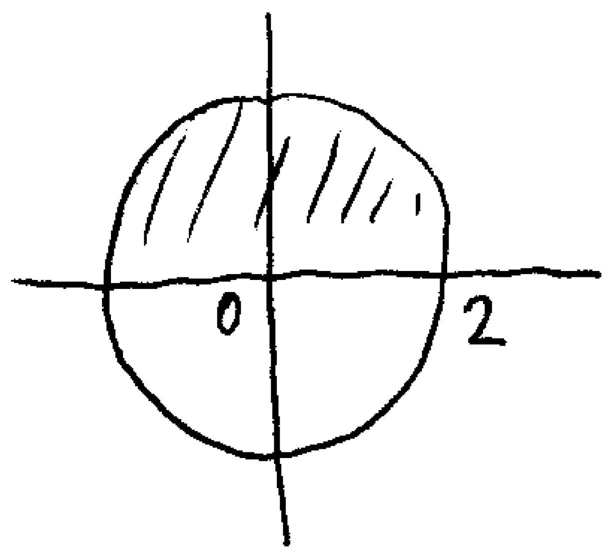
1) Vypočtěte $\iint\limits_M x\,dx\,dy$, kde $M:\ x^2+y^2\le 4$, $y\ge 0$



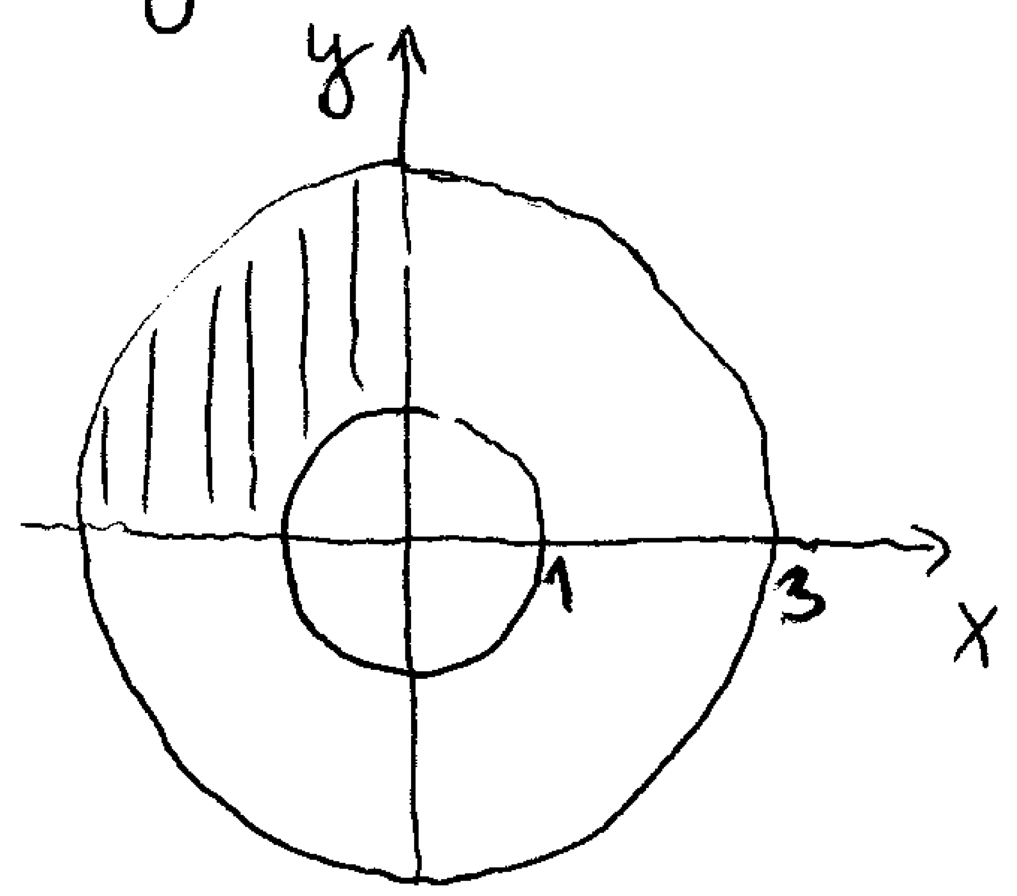
$0\le\varrho\le 2$ $\qquad x=\varrho\cos\varphi$

$0\le\varphi\le\pi$

$$\iint\limits_M x\,dx\,dy = \int_0^{\pi}\left(\int_0^2 \varrho\cos\varphi\cdot\varrho\,d\varrho\right)d\varphi = \int_0^{\pi}\cos\varphi\,d\varphi\cdot\int_0^2\varrho^2\,d\varrho =$$

$$= \left[\sin\varphi\right]_0^{\pi}\cdot\left[\frac{\varrho^3}{3}\right]_0^2 = (\sin\pi-\sin 0)\cdot\left(\frac{8}{3}-0\right) = 0$$

2) Vypočtěte $\iint\limits_M (x^2+y^2)\,dx\,dy$, kde $M:\ 1\le x^2+y^2\le 9$, $y\ge 0$, $x\le 0$



$1\le\varrho\le 3$

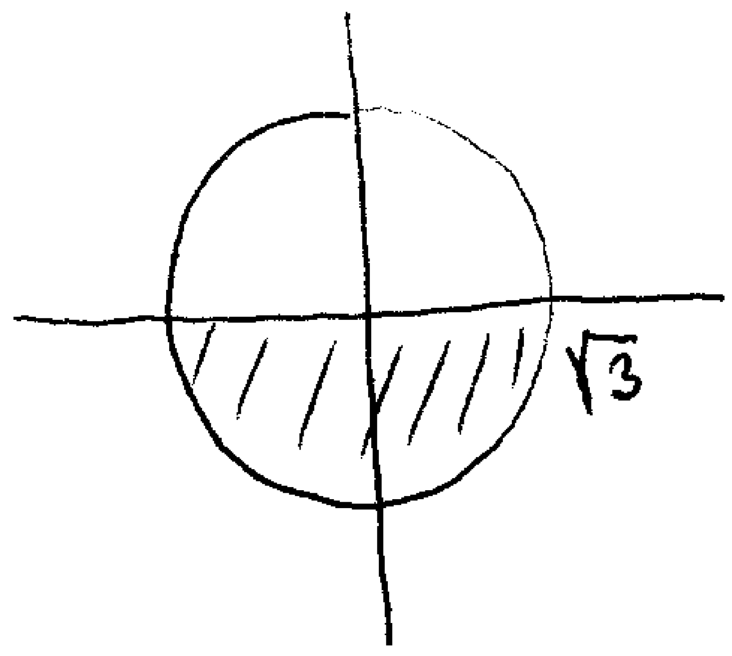
$\frac{\pi}{2}\le\varphi\le\pi$

$$\iint\limits_M (x^2+y^2)\,dx\,dy = \int_{\frac{\pi}{2}}^{\pi}\left(\int_1^3 (\varrho^2\cos^2\varphi+\varrho^2\sin^2\varphi)\,\varrho\,d\varrho\right)d\varphi =$$

$$\int_{\frac{\pi}{2}}^{\pi}\left(\int_1^3 \varrho^3(\cos^2\varphi+\sin^2\varphi)\,d\varrho\right)d\varphi = \int_{\frac{\pi}{2}}^{\pi}\left(\int_1^3\varrho^3\,d\varrho\right)d\varphi = \int_{\frac{\pi}{2}}^{\pi}d\varphi\cdot\int_1^3\varrho^3\,d\varphi$$

$$= \left[\varphi\right]_{\frac{\pi}{2}}^{\pi}\cdot\left[\frac{\varrho^4}{4}\right]_1^3 = \left(\pi-\frac{\pi}{2}\right)\cdot\left(\frac{81}{4}-\frac{1}{4}\right) = \frac{\pi}{2}\cdot\frac{80}{4} = 10\pi$$

3) Vypočítejte $\iint_M (x+y)\,dx\,dy$, kde M: $x^2+y^2 \le 3$, $y \le 0$



$0 \le \rho \le \sqrt{3}$

$\pi \le \varphi \le 2\pi$

$$\iint_M (x+y)\,dx\,dy = \int_{\pi}^{2\pi} \left(\int_0^{\sqrt{3}} (\rho\cos\varphi + \rho\sin\varphi)\,\rho\,d\rho \right) d\varphi =$$

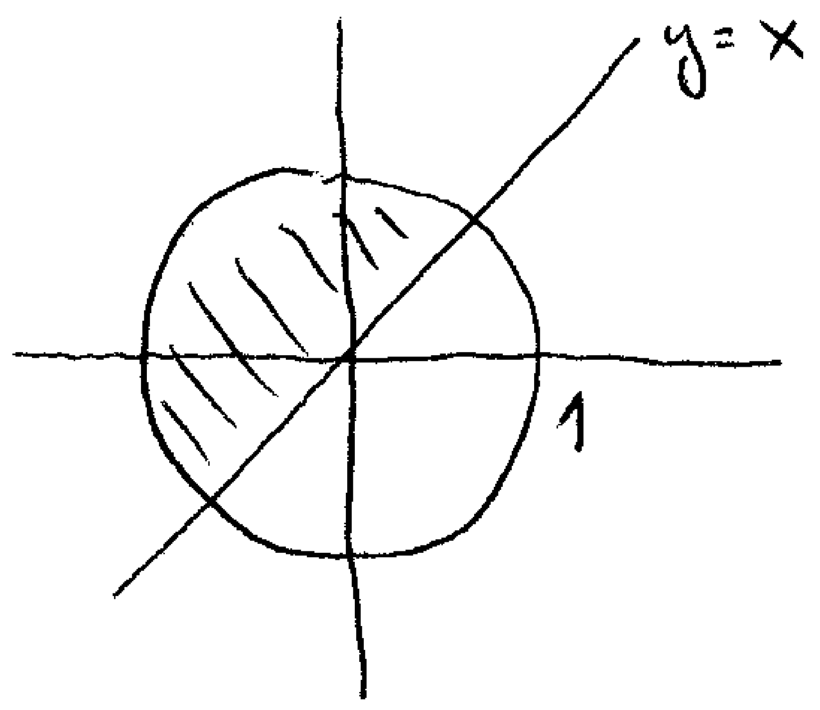
$$= \int_{\pi}^{2\pi} \left(\int_0^{\sqrt{3}} \rho^2 (\cos\varphi + \sin\varphi)\,d\rho \right) d\varphi = \int_{\pi}^{2\pi} (\cos\varphi + \sin\varphi)\,d\varphi \cdot \int_0^{\sqrt{3}} \rho^2\,d\rho =$$

$$= \left[\sin\varphi - \cos\varphi\right]_{\pi}^{2\pi} \cdot \left[\frac{\rho^3}{3}\right]_0^{\sqrt{3}} = \left[(\sin 2\pi - \cos 2\pi) - (\sin\pi - \cos\pi)\right] \cdot \left(\frac{3\sqrt{3}}{3} - 0\right)$$

$$= \left[(0-1) - (0-(-1))\right] \cdot \sqrt{3} = -2\sqrt{3}$$

4) Transformujte integrál $\iint_M e^{-x^2-y^2}\,dx\,dy$, kde M: $x^2+y^2 \le 1$, $y \ge x$

do polárních souřadnic a dále nepočítejte.



$0 \le \rho \le 1$

$\frac{\pi}{4} \le \varphi \le \frac{5}{4}\pi$

$$-x^2-y^2 = -(x^2+y^2) =$$
$$= -(\rho^2\cos^2\varphi + \rho^2\sin^2\varphi)$$
$$= -\rho^2(\cos^2\varphi + \sin^2\varphi) = -\rho^2$$

$$\iint_M e^{-x^2-y^2}\,dx\,dy = \int_{\frac{\pi}{4}}^{\frac{5}{4}\pi} \left(\int_0^1 e^{-\rho^2}\,\rho\,d\rho \right) d\varphi$$