Q3.11 *Inverse scattering about* $-\infty$. Find the equation for $L(x, z)$ if

$$\psi_-(x; k) = e^{-ikx} + \int_{-\infty}^{x} L(x, z) e^{-ikz} \, dz$$

is a solution of $\psi'' + \{\lambda - u(x)\}\psi = 0$; see equation (3.39). What boundary conditions must $L(x, z)$ satisfy?

Q3.12 *Poles of the transmission coefficient.*

(i) Use the identity (3.43) to prove that a^{-1} has zeros at $k = i\kappa_n$.

(ii) Use the identity (3.44), et seq., to show that $a(k)$ has *simple* poles at $k = i\kappa_n$.

Q3.13 *Integral equations.* Find the solutions of the integral equations

(i) $K(x, z) + e^{-(x+z)} + \int_{x}^{\infty} K(x, y) e^{-(y+z)} \, dy = 0;$

(ii) $\phi(x, z) + xz + \int_{0}^{1} \phi(x, y) yz \, dy = 0;$

(iii) $K(x, z) - e^{-(x+z)} - \int_{-z}^{x} K(x, y) e^{-(y+z)} \, dy = 0;$

(iv) $\phi(x) = 1 + \int_{0}^{\pi} \phi(y) \sin(x + y) \, dy.$

Q3.14 *Neumann series.* Now use a Neumann series to find the solution to Q3.13(i).

Q3.15 *Inverse scattering.* Reconstruct the potential function, $u(x)$, for which the reflection coefficient is

$$b(k) = -\beta/(\beta + ik), \qquad \beta < 0.$$

Q3.16 *Inverse scattering with zero reflection coefficient.* For the case of three discrete eigenvalues, with $b(k) = 0$ for all k, find an expression for $|A|$ (see example (ii), §3.4) so that the potential function can be written as

$$u(x) = -2 \frac{d^2}{dx^2} \log |A|.$$