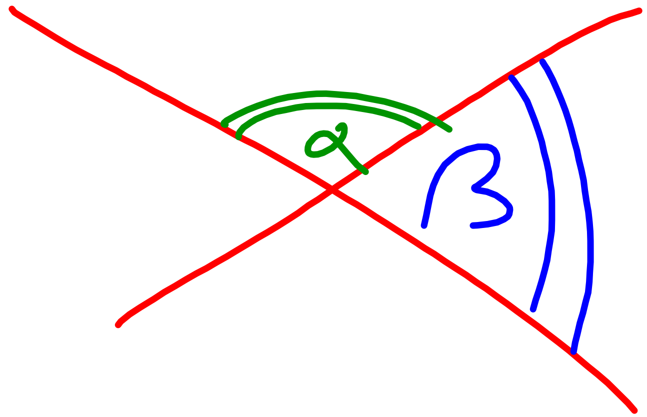


dub 2-16:12

$$\operatorname{dist}(M,N) = \operatorname{dist}(p-q, \operatorname{Dir} M + \operatorname{Dir} N)!$$

$$\| \; \inf\{\|x-y\| : x\in M, y\in N\} =$$

$x = p + u,\ u \in \operatorname{Dir} M$

$y = q + v,\ v \in \operatorname{Dir} N$

$$= \inf\{\|(p+u)-(q+v)\| : u \in \operatorname{Dir} M,\ v \in \operatorname{Dir} N\}$$

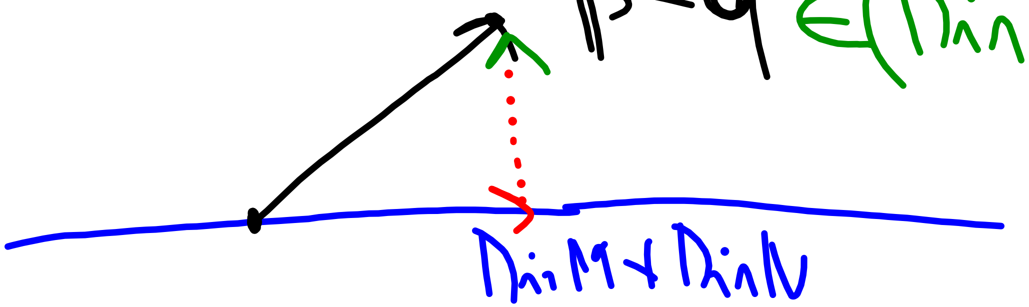
$$= \inf\{\|(p-q)-(v-u)\| : u \in \operatorname{Dir} M,\ v \in \operatorname{Dir} N\}$$

$w \in \operatorname{Dir} M + \operatorname{Dir} N$

(a) $p \in M,\ q \in N$ mají příčku M a $N \iff p - q \in (\mathrm{Din}\,M + \mathrm{Din}\,N)^{\perp}$

$$\|p-q\| \le \inf\{\|x-y\|;\ x \in M,\ y \in N\} = \inf\{\|(p-q)-w\| : w \in \mathrm{Din}\,M + \mathrm{Din}\,N\}$$

$p - q \in (\mathrm{Din}\,M + \mathrm{Din}\,N)^{\perp}$



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(b) $\forall p \in M, q \in N$
$\forall u \in \mathrm{Dir}\, M, v \in \mathrm{Dir}\, N$
$p+u$ a $q+v$ krát přímek M a N $\overset{2.}{\Longleftrightarrow}$ $(v-u)$ je kolmý průmět $p-q$ do $\mathrm{Dir}\, M + \mathrm{Dir}\, N$.

$$\Updownarrow$$

$$(p+u)-(q+v) \in (\mathrm{Dir}\, M + \mathrm{Dir}\, N)^{\perp}$$

$$\| \qquad \in \mathrm{Dir}\, M + \mathrm{Dir}\, N$$

$$(p-q)-(v-u) = w$$

$$(p-q) = (v-u) + w$$

(c) $\exists\, p \in M,\ q \in N$, které tvoří příčku M a N

$(p - q)_{\mathrm{Dir}\,M + \mathrm{Dir}\,N} = w$ ∈ Dir M + Dir N

kde má pnajko

$w = \bar{u} + \bar{v}$, $\bar{u} \in \mathrm{Dir}\,M$

$v = \bar{v}$, $u = -\bar{u}$, $\bar{v} \in \mathrm{Dir}\,N$

$w = \underline{v} - \underline{u}$, $v \in \mathrm{Dir}\,N$, $u \in \mathrm{Dir}\,M$

$$M \cap N \neq \emptyset \iff \mathrm{dist}(M,N) = 0$$

$\Rightarrow$: $p \in M \cap N$ $\quad \mathrm{dist}(M,N) \leq \|p - p\| = 0$

$\Leftarrow$: Nechť $\mathrm{dist}(M,N) = 0$

$\exists\, p \in M,\ q \in N \quad \|p - q\| = 0$

$\Rightarrow p = q.$

$$\operatorname{dist}(M, N) = \operatorname{dist}\left(p - q, \left[u_1, \dots, u_m, v_1, \dots, v_n\right]\right)$$

$$S = \underbrace{\left[u_1, \dots, u_m, v_1, \dots, v_n\right]}_{\alpha}$$

$$G(\alpha) \cdot c = \langle p - q, \alpha^T \rangle$$

$$(p-q)_S = \underbrace{\sum_{i=1}^{m} c_i u_i}_{\in U} + \underbrace{\sum_{j=1}^{n} c_{m+j} v_j}_{\in V}$$

$p = (1, 1, 2, -2)^T \in M$

$q = (0, 0, 5, -1)^T \in N$

$p - q = (1, 1, -3, -1)^T$

$\langle p - q, e_1 + e_2 \rangle = 1 + 1 = \underline{\underline{2}}.$

$\langle e_1 + e_2, e_1 + e_2 \rangle = 1 + 1 = 2$

$\vdots$

$\downarrow$

dub 2-17:37

$$\left(\begin{array}{cccc|c} \underline{2} & 2 & 1 & 1 & \underline{2} \\ 2 & 3 & 1 & 2 & -1 \\ 1 & 1 & 2 & 2 & 0 \\ 1 & 2 & 2 & 3 & -3 \end{array}\right) \sim \cdots$$

$$\cdots \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 13/3 \\ 0 & 1 & 0 & 1 & -3 \\ 0 & 0 & 1 & 1 & -\frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$c_v = \left(13/3 + v,\ -3 - v,\ -\tfrac{2}{3} - v,\ v\right)$$

$$u_1 = (4/3, 4/3, -3-1, 0)^T$$

$$v_1 = \left(0, -\frac{2}{3}, 1, -\frac{5}{3}\right)^T$$

$$u_1 + v_1 = \left(4/3, 2/3, -3, -\frac{5}{3}\right)^T$$

$$p_1 = (1,1,2,-2)^T - u_1 = \left(-\frac{1}{3}, -\frac{1}{3}, 5+1, -2\right)$$

$$q_1 = (0,0,5,-1) + v_1 = \left(0, -\frac{2}{3}, 5+1, -5/3\right)$$

$$\|p_1 - q_1\| = \left\|\left(-\frac{1}{3}, \frac{1}{3}, 0, -\frac{1}{3}\right)\right\| = \sqrt{3 \cdot \frac{1}{9}} = \frac{1}{\sqrt{3}}.$$

$$\measuredangle(S,T) = \inf\{\measuredangle(x,T) : \underset{x \neq 0}{x} \in S \cap (S \cap T)^{\perp}$$

$x \neq 0 \qquad x_T \in T$

$x - x_T \in T^{\perp} \qquad \underline{x \in S \cap (S \cap T)^{\perp}}$

$$x_T = x - (x - x_T) \in (S \cap T)^{\perp} \cap T$$

$\in (S \cap T)^{\perp} \qquad \in T^{\perp}$

$\geq$

$$\measuredangle(S,T) = \inf\{\measuredangle(x,y),\ \overset{\neq 0}{x} \in S \cap (S \cap T)^{\perp},\ y \in T \cap (S \cap T)^{\perp},$$

$$\Rightarrow \measuredangle(S,T) \leq \inf\{\measuredangle(x,T) ;\ \underset{0 \neq}{x} \in S \cap (S \cap T)^{\perp}$$

$$\inf\left\{\arccos\frac{\|x_T\|}{\|x\|}\ ;\ \underset{\neq 0}{x}\in S\cap(S\cap T)^{\perp}\right\}$$

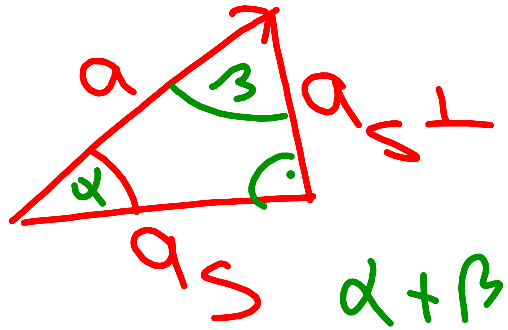
$$\inf\left\{\measuredangle(x,y)\ ;\ \underset{\neq 0}{x}\in S\cap(S\cap T)^{\perp},\ \underset{\neq 0}{y}\in T\cap(S\cap T)^{\perp}\right\}$$

$$\frac{|\langle x,y\rangle|}{\|x\|\,\|y\|}\leq\frac{\|x_T\|}{\|x\|}$$

dub 2-18:39

$$\sphericalangle([a]^\perp, S) = \frac{\pi}{2} - \sphericalangle(a, S) = $$

$$= \sphericalangle(a, S^\perp)$$



$\alpha + \beta = \frac{\pi}{2}$

$$a_S = 0 \Leftrightarrow a \in S^\perp$$

$$\Leftrightarrow [a] \le S^\perp \Leftrightarrow S = S^{\perp\perp} \le [a]^\perp$$

$$[a]^\perp \le S \Leftrightarrow S^\perp \le [a]^{\perp\perp} = [a]$$

$$\sphericalangle([a]^\perp, S) = 0 \qquad \sphericalangle(a, S) = \frac{\pi}{2}$$

$a_S \neq 0$, $[a]$, $S^\perp$ ani

$[a]^\perp$, S nejsou ve vztahu inkluze

$$a_S \longmapsto a_{S_T} \in T = [a]^\perp$$

$$\angle([a]^\perp, S) = \arccos \frac{\|a_{S_T}\|}{\|a_S\|} =$$

$$= \frac{\pi}{2} - \angle(a, S)$$

$$([a]^\perp \cap S)^\perp = [a] + S^\perp$$

∈S ∈S^⊥

$$a_S = a - (a - a_S) \in [a] + S^\perp$$

$$a_S \in ([a]^\perp \cap S)^\perp$$

$$\sphericalangle([a]^\perp, S) \le \sphericalangle(a_{ST}, a_S) =$$

$$a_{ST} \in T = [a]^\perp \qquad = \arccos \frac{\|a_{ST}\|}{\|a_S\|}$$

$$a_S \in S$$

$$\forall x \in [a]^\perp \cap ([a]^\perp \cap S)^\perp$$

$$x \neq 0 \qquad \frac{\|x_S\|}{\|x\|} \le \frac{\|a_{ST}\|}{\|a_S\|}$$

$$\langle x, a_s \rangle = \langle x_s, a_s \rangle$$

$$x \in [a]^\perp \cap ([a]^\perp \cap S)^\perp$$

$$x = x_s + (x - x_s)$$

$$\in S^\perp$$

$$a_s \in S \qquad \langle a_s, x - x_s \rangle = 0$$

$$\langle x, a_s \rangle = \langle x_s + (x - x_s), a_s \rangle$$

$$= \langle x_s, a_s \rangle + 0 = \langle x_s, a_s \rangle$$

dub 2-19:09

$x \in ([a]^\perp \cap S)^\perp = [a] + S^\perp$

x_S a a_S jsou LZ ✓

$x = \boxed{k \cdot a} + u, \quad u \in S^\perp$

$k \in \mathbb{R}$

$k \cdot a = x_S \in S$

$a_S \qquad (k \cdot a)_S = k \cdot a_S = x_S$

$\langle x_S, a_S \rangle = \|x_S\| \, \|a_S\|$

dub 2-19:12

$$\frac{\| x_s \|}{\| x \|} = \frac{\| x_s \| \cdot \| a_s \|}{\| x \| \, \| a_s \|} = $$

$$= \frac{\langle x_s, a_s \rangle}{\| x \| \, \| a_s \|} \leq \frac{\| a_{ST} \|}{\| a_s \|} \leq \underline{\text{ok}}.$$

dub 2-19:18

$$\measuredangle([a]^{\perp},[b]^{\perp}) =$$

$$= \frac{\pi}{2} - \measuredangle([a],[b]^{\perp}) =$$

$$= \frac{\pi}{2} - \left(\frac{\pi}{2} - \measuredangle([a],[b])\right) =$$

$$= \measuredangle(a,[b]) = \min(\measuredangle(a,b), \measuredangle(a,-b))$$

dub 2-19:21

$$\underbrace{(A^T \cdot A)} \cdot x = A^T b$$

$$\alpha = (r_1(A), \ldots, r_m(A))$$

$$A^T \cdot A = G(\alpha) \qquad S = [r_1(A) \ldots r_m(A)]$$

$$S \ni x_1 r_1(A) + \ldots + x_m r_m(A) = Ax$$

$$G(\alpha) \cdot x = \langle b, \alpha \rangle^T$$

$$Ax = b_S \Leftrightarrow A^T A x = \underline{\underline{A^T \cdot b}}$$

$\mathcal{R}(A|b)$ $\quad A x = b$

x a 0 budou tvořit příčku

af. podprostorů $\mathcal{R}(A)$ a $\{0\}$

$\iff x \in \mathcal{R}(A)^{\perp}$

$u \in \mathcal{R}(A) \iff A \cdot u = 0 \iff$

$\forall i \; 1 \le i \le m \quad n{:}(A) \cdot u = 0$

$x \in \mathcal{R}(A)^{\perp} \iff u^T \cdot x = 0 \; \forall u \in \mathcal{R}(A)$

$$\Rightarrow \quad r_i(A)^T \in R(A)^\perp$$

$$[r_1(A)^T, \dots, r_m(A)^T] \subseteq R(A)^\perp$$

$$h(A) = \dim [r_1(A), \dots, r_m(A)] = \dim [r_1(A), \dots, r_m(A)]$$

$$R(A)^\perp \oplus R(A) = \mathbb{R}^m$$

$$h(A) \qquad m - h(A)$$

$$\Rightarrow$$

$$x \in R(A)^T \Leftrightarrow x = \sum_{i=1}^{m} z_i \, r_i(A)^T$$

$$x = A^T z.$$