$x \in U$, $y \in V$

$F(x,y) = (x)_\alpha^T [F]_{\alpha,\beta} \cdot (y)_\beta$

$x = \alpha (x)_\alpha$, $y = \beta (y)_\beta$

$= \sum_{i=1}^{m} x_i u_i$ $\qquad = \sum_{j=1}^{n} y_j v_j$

$F\left(\sum x_i u_i, \sum y_j v_j\right) = \sum_i \sum_j x_i y_j F(u_i, v_j)$

$F(x,y) = (x)_\alpha^T B (y)_\beta$

$a_{ij} = F(u_i, v_j) = (0, \dots, 1, \dots, 0) B \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}_j = b_{ij}$

úno 19-9:58

$x \in V_1,\ y \in V_2$

$F(x,y) = (x)^T_{\alpha_1} [F]_{\alpha_1,\alpha_2} (y)_{\alpha_2}$

$F(x,y) = (x)^T_{\beta_1} [F]_{\beta_1,\beta_2} (y)_{\beta_2}$

$P_{\alpha_2,\beta_2} (y)_{\beta_2} = (y)_{\alpha_2}$

$P_{\alpha_1,\beta_1} (x)_{\beta_1} = (x)_{\alpha_1}$

$F(x,y) = (x)^T_{\beta_1} P^T_{\alpha_1,\beta_1} [F]_{\alpha_1,\alpha_2} P_{\alpha_2,\beta_2} (y)_{\beta_2}$

(a) $F^*: U \to V^*$ & $L2$

$x \in U \longmapsto \varphi_x : V \to K$

$\mathbb{R}$

$\varphi_x(y) = F(x, y)$

$F^*(x + \mathbb{R}) = F^*(x) + F^*(\mathbb{R})$

$\varphi_{x+\mathbb{R}}(y) = \varphi_x(y) + \varphi_{\mathbb{R}}(y) \quad \forall y \in V$

$\parallel$

$F(x+\mathbb{R}, y) = F(x, y) + F(\mathbb{R}, y)$

α báze U, β báze V, $\dim V = n$

$A = [F]_{\alpha,\beta}$ $\quad m = \dim U = \dim \operatorname{Ker} F^* + \underbrace{\dim \operatorname{Im} F^*}_{h(F^*)}$

$\operatorname{Ker} F^* = \{x \in U : F^*(x) = o\}$

$= \{x \in U : (\forall y \in V)\ \varphi_x(y) = 0\}$

$= \{x \in U : (\forall y \in V)\ F(x,y) = 0\}$

$= \{x \in U : (\forall y \in V)\ (x)_\alpha^T A (y)_\beta = 0\}$

$= \{x \in U : (x)_\alpha^T A = o\}$ $\qquad \boxed{A^T c = o}$

$\dim \operatorname{Ker} F^* = m - h(A)$

$$m = \dim \operatorname{Ker} F^* + h(F^*)$$

$$\dim \operatorname{Ker} F^* = m - h(A)$$

$$\Rightarrow h(A) = h(F^*)$$

$$\| $$

$$h(F)$$

F^* je prostá $\Leftrightarrow \operatorname{Ker} F^* = \{0\} \Leftrightarrow$

$\dim \operatorname{Ker} F^* = 0 \Leftrightarrow m = h(A)$

F^* je surjekcia $\Leftrightarrow \operatorname{Im} F^* = V^* \Leftrightarrow$

$h(A) = \dim \operatorname{Im} F^* = \dim V = n$

$\cancel{1+1=0}$

$$F_0(x,y) = \frac{1}{2}\left(F(x,y) + F(y,x)\right)$$

symetrická

$$F_1(x,y) = \frac{1}{2}\left(F(x,y) - F(y,x)\right)$$

antisym. forma

$$F_0 + F_1 = F = G_0 + G_1$$

sym. ant.

$$\Rightarrow F_0 = G_0, \quad F_1 = G_1$$

$$F_0(x,y) = \frac{1}{2}\left(F(x,y)+F(y,x)\right)$$

$$= \frac{1}{2}\left(G_0(x,y) + G_1(x,y) + G_0(y,x) + G_1(y,x) \right)$$

$$= \frac{1}{2} \cdot 2G_0(x,y) \Rightarrow F_0 = G_0$$

$$\underbrace{}_{=\,0}$$

$$F = F_0 + F_1 = G_0 + G_1 \Rightarrow F_1 = G_1$$

$$g(x) = F(x,x) = F_0(x,x)$$

$$= G_0(x,x)$$

sym...

$$G_0(x,y) = F_0(x,y) \quad \forall x,y \in V$$

$$G_0(x+y,x+y) = G_0(x,x) + 2G_0(x,y) + G_0(y,y)$$

$$\|$$

$$F_0(x+y,x+y) = F_0(x,x) + 2F_0(x,y) + F_0(y,y)$$

$$A = I_n^T A I_m$$

$$B = P^T A P \qquad (P^T)^{-1} \quad P^{-1}$$

$$(P^T)^{-1} B P^{-1} = A$$

$$B = P^T A P, \quad C = Q^T B Q$$

$$C = Q^T P^T A P Q$$

$$(P \cdot Q)^T$$

$$B = P^T A P \qquad\qquad B = B^T$$

$$\parallel \qquad\qquad \parallel$$

$$B^T = P^T A^T P$$

$a_{ii} \neq 0$

$\left(- \frac{a_{ij}}{a_{ii}} \right)$

$a_{ij} \neq 0 \neq a_{ji}$