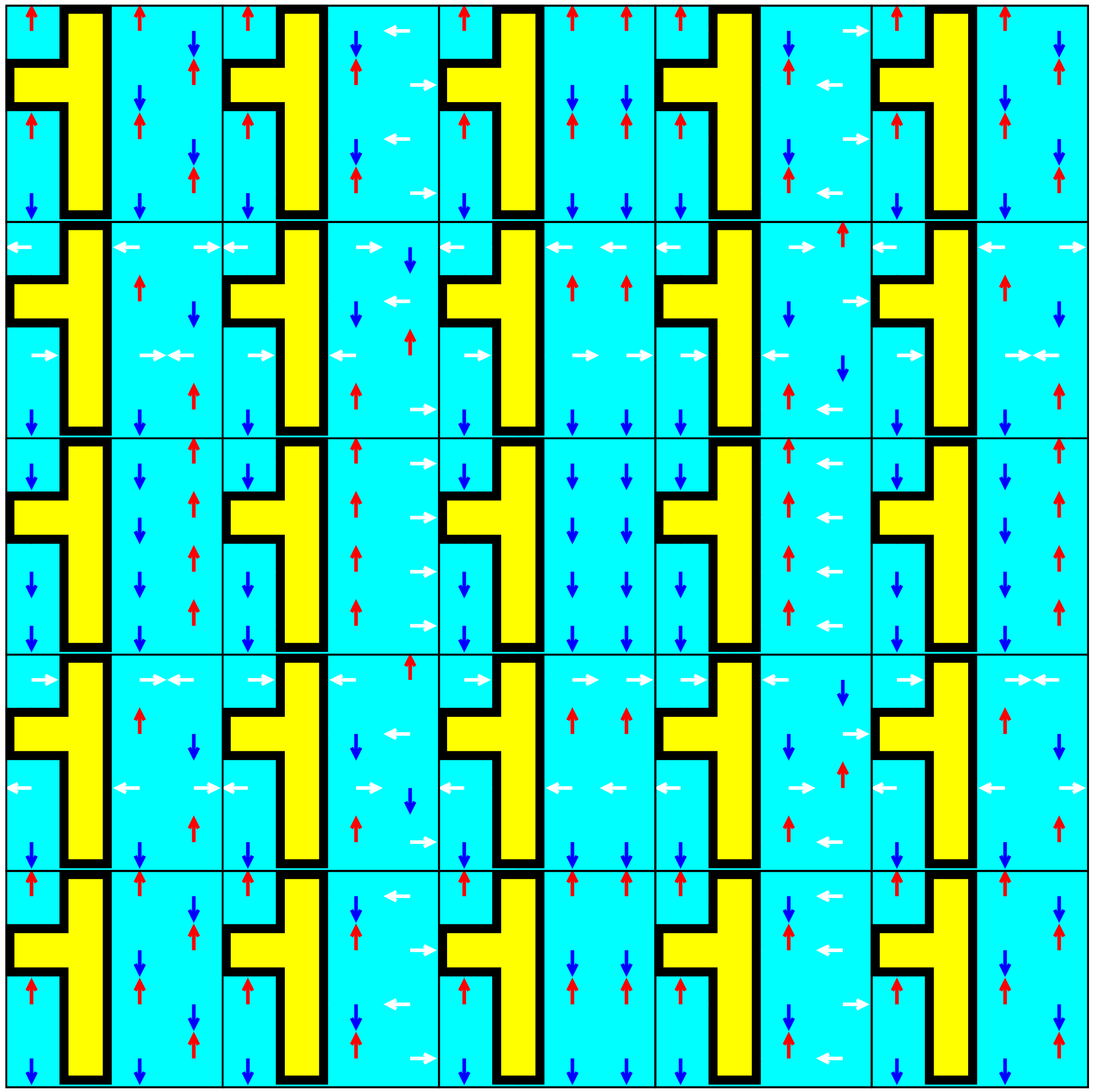
0	0	0	0	0
0	0	0	0	0
0	−4	8	−4	0
0	0	0	0	0
0	0	0	0	0

k space



0	0	0	0	0
+1	+1	+1	+1	+1
+2	−2	10	−2	+2
+1	+1	+1	+1	+1
0	0	0	0	0

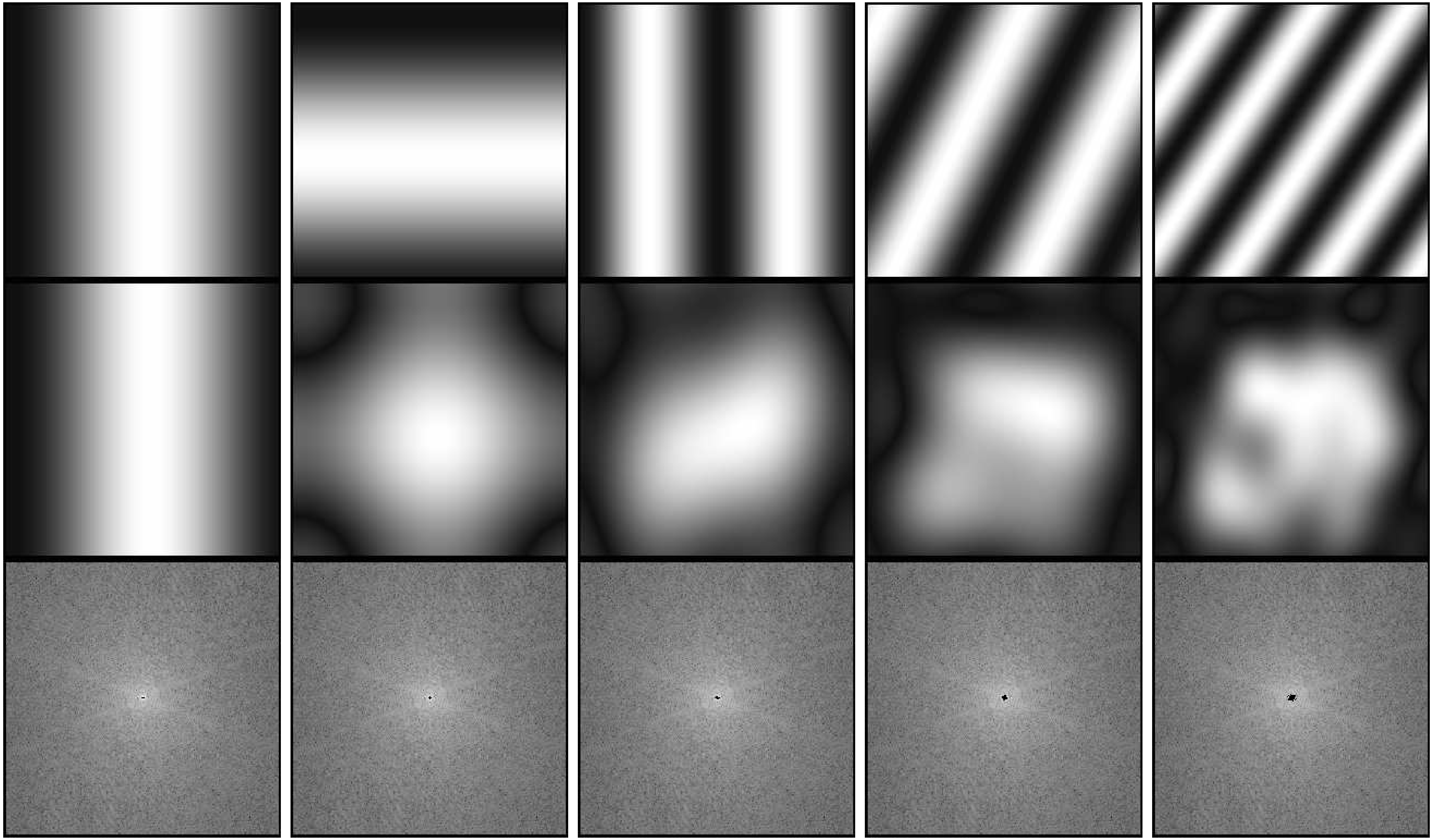
k space

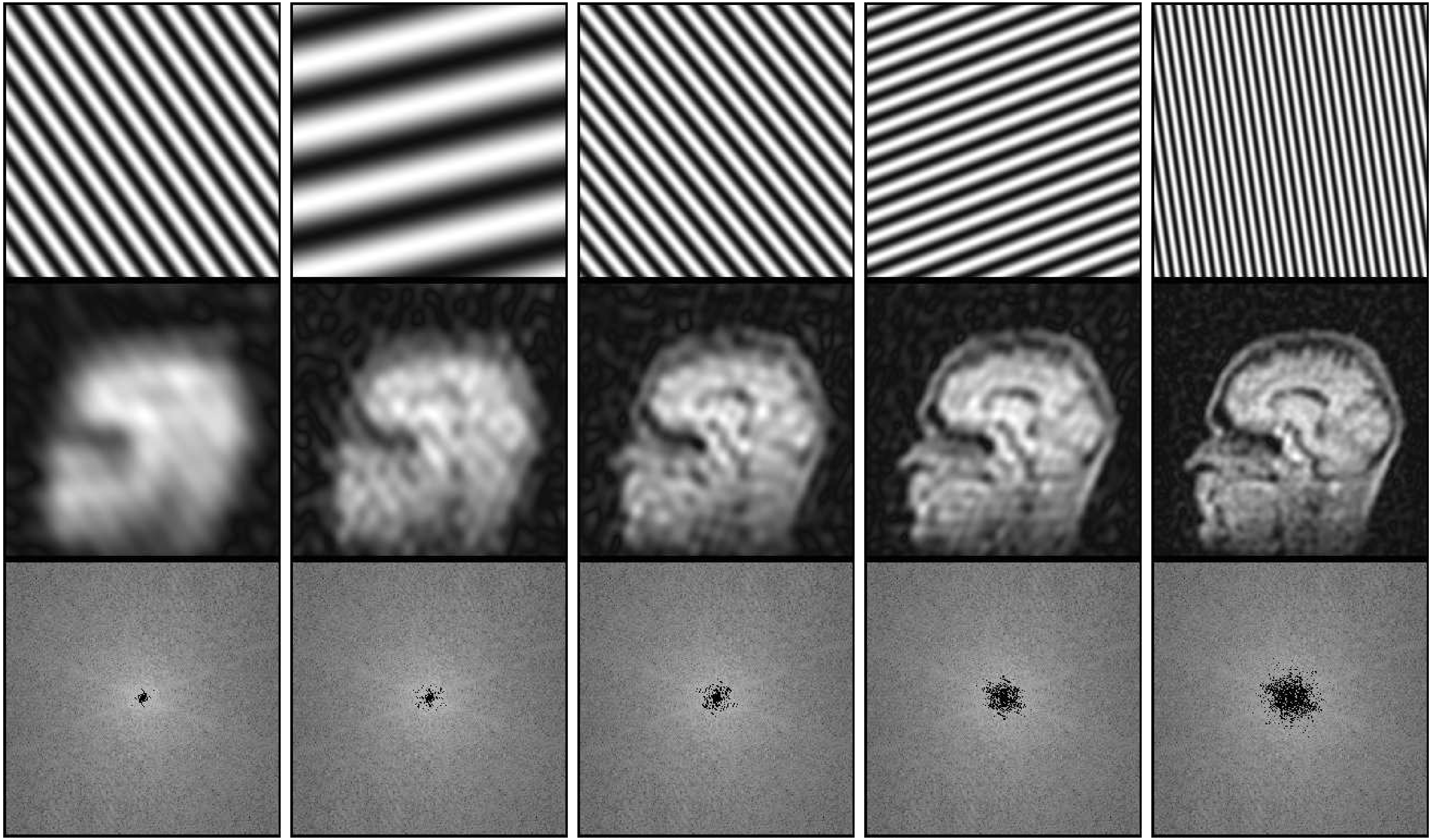


−1	−1	−1	−1	−1
+1	+1	+1	+1	+1
+3	−1	11	−1	+3
+1	+1	+1	+1	+1
−1	+1	−1	−1	+1

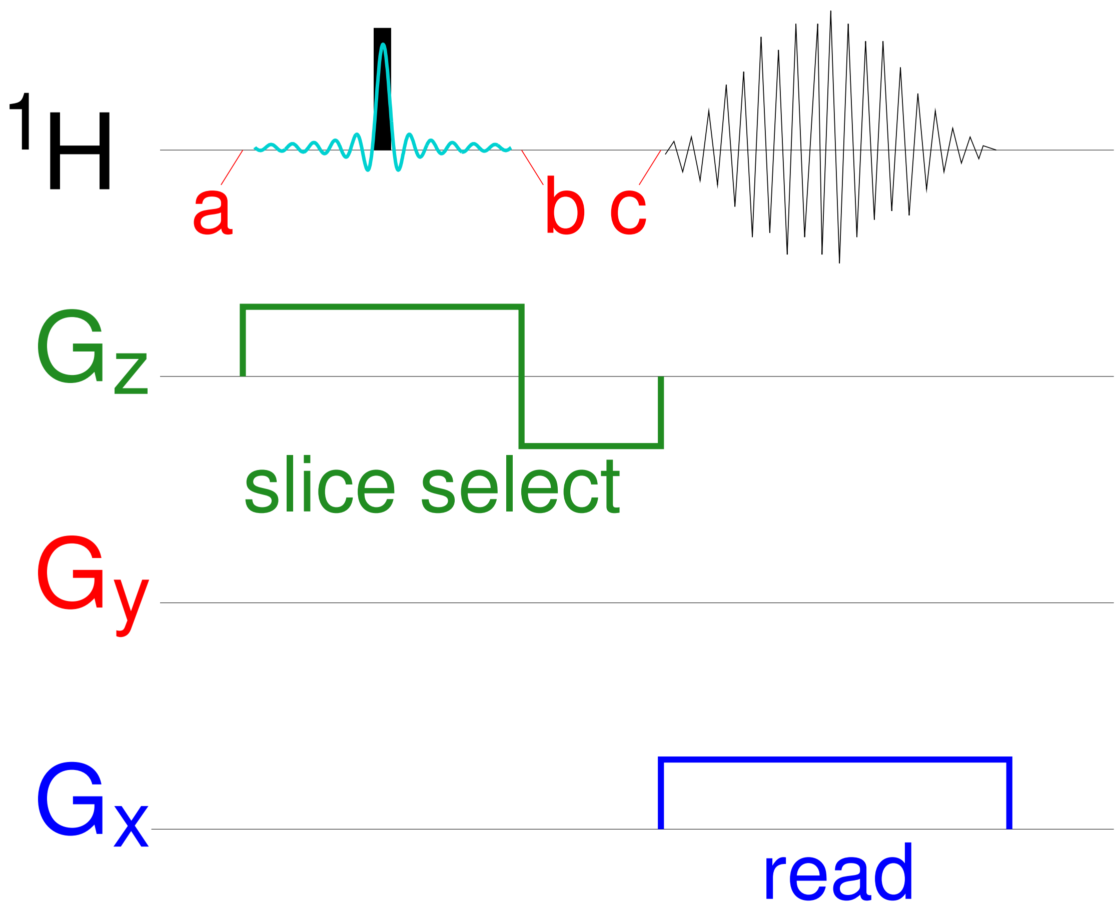
See Figure 15 in

http://eprints.drcmr.dk/37/1/MRI_English_a4.pdf





1D imaging in the slice



Frequency encoding gradient

$$\langle M_+\rangle(k_x) = \int_0^L K e^{i\Omega t - R_2 t} \mathcal{N}(x) e^{-i k_x x} dx$$

$$\approx \int_{-\infty}^{\infty} \underbrace{K e^{i\Omega t - R_2 t}}_{K'} \mathcal{N}(x) e^{-i k_x x} dx$$

Signal $\langle M_+\rangle(k_x)$ is *Fourier transform* of *spin density* $\mathcal{N}(x)$

$\Rightarrow$ *Spin density* $\mathcal{N}(x)$ can be reconstructed

by Fourier transformation of the signal $\langle M_+\rangle(k_x)$

Frequency encoding gradients

$$\langle M_+ \rangle (k_x) \approx K' \int_{-\infty}^{\infty} \mathcal{N}(x) \mathrm{e}^{-\mathrm{i} k_x x} \mathrm{d}x$$

$$\Delta t \Delta f = \frac{1}{N}$$

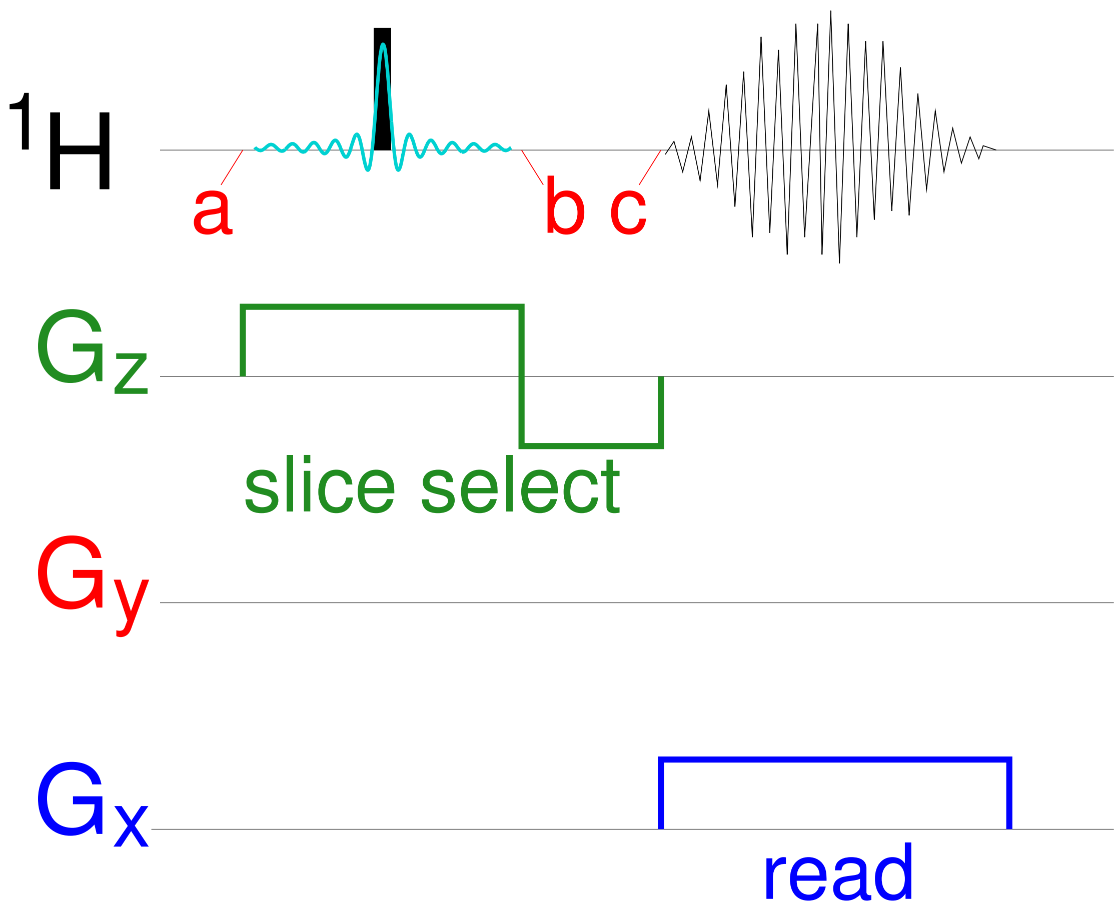
$$k_x = \gamma G_x t = n \cdot \Delta k_x \qquad x = j \Delta x$$

$$\Delta k_x = \gamma G_x \Delta t = \frac{\gamma G_x}{N \Delta f}$$

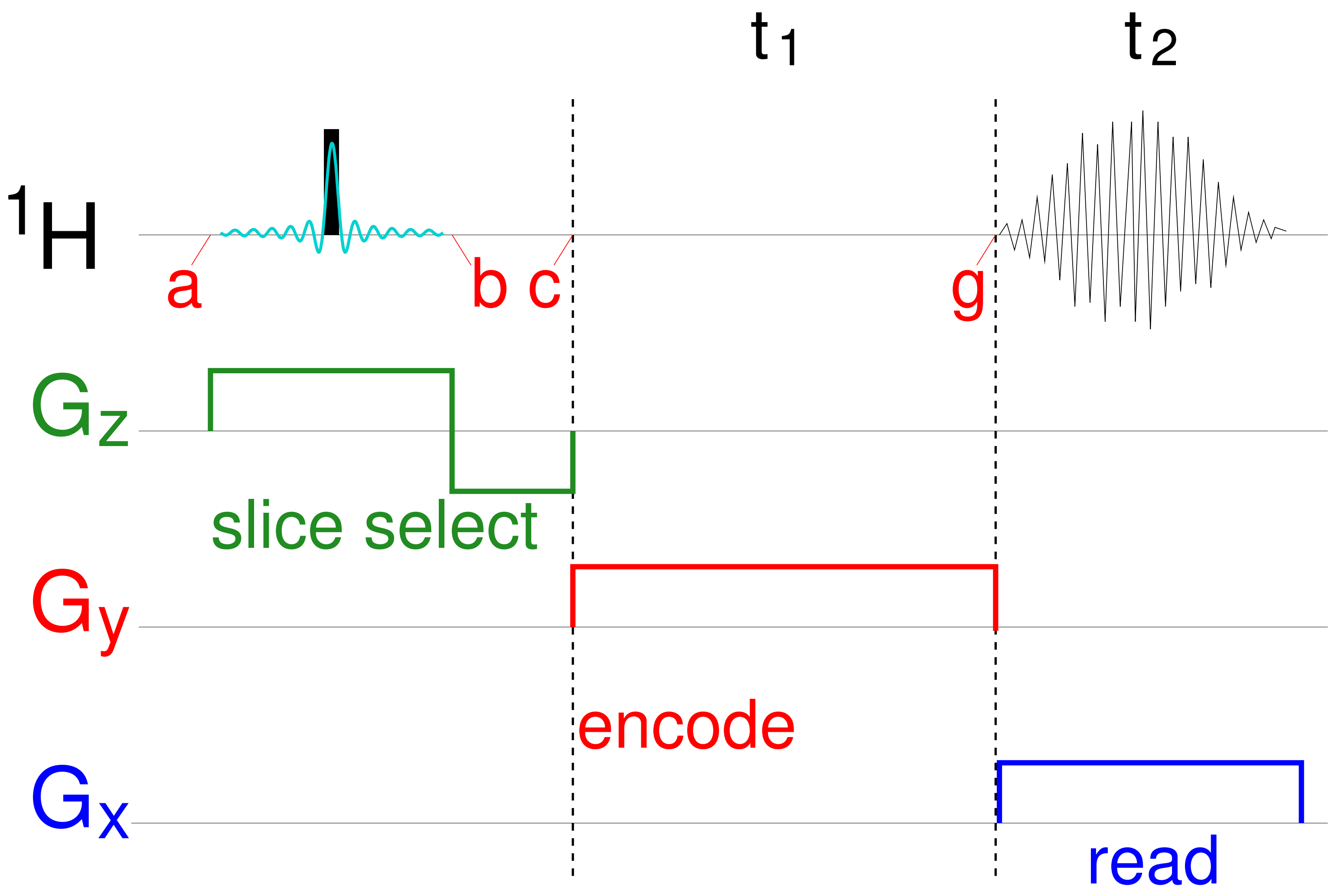
$$\mathcal{N}(x) = \frac{\Delta k_x}{K'} \sum_{n=0}^{N-1} \langle M_+ \rangle (k_x) \mathrm{e}^{\mathrm{i} 2\pi \frac{j \cdot n}{N}}$$

Better resolution than slice thickness

1D imaging in the slice



2D imaging in the slice



2D frequency encoding possible

Two frequency encoding gradients

$$\langle M_+\rangle(k_x,k_y) \approx K' \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathcal{N}(x,y)\mathrm{e}^{-\mathrm{i}(k_x x + k_y y)}\mathrm{d}x\mathrm{d}y$$

$$\Delta t_2 \Delta f_2 = \frac{1}{N_x} \quad \Delta t_1 \Delta f_1 = \frac{1}{N_y}$$

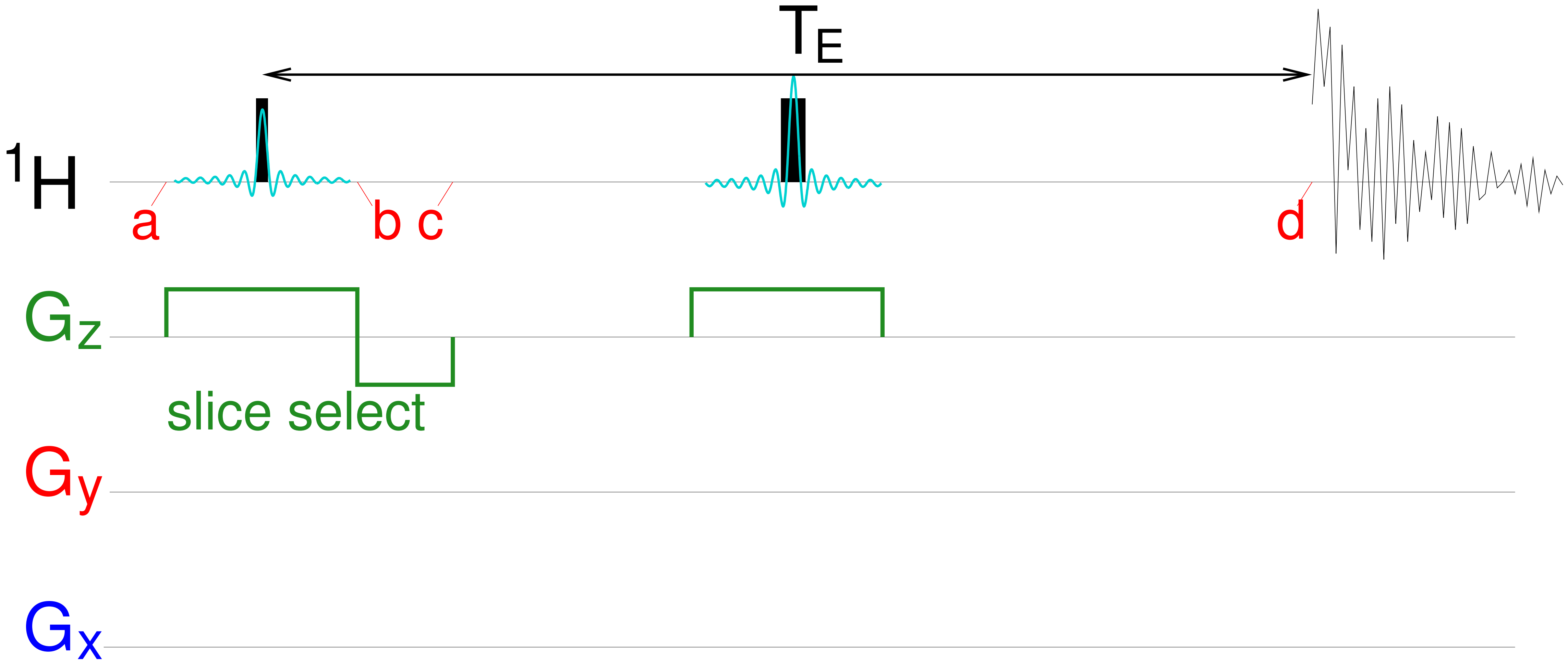
$$\begin{aligned} k_x &= \gamma G_x t_2 = n_x \cdot \Delta k_x & x = j_x \Delta x \\ k_y &= \gamma G_y t_1 = n_y \cdot \Delta k_y & y = j_y \Delta y \\ \Delta k_x &= \gamma G_x \Delta t_2 = \frac{\gamma G_x}{N_x \Delta f_2} \\ \Delta k_y &= \gamma G_y \Delta t_1 = \frac{\gamma G_y}{N_y \Delta f_1} \end{aligned}$$

$$\mathcal{N}(x,y) = \frac{\Delta k_x \Delta k_y}{K'} \sum_{n_x=0}^{N_x-1} \sum_{n_y=0}^{N_y-1} \langle M_+\rangle(k_x,k_y)\mathrm{e}^{\mathrm{i}2\pi\left(\frac{j_x \cdot n_x}{N_x} + \frac{j_y \cdot n_y}{N_y}\right)}$$

Frequency and phase encoding gradients

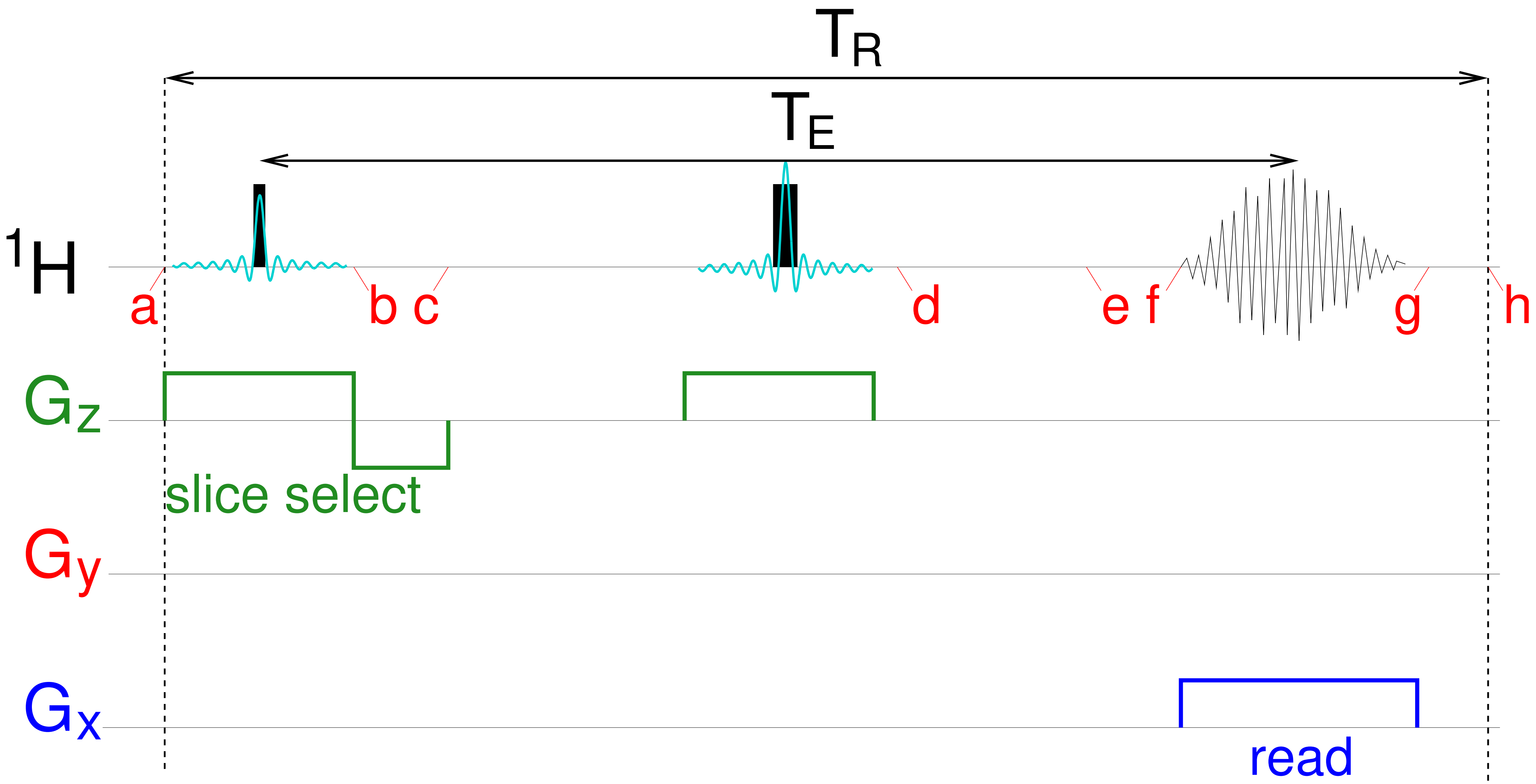
$$\Delta k_y = \begin{cases} \gamma G_y \Delta t_1 = \frac{\gamma G_y}{N_y \Delta f_1} \\ \\ \gamma t_x \Delta G_y \end{cases}$$

MRI spin echo experiment



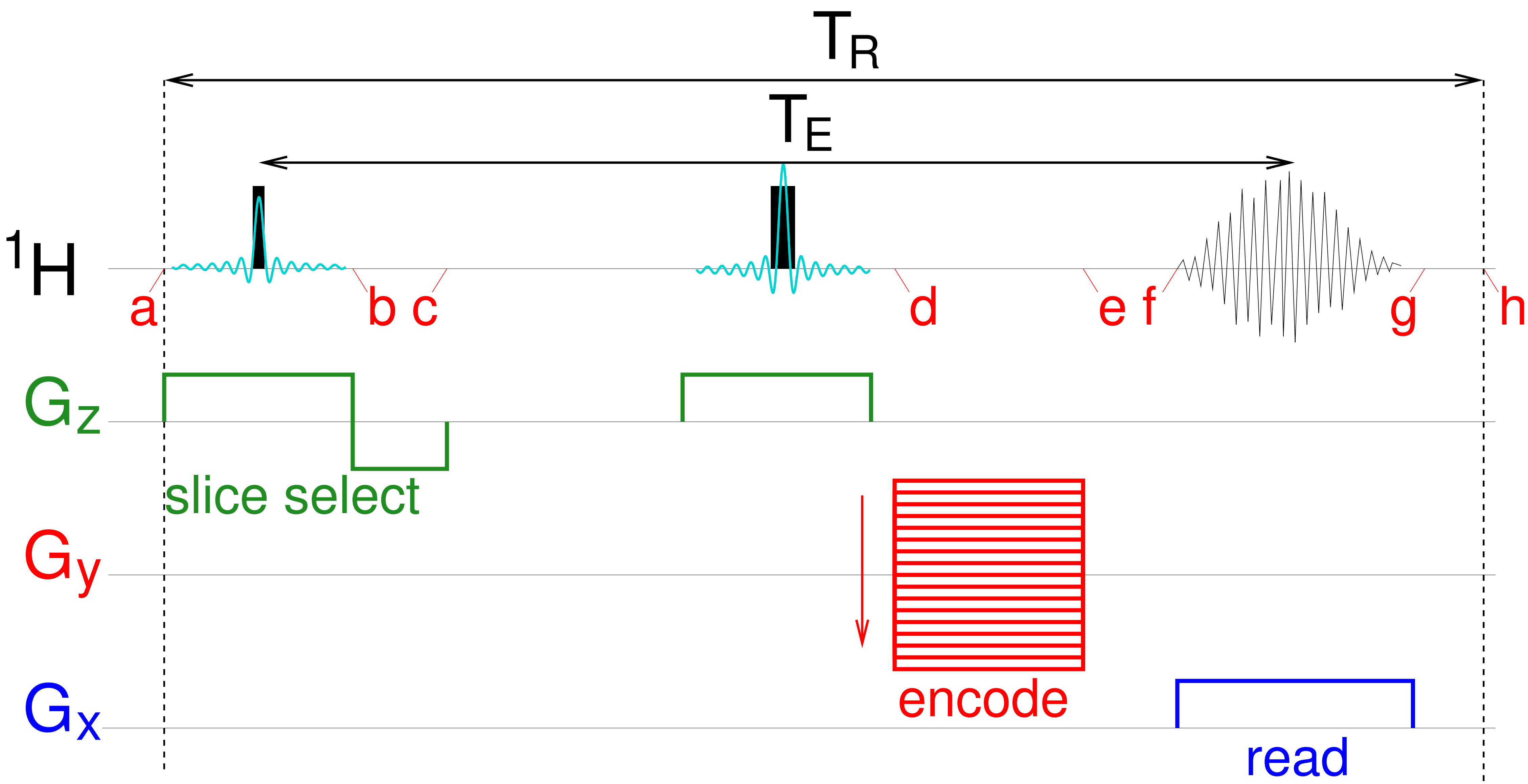
Phase encoding typical in MRI

MRI spin echo experiment



Frequency encoding in x

MRI spin echo experiment



Phase encoding in y

Frequency and phase encoding gradients

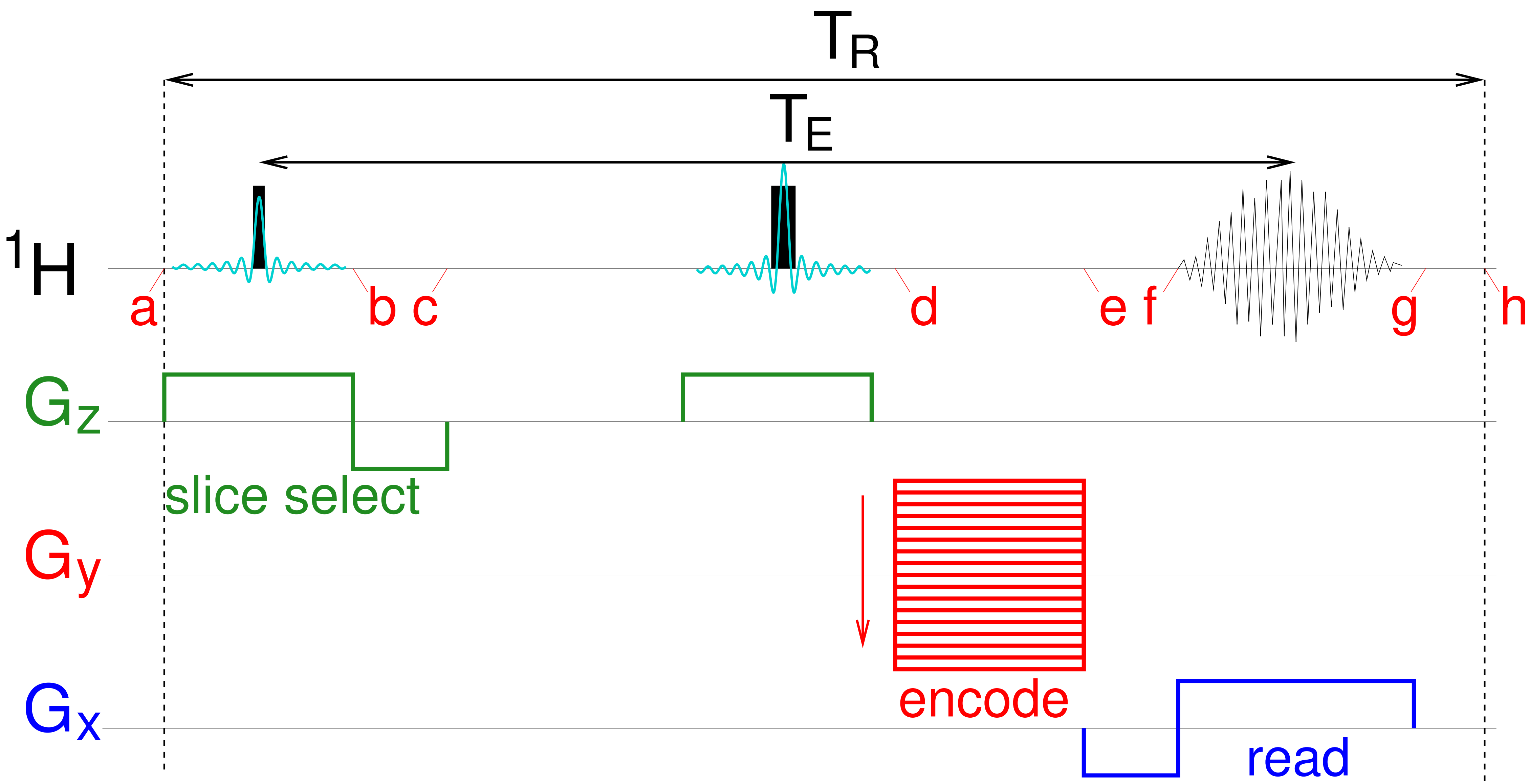
$$\langle M_+\rangle(k_x,k_y) \approx K' \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathcal{N}(x,y)\mathrm{e}^{-\mathrm{i}(k_x x + k_y y)}\mathrm{d}x\mathrm{d}y$$

$$\Delta t \Delta f = \frac{1}{N_x}$$

$$\begin{aligned} k_x &= \gamma G_x t = n_x \cdot \Delta k_x \qquad x = j_x \Delta x \\ k_y &= \gamma G_y t_y = n_y \cdot \Delta k_y \qquad y = j_y \Delta y \\ \Delta k_x &= \gamma G_x \Delta t = \frac{\gamma G_x}{N_x \Delta f} \\ \Delta k_y &= \gamma t_y \Delta G_y \end{aligned}$$

$$\mathcal{N}(x,y) = \frac{\Delta k_x \Delta k_y}{K'} \sum_{n_x=0}^{N_x-1} \sum_{n_y=-\frac{N_y}{2}}^{\frac{N_y}{2}-1} \langle M_+\rangle \mathrm{e}^{\mathrm{i}2\pi\left(\frac{j_x \cdot n_x}{N_x} + \frac{j_y \cdot n_y}{N_y}\right)}$$

MRI spin echo experiment



Pre-phase gradient in x $(-x \to x)$

Frequency and phase encoding gradients

$$\langle M_+\rangle(k_x,k_y) \approx K' \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathcal{N}(x,y)\mathrm{e}^{-\mathrm{i}(k_x x+k_y y)}\mathrm{d}x\mathrm{d}y$$

$$\Delta t \Delta f = \frac{1}{N_x}$$

$$k_x = \gamma G_x t = n_x \cdot \Delta k_x \qquad x = j_x \Delta x$$

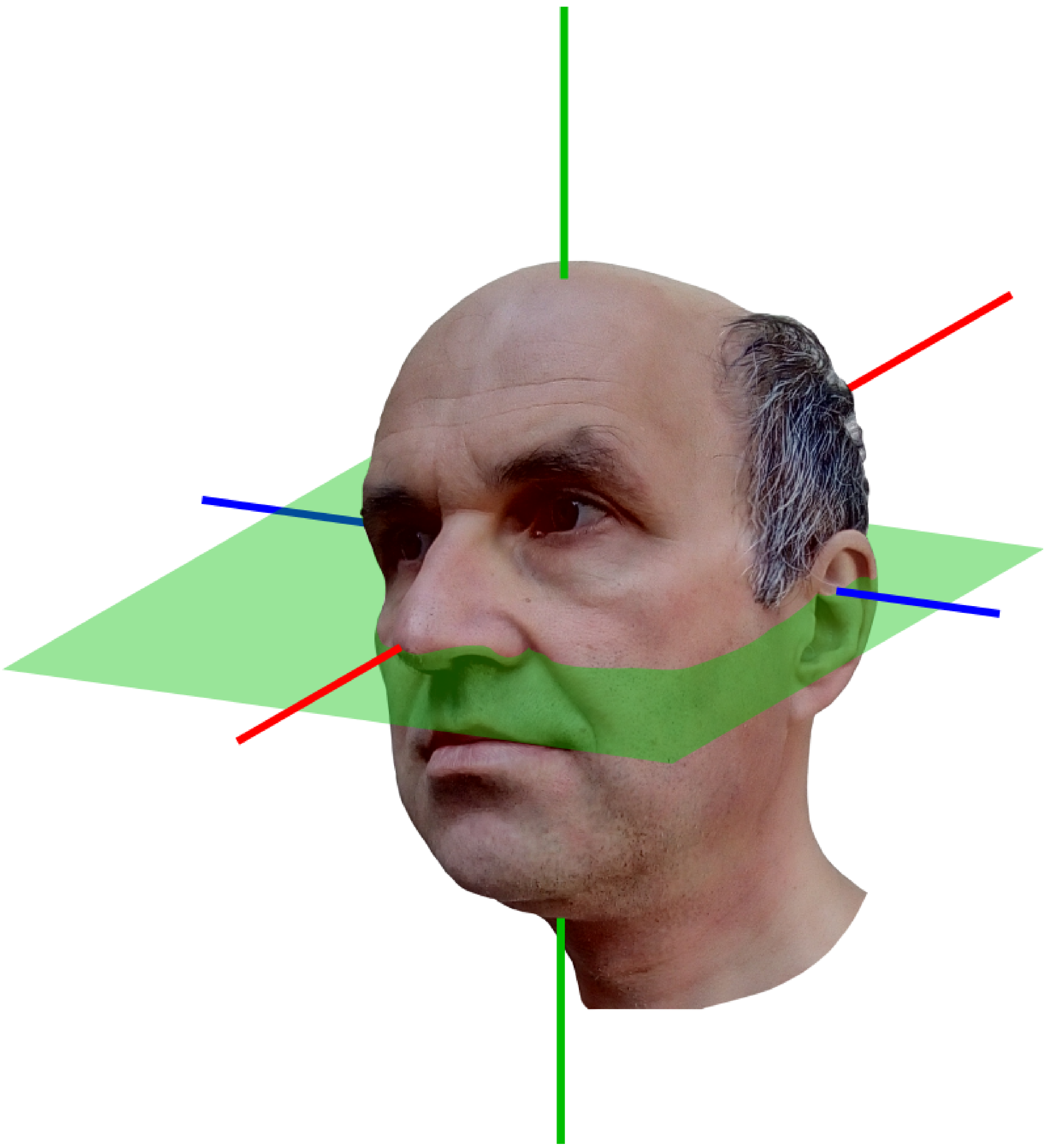
$$k_y = \gamma G_y t_y = n_y \cdot \Delta k_y \qquad y = j_y \Delta y$$

$$\Delta k_x = \gamma G_x \Delta t = \frac{\gamma G_x}{N_x \Delta f}$$

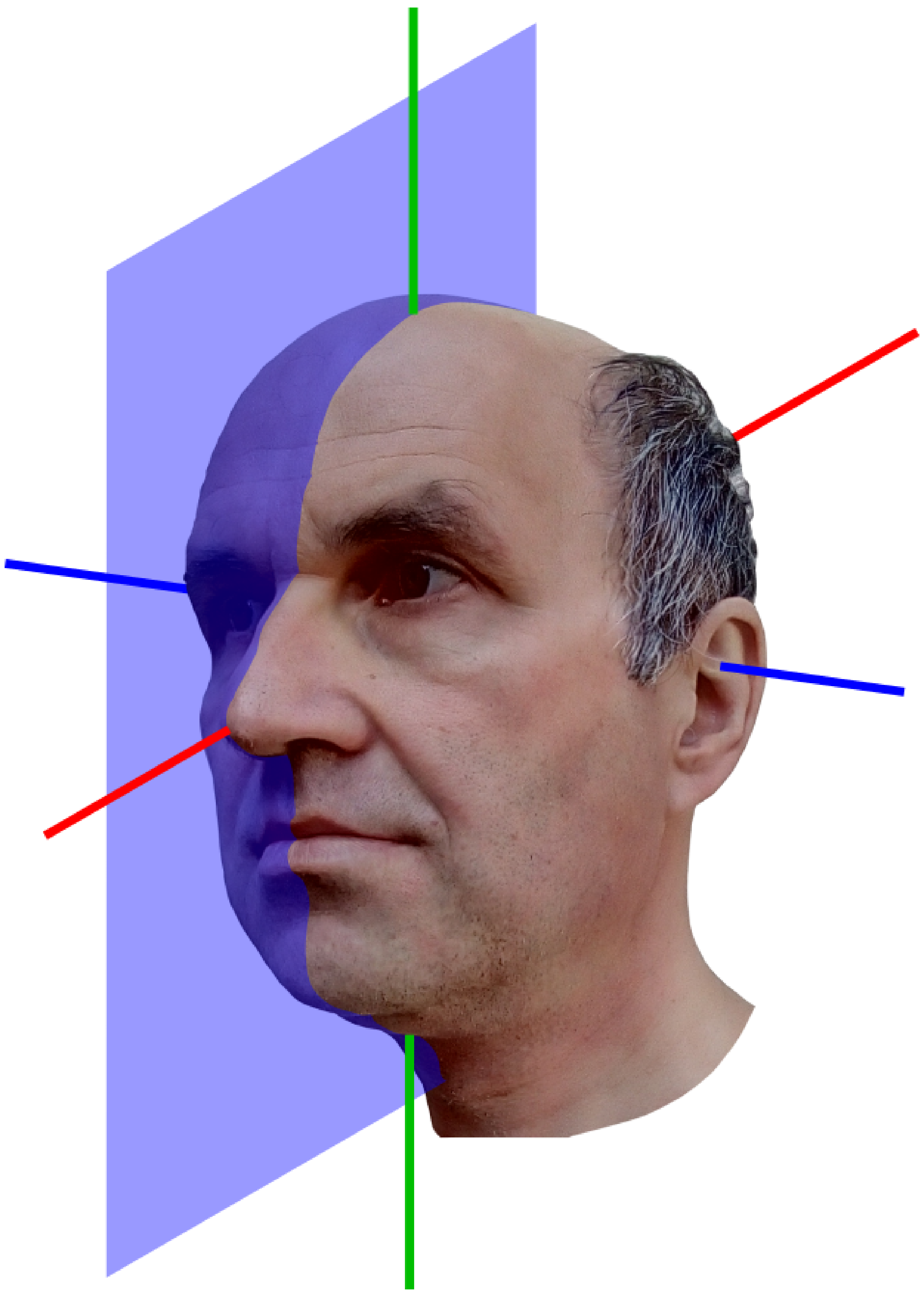
$$\Delta k_y = \gamma t_y \Delta G_y$$

$$\mathcal{N}(x,y) = \frac{\Delta k_x \Delta k_y}{K'} \sum_{n_x=-\frac{N_x}{2}}^{\frac{N_x}{2}-1} \sum_{n_y=-\frac{N_y}{2}}^{\frac{N_y}{2}-1} \langle M_+\rangle \mathrm{e}^{\mathrm{i}2\pi\left(\frac{j_x \cdot n_x}{N_x}+\frac{j_y \cdot n_y}{N_y}\right)}$$

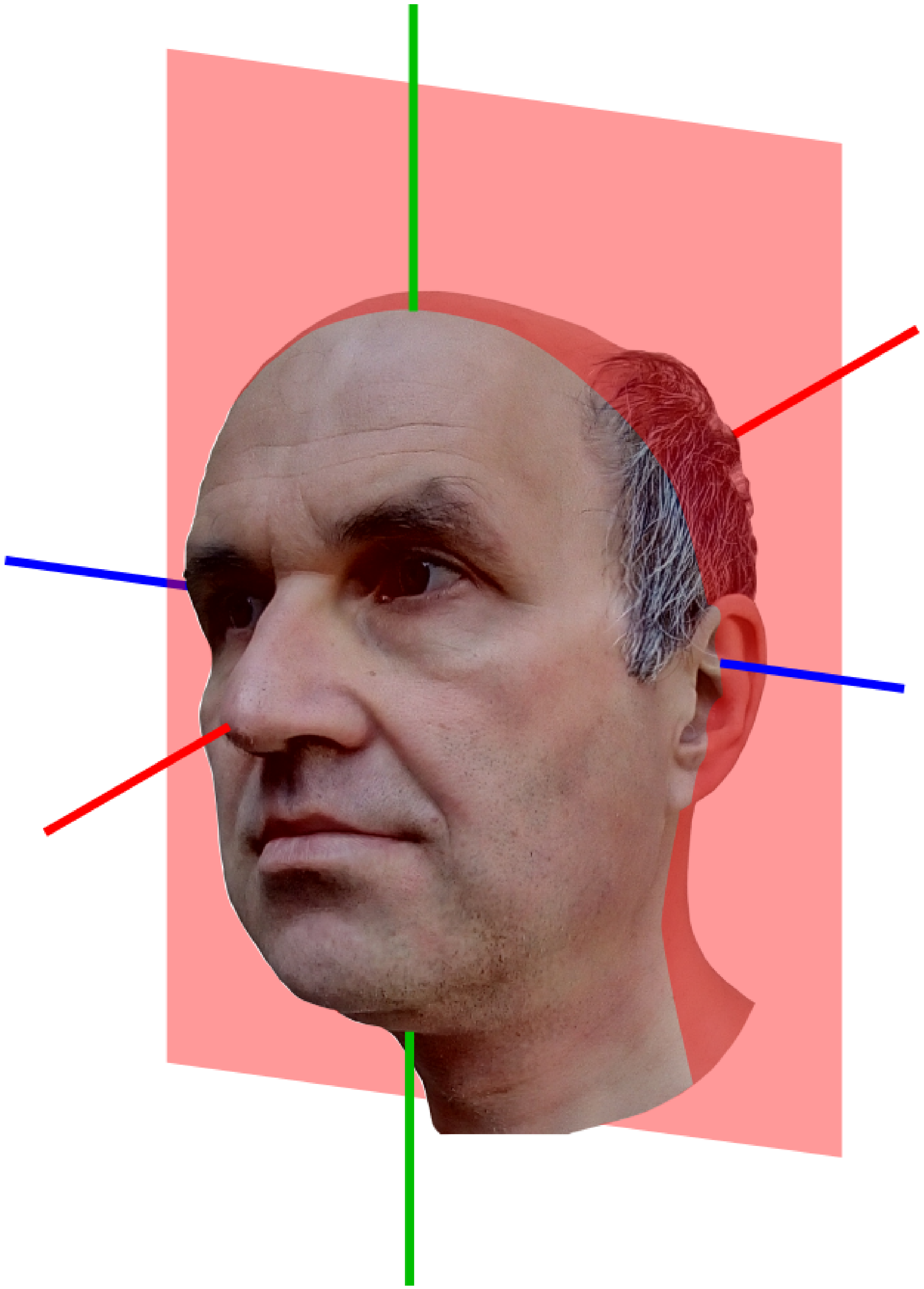
Axial slice selection by G_z



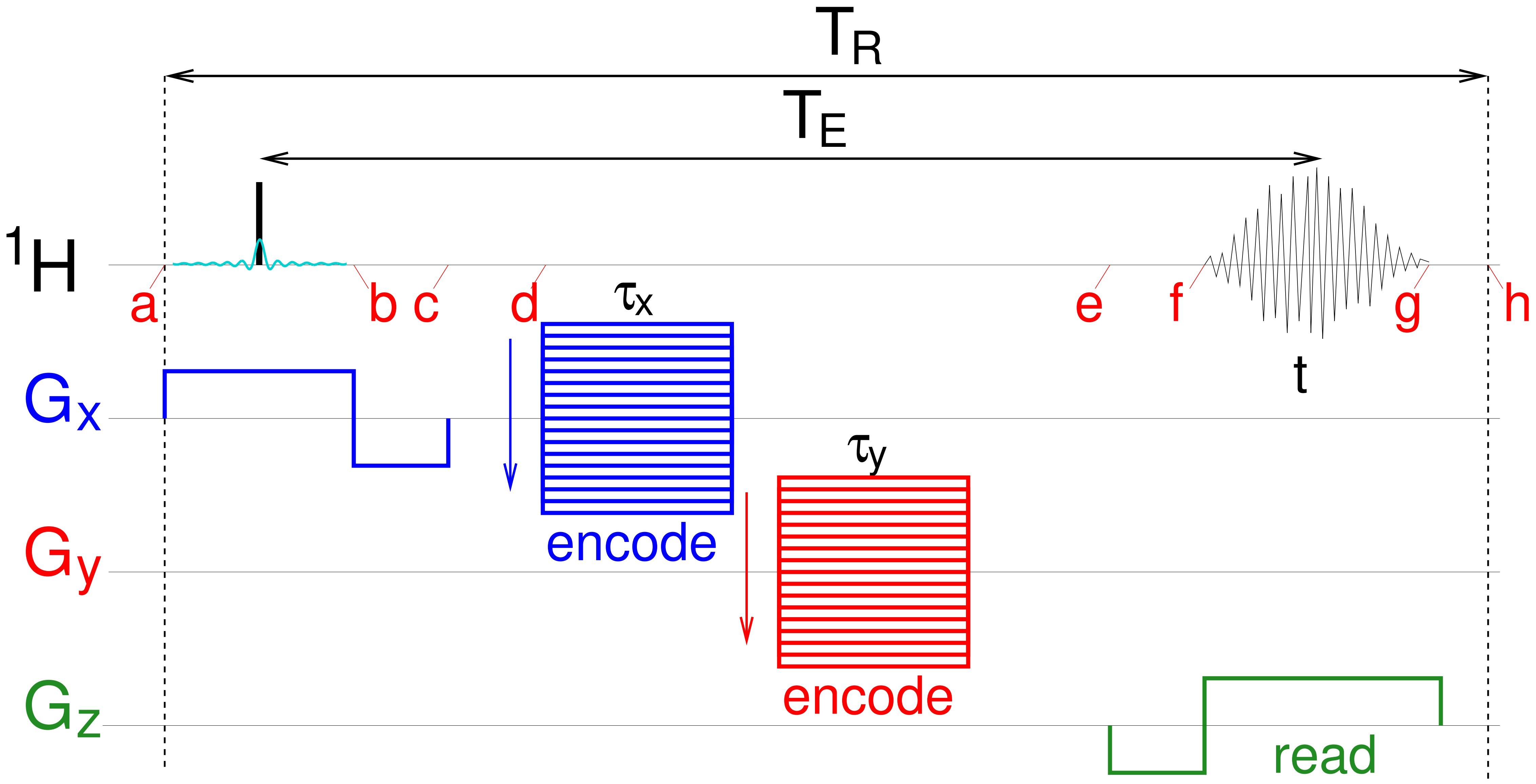
Sagittal slice selection by G_x



Coronal slice selection by G_y



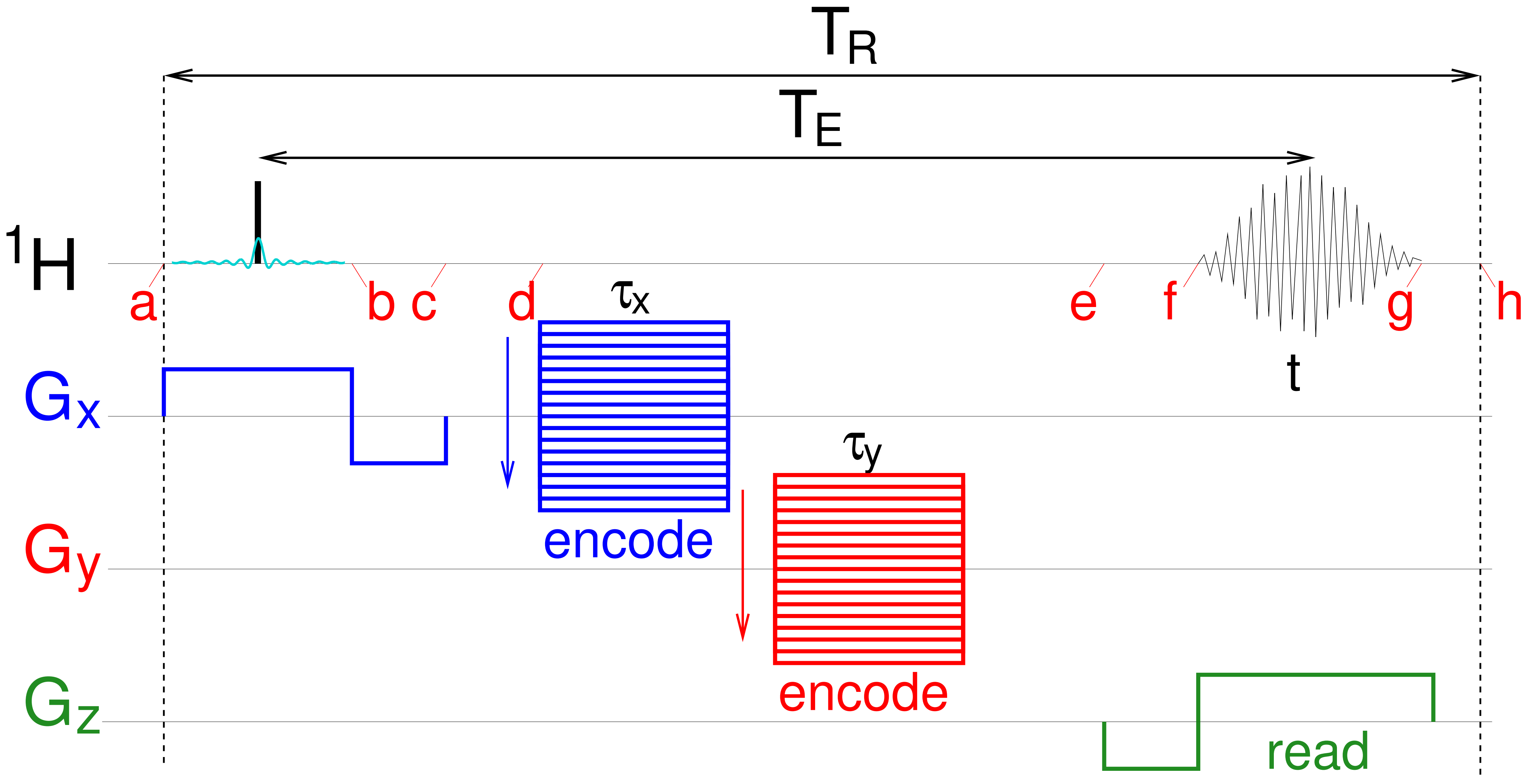
3D gradient echo imaging



High resolution in all dimensions

More time consuming

3D gradient echo imaging



Short ($\sim 10^\circ$) pulse to save time

$\Rightarrow$ several measurements before return to equilibrium

Two phase encoding gradients

$$\langle M_+\rangle(\vec{k}) \approx K' \int_V \mathcal{N}(\vec{r})\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{r}}\mathrm{d}V$$

$$\langle M_+\rangle(k_x, k_y, k_z) \approx K' \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathcal{N}(x, y, z)\mathrm{e}^{-\mathrm{i}(k_x x + k_y y + k_z z)}\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$k_x = \gamma G_x t_x = n_x \Delta k_x \quad x = j_x \Delta x$$

$$k_y = \gamma G_y t_y = n_y t \Delta k_y \quad y = j_y \Delta y$$

$$k_z = \gamma G_z t = n_z t \Delta k_z \quad z = j_z \Delta z$$

$$\Delta k_x = \gamma t_x \Delta G_x$$

$$\Delta k_y = \gamma t_y \Delta G_y$$

$$\Delta k_z = \gamma G_z \Delta t = \frac{\gamma G_z}{N_z \Delta f}$$

$$\mathcal{N}(x, y, z) = \frac{\Delta k_x \Delta k_y \Delta k_z}{K'} \sum_{n_x}\sum_{n_y}\sum_{n_z} \langle M_+\rangle \mathrm{e}^{\mathrm{i}2\pi\left(\frac{j_x \cdot n_x}{N_x} + \frac{j_y \cdot n_y}{N_y} + \frac{j_z \cdot n_z}{N_z}\right)}$$

Contrast and weighting

Contrast is more important than intensity

$$\langle M_+\rangle(k_x) = \frac{\gamma\hbar}{2}\mathrm{e}^{\mathrm{i}\Omega t}\frac{\gamma\hbar B_0}{2k_\mathrm{B}T}\mathrm{e}^{-R_2 t}\mathcal{N}(x)\mathrm{e}^{-\mathrm{i}\overbrace{\gamma G_x t x}^{\phi}}$$

$$\langle M_+\rangle(k_x) = \frac{\gamma\hbar}{2}\mathrm{e}^{\mathrm{i}\Omega t}\frac{\gamma\hbar B_0}{2k_\mathrm{B}T}\left(1-\mathrm{e}^{-R_1 T_\mathrm{R}}\right)\mathrm{e}^{-R_2 t}\mathcal{N}(x)\mathrm{e}^{-\mathrm{i}\overbrace{\gamma G_x t x}^{\phi}}$$

if not started from thermodynamic equilibrium

$$\langle M_+\rangle(\vec{k}) \propto \int\limits_V \left(1-\mathrm{e}^{-R_1 T_\mathrm{R}}\right)\,\mathrm{e}^{-R_2 T_\mathrm{E}}\,\mathcal{N}(\vec{r})\,\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{r}}\mathrm{d}V$$

- T_1 weighting: difference in $R_1 \equiv 1/T_1$, short T_R and T_E
- T_2 weighting: difference in $R_2 \equiv 1/T_2$, long T_R and T_E
- spin density weighting: difference in $\mathcal{N}$, long T_R, short T_E