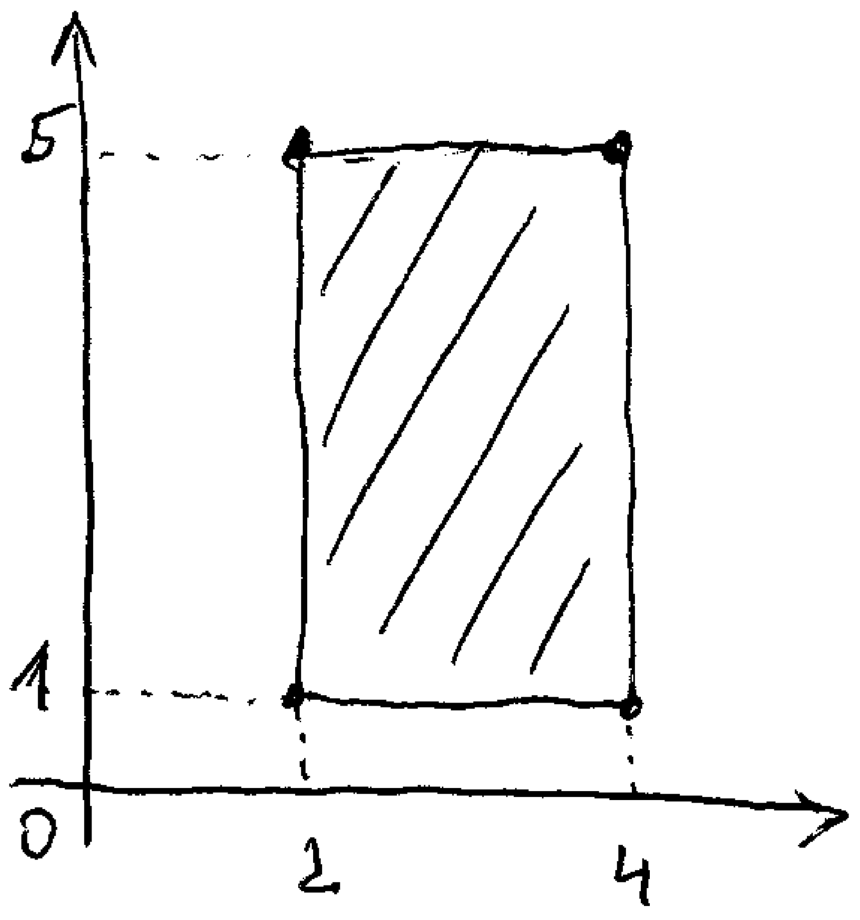
1) 

obdélník

A [2,1]

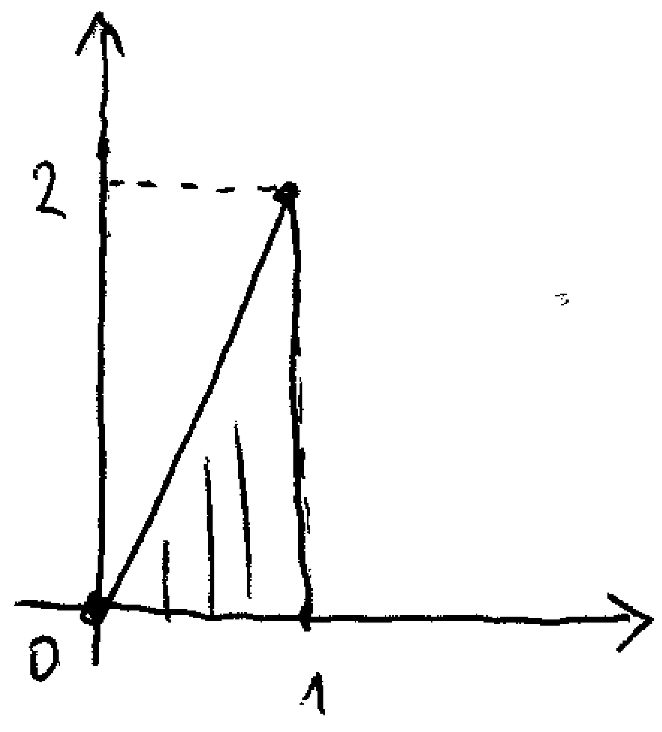
B [4,1]

C [4,5]

D [2,5]

M : $2 \le x \le 4$

$1 \le y \le 5$

2) 

trojúhelník

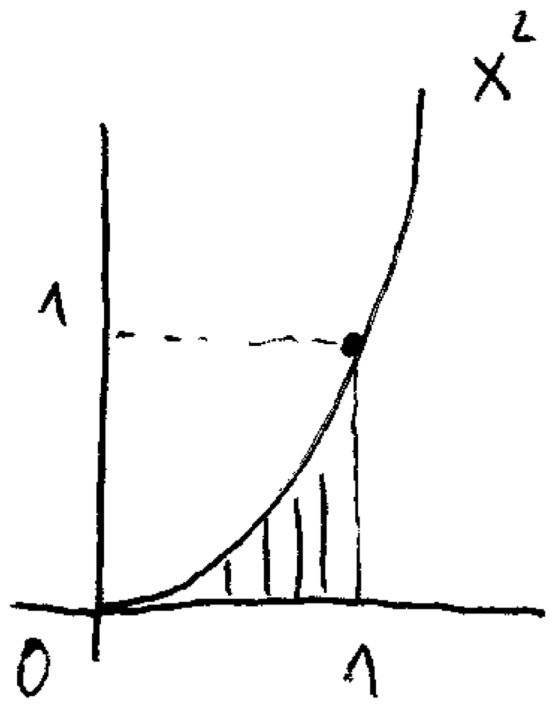
A [0,0]

B [1,0]

C [1,2]

M : $0 \le x \le 1$

$0 \le y \le 2x$

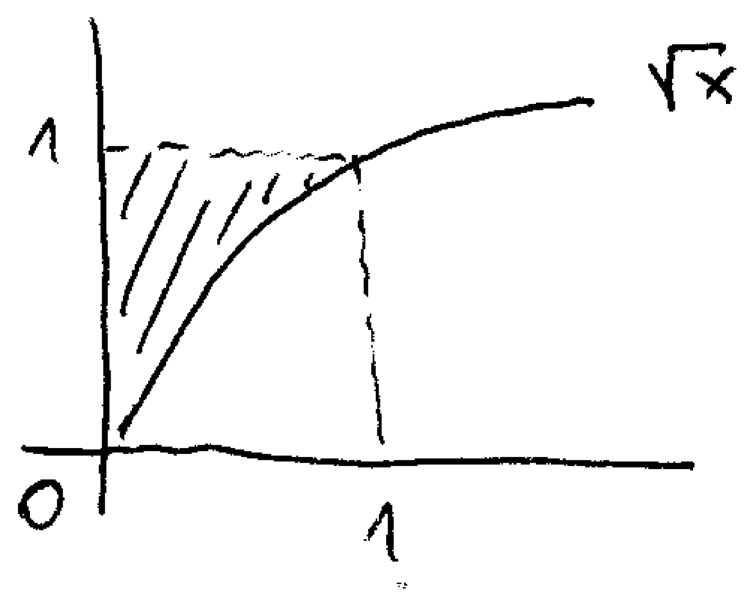
3) 

M ohraničená křivkami

$y = 0,\ x = 1,\ y = x^2$

$0 \le x \le 1$

$0 \le y \le x^2$

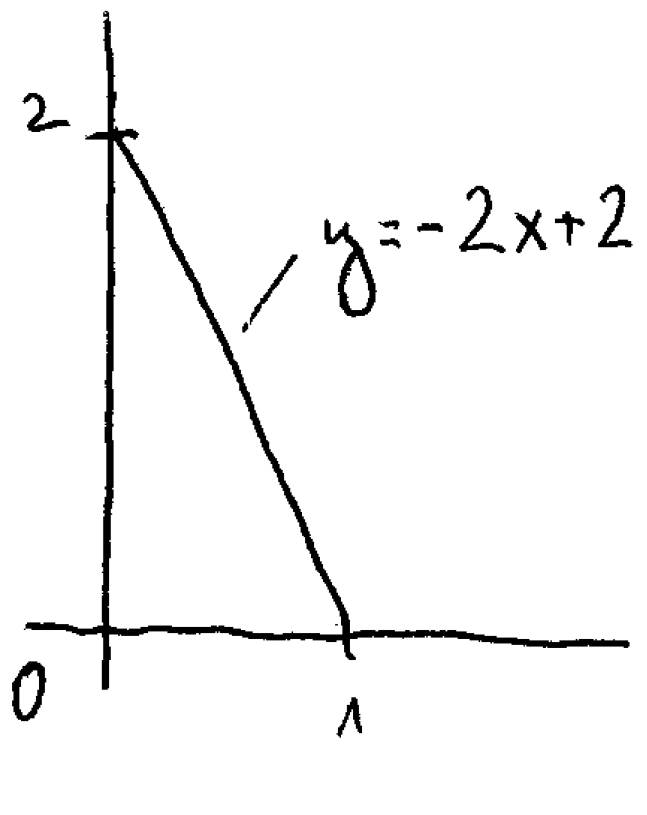
4) 

M ohraničená křivkami

$y = 1,\ x = 0,\ y = \sqrt{x}$

$0 \le x \le 1$

$\sqrt{x} \le y \le 1$

5) 

M trojúhelník

A [0,0],

B [1,0]

C [0,2]

$0 \le x \le 1$

$0 \le y \le -2x+2$

Druhá možnost

$0 \le y \le 2$

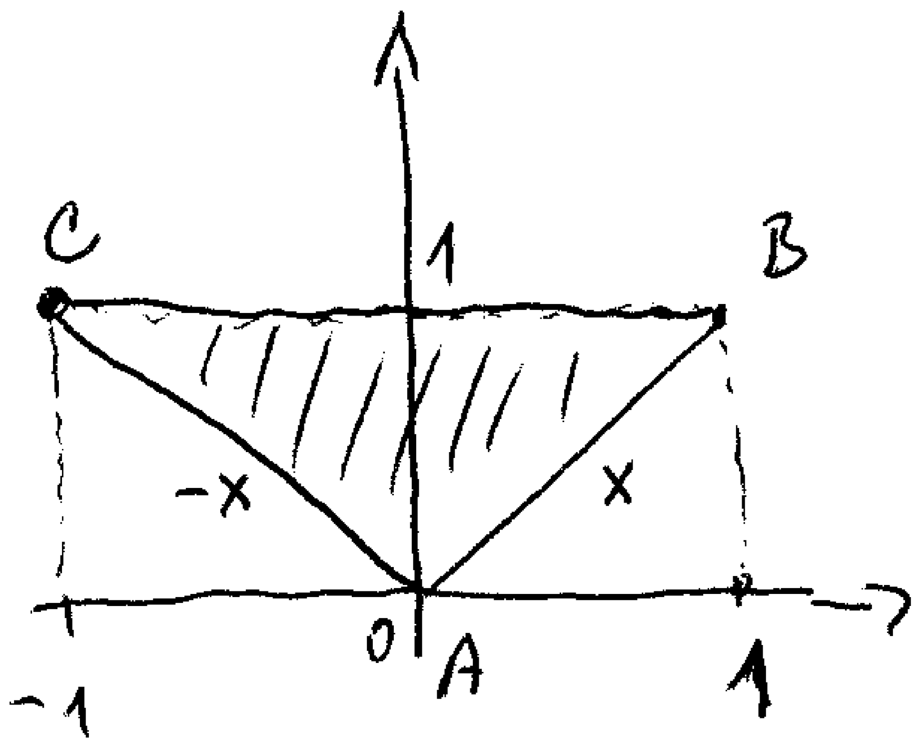
$0 \le x \le 1 - \frac{1}{2}y$

$y = -2x + 2$

$2x = 2 - y$

$x = 1 - \frac{1}{2}y$

6)



M trojúhelník

$A[0,0]$

$B[1,1]$

$C[-1,1]$

$M = M_1 \cup M_2$

$-1 \le x \le 0$ $\qquad$ $0 \le x \le 1$

$-x \le y \le 1$ $\qquad$ $x \le y \le 1$

M_1 $\qquad$ M_2

Druhá možnost

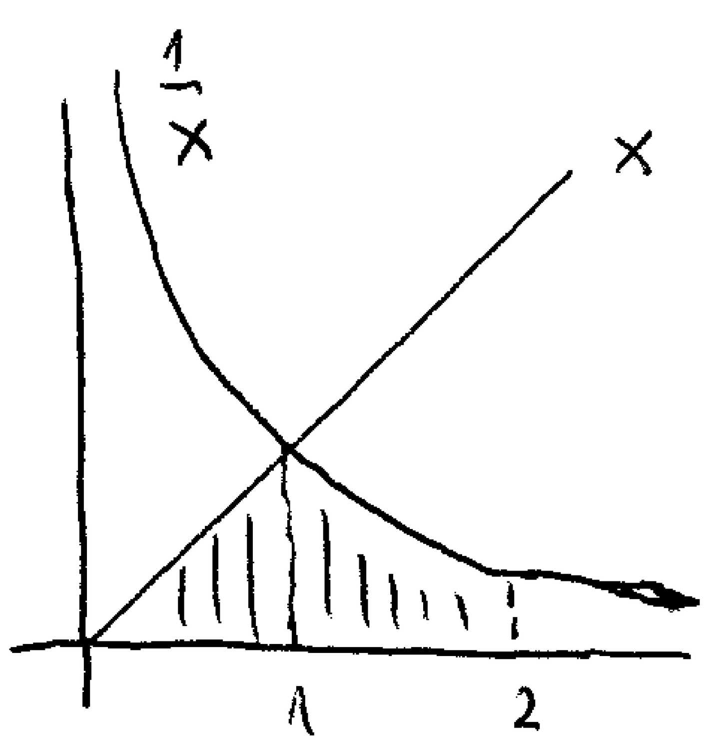
$y = -x \rightarrow x = -y$

$y = x \rightarrow x = y$

$0 \le y \le 1$

$-y \le x \le y$

7)



M ohraničená křivkami:

$y = 0,\ y = x,\ y = \frac{1}{x}$

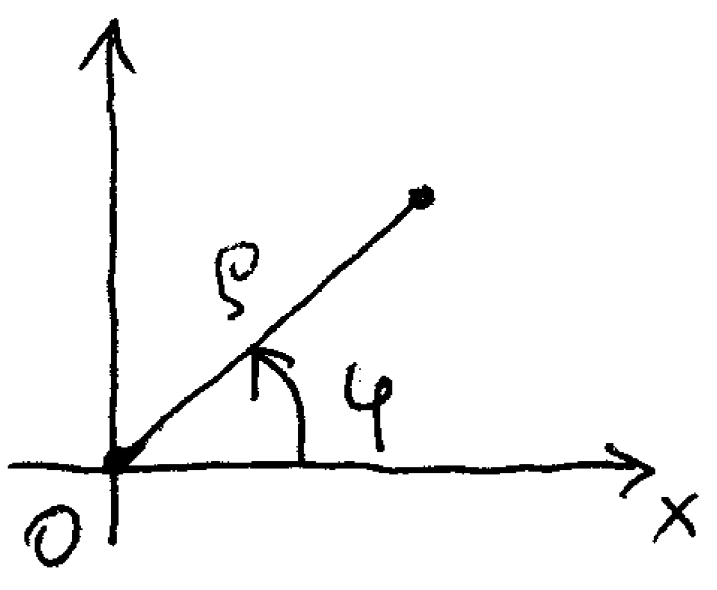
$x = 2$

$M = M_1 \cup M_2$

$0 \le x \le 1$ $\qquad$ $1 \le x \le 2$

$0 \le y \le x$ $\qquad$ $0 \le y \le \frac{1}{x}$

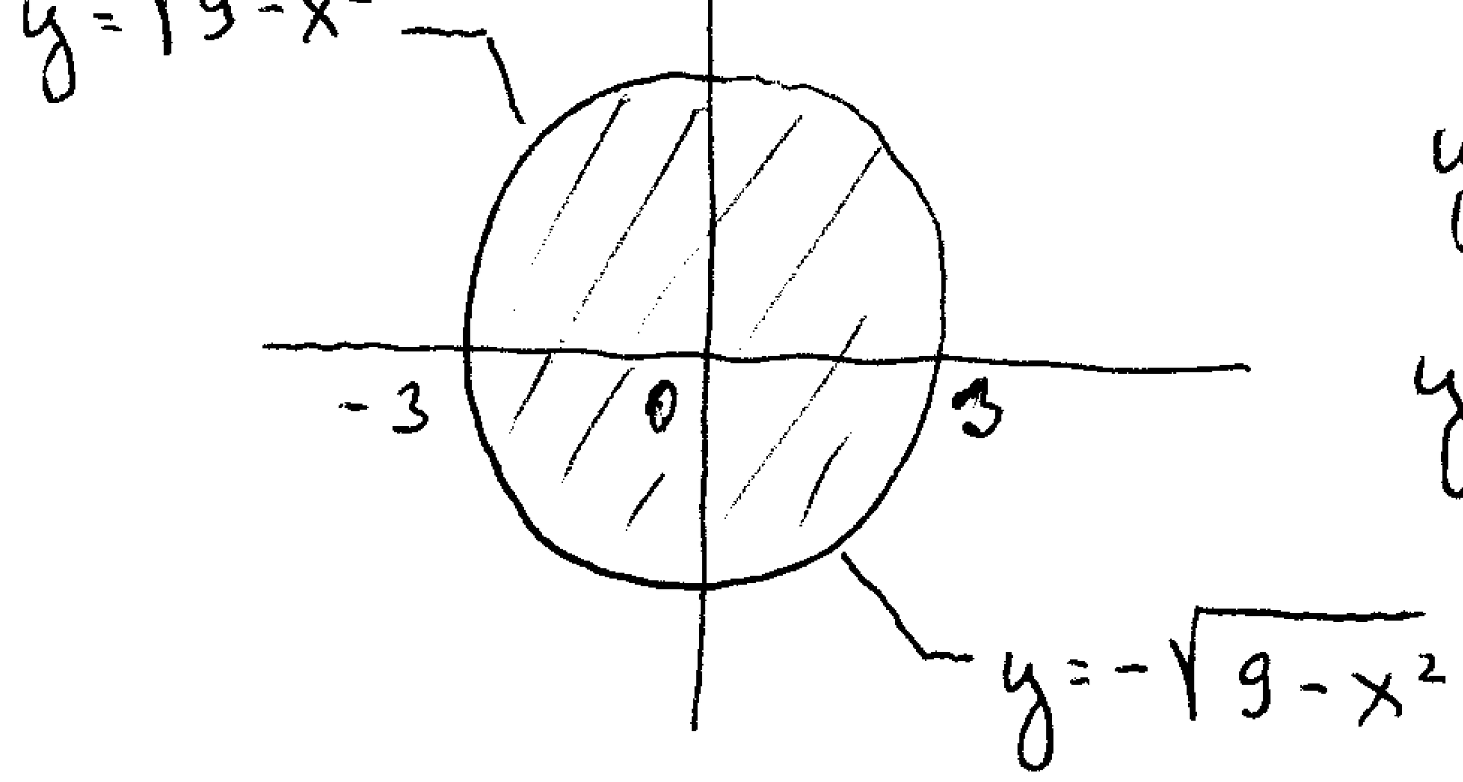
Polární souřadnice



$x = \rho \cos \varphi$

$y = \rho \sin \varphi$

8) Kruh, $S = [0,0]$, $r = 3$



$x^2 + y^2 = 9$

$y^2 = 9 - x^2$

$y = \pm\sqrt{9 - x^2}$

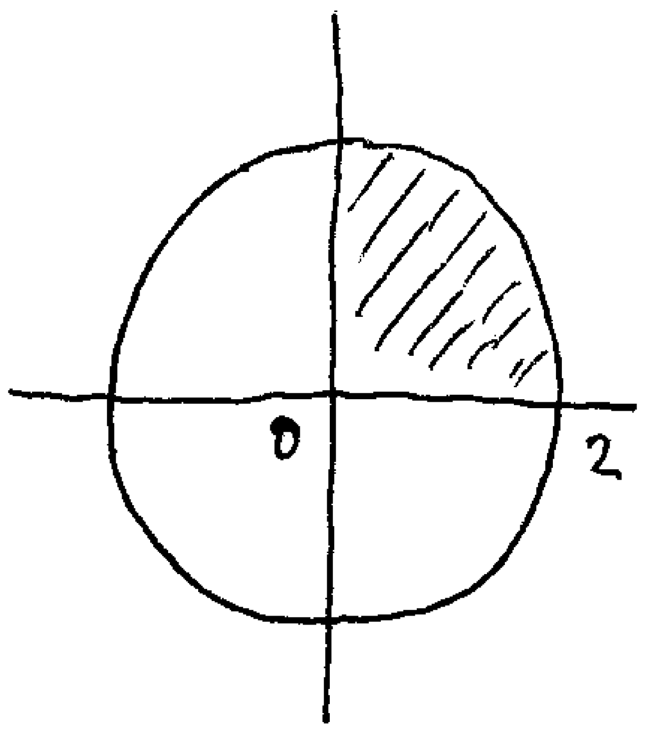
kartézské vyjádření

$$\boxed{\begin{array}{c} -3 \le x \le 3 \\ -\sqrt{9-x^2} \le y \le \sqrt{9-x^2} \end{array}}$$

$$\boxed{\begin{array}{c} 0 \le \rho \le 3 \\ 0 \le \varphi \le 2\pi \end{array}}$$

„polární" vyjádření

9)

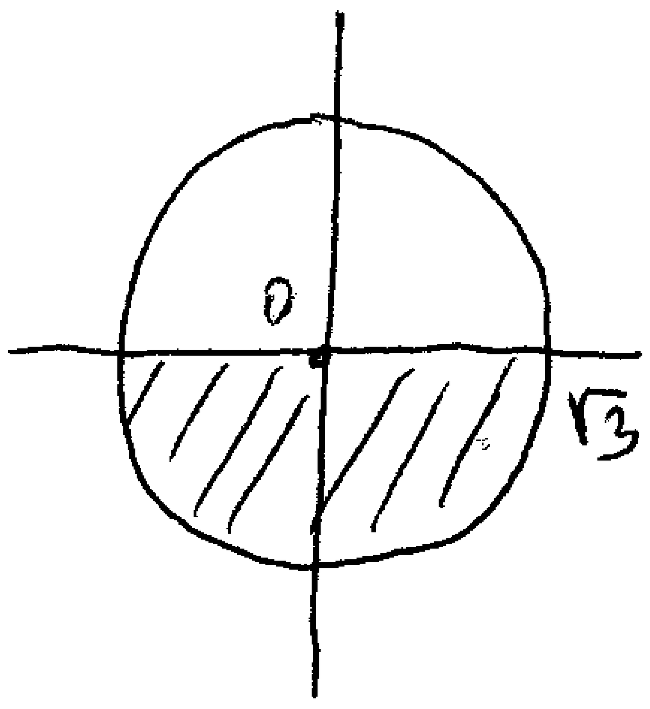


M část kruhu $x^2+y^2 \le 4$, pro $x \ge 0$, $y \ge 0$

$0 \le \varrho \le 2$

$0 \le \varphi \le \frac{\pi}{2}$

10)

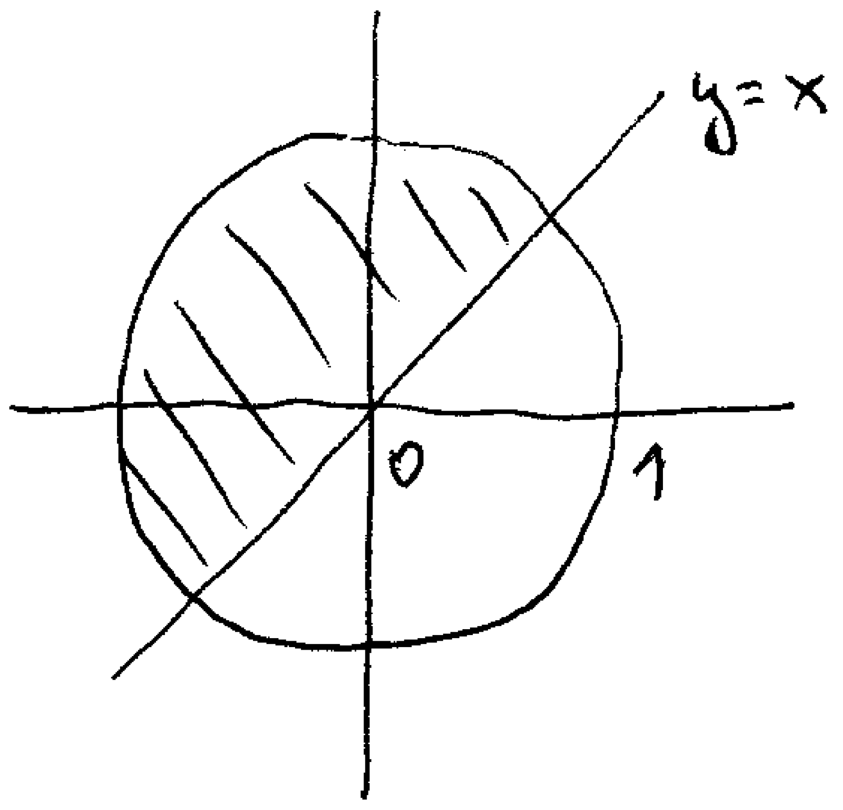


M část kruhu $x^2+y^2 \le 3$, pro $y \le 0$

$0 \le \varrho \le \sqrt{3}$

$\pi \le \varphi \le 2\pi$

11)

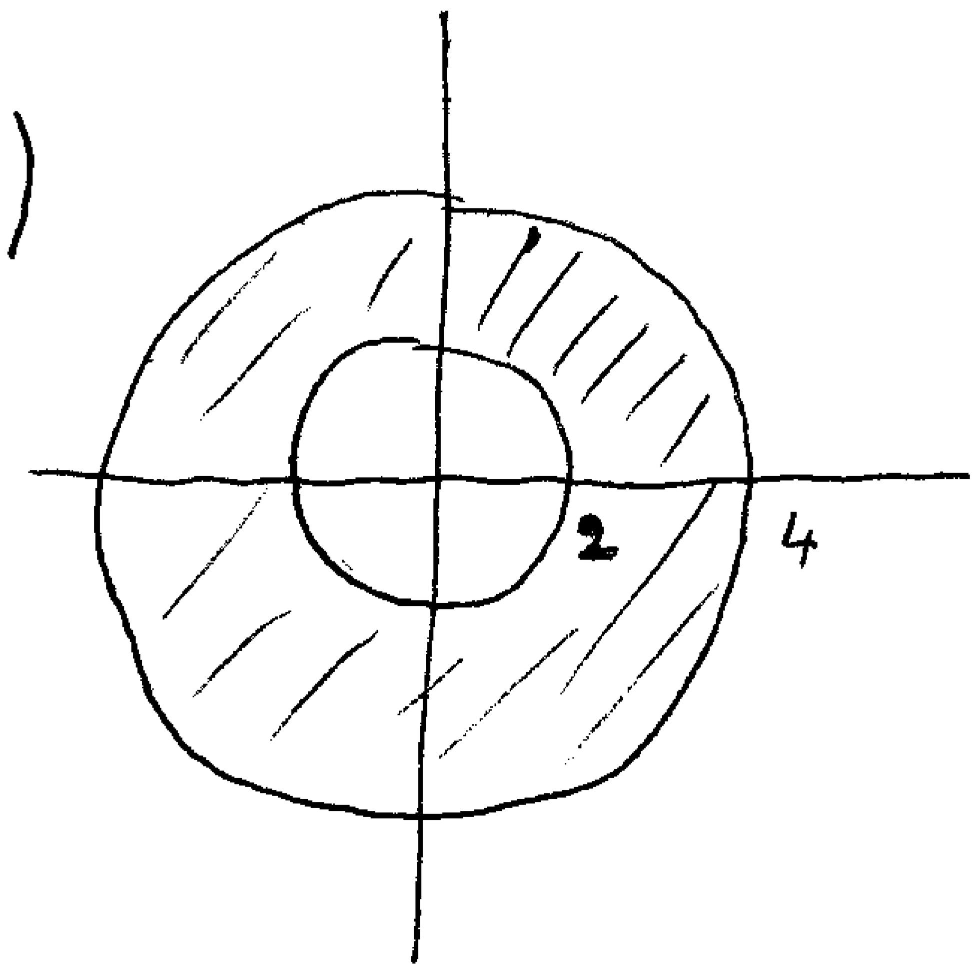


M je ohraničena křivkami $x^2+y^2=1$, $y=x$ a platí $y \ge x$

$0 \le \varrho \le 1$

$\frac{\pi}{4} \le \varphi \le \frac{5}{4}\pi$

12)

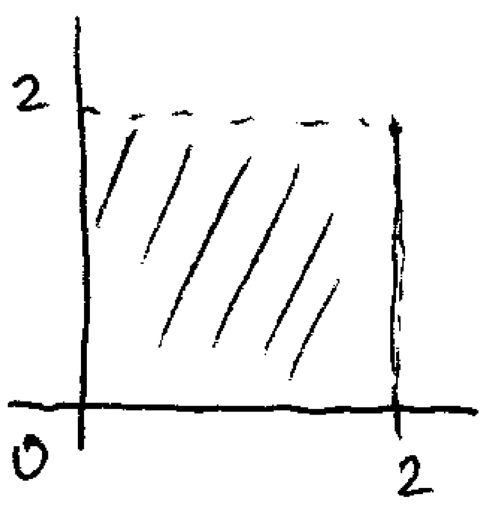


$2 \le \varrho \le 4$

$0 \le \varphi \le 2\pi$

Vypočtěte dvojné integrály

1) $\iint_M x y^2 \, dx \, dy$, kde M je čtverec s vrcholy
$A=[0,0]$, $B=[2,0]$, $C=[2,2]$, $D=[0,2]$



M: $0 \leq x \leq 2$
$0 \leq y \leq 2$

$$\iint_M x y^2 \, dx \, dy = \int_0^2 \left(\int_0^2 x y^2 \, dy \right) dx = \int_0^2 \left[x \frac{y^3}{3} \right]_0^2 dx = \int_0^2 \left(\frac{8}{3} x - 0 \right) dx$$

$$= \left[\frac{8}{3} \frac{x^2}{2} \right]_0^2 = \frac{4}{3} \cdot 4 - \frac{4}{3} \cdot 0 = \frac{16}{3}$$

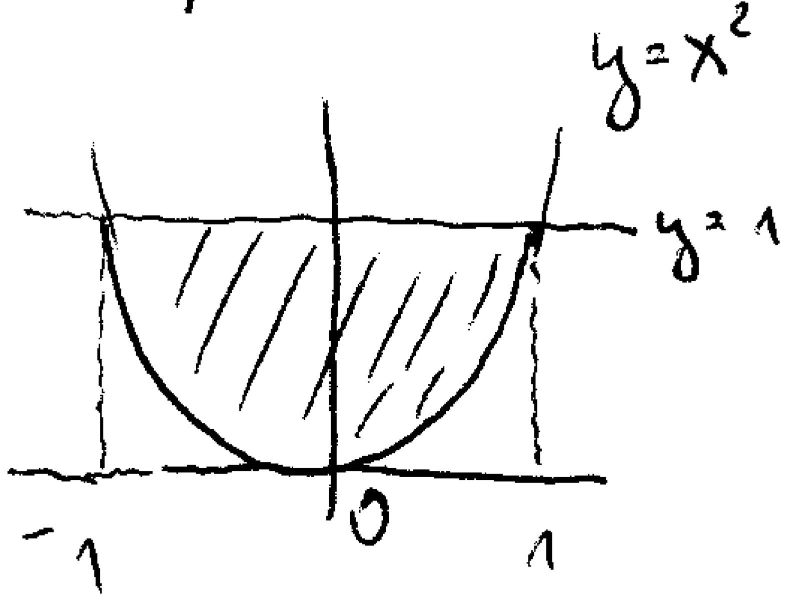
$$\iint_M x y^2 \, dx \, dy = \int_0^2 \left(\int_0^2 x y^2 \, dx \right) dy = \int_0^2 \left[\frac{x^2}{2} y^2 \right]_0^2 dy = \int_0^2 \left[2y^2 - 0 \right] dy$$

$$= \left[2 \frac{y^3}{3} \right]_0^2 = \frac{16}{3}$$

$$\iint_M x y^2 \, dx \, dy = \int_0^2 x \, dx \cdot \int_0^2 y^2 \, dy = \left[\frac{x^2}{2} \right]_0^2 \cdot \left[\frac{y^3}{3} \right]_0^2 = (2 - 0) \cdot \left(\frac{8}{3} - 0 \right) = \frac{16}{3}$$

---

2) $\iint_M x^2 y \, dx \, dy$; M ohraničená křivkami $y = x^2$, $y = 1$

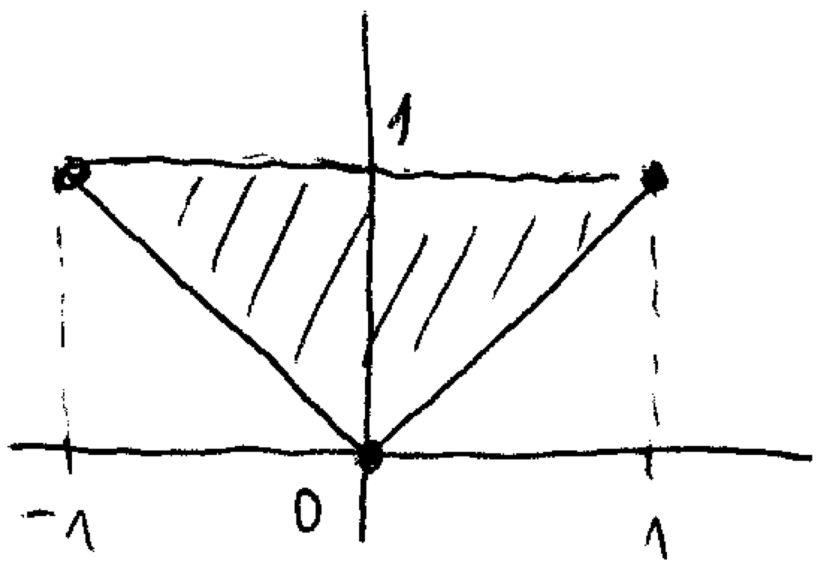


M: $-1 \leq x \leq 1$
$x^2 \leq y \leq 1$

$$\iint_M x^2 y \, dx \, dy = \int_{-1}^1 \left(\int_{x^2}^1 x^2 y \, dy \right) dx = \int_{-1}^1 \left[x^2 \frac{y^2}{2} \right]_{x^2}^1 dx = \int_{-1}^1 \left(\frac{1}{2} x^2 - \frac{1}{2} x^6 \right) dx$$

$$= \left[\frac{x^3}{6} - \frac{x^7}{14} \right]_{-1}^1 = \left(\frac{1}{6} - \frac{1}{14} \right) - \left(\frac{-1}{6} + \frac{1}{14} \right) = \frac{4}{21}$$

3) $\iint_M x\,dx\,dy$, M je trojúhelník s vrcholy $A[0,0]$, $B[1,1]$, $C=[-1,1]$



a) $\iint_M x\,dx\,dy = \iint_{M_1} x\,dx\,dy + \iint_{M_2} x\,dx\,dy$

$M_1: \; -1 \le x \le 0$, $\; -x \le y \le 1$ $\qquad$ $M_2: \; 0 \le x \le 1$, $\; x \le y \le 1$

$$\iint_{M_1} x\,dx\,dy = \int_{-1}^{0} \left(\int_{-x}^{1} x\,dy \right) dx = \int_{-1}^{0} [xy]_{-x}^{1}\,dx = \int_{-1}^{0} (x + x^2)\,dx = \left[\frac{x^2}{2} + \frac{x^3}{3} \right]_{-1}^{0} =$$

$$= 0 - \left(\frac{1}{2} - \frac{1}{3} \right) = -\frac{1}{6}$$

$$\iint_{M_2} x\,dx\,dy = \int_{0}^{1} \left(\int_{x}^{1} x\,dy \right) dx = \int_{0}^{1} [xy]_{x}^{1}\,dx = \int_{0}^{1} (x - x^2)\,dx = \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_{0}^{1} = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

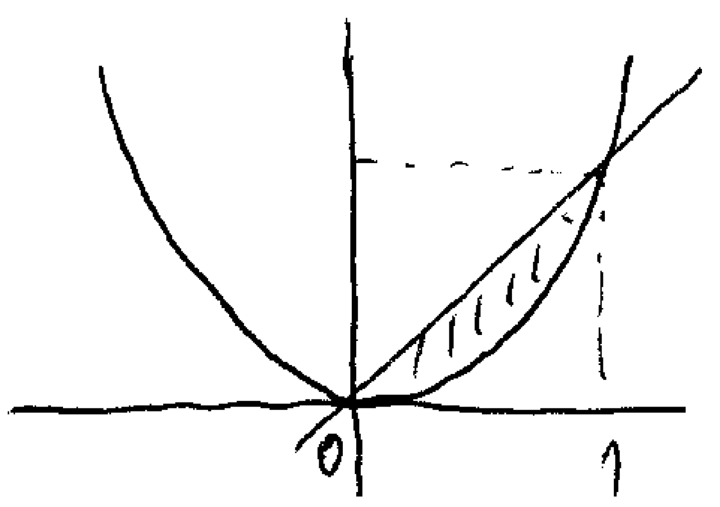
$$\iint_M x\,dx\,dy = -\frac{1}{6} + \frac{1}{6} = 0$$

b) $M: \; 0 \le y \le 1$, $\; -y \le x \le y$

$$\iint_M x\,dx\,dy = \int_0^1 \left(\int_{-y}^{y} x\,dx \right) dy = \int_0^1 \left[\frac{x^2}{2} \right]_{-y}^{y} dx$$

$$= \int_0^1 \left[\frac{y^2}{2} - \frac{(-y)^2}{2} \right] dx = \int_0^1 0\,dx = 0$$

4) $\iint_M xy^2\,dx\,dy$, M je ohraničená křivkami $y = x$, $y = x^2$



$0 \le x \le 1$
$x^2 \le y \le x$

$$\iint_M xy^2\,dx\,dy = \int_0^1 \left(\int_{x^2}^{x} xy^2\,dy \right) dx = \int_0^1 \left[x \frac{y^3}{3} \right]_{x^2}^{x} dx =$$

$$= \int_0^1 \left(\frac{x^4}{3} - \frac{x^7}{3} \right) dx = \left[\frac{x^5}{15} - \frac{x^8}{24} \right]_0^1 = \frac{1}{15} - \frac{1}{24} = \frac{1}{40}$$