$$\|x+y\| \leq \|x\| + \|y\|$$

$$\|cx\| = |c| \cdot \|x\|$$

$$\|(-1)x\| = 1 \cdot \|x\|$$

$$\|x+(-1)x\| \leq \|x\| + \|x\| = 2\|x\|$$

$$\|0\| \quad 0 = \|0\| \leq \|x\|$$

$$\|0\| = \|0 \cdot 0\| = |0| \cdot \|0\| = 0$$

$$\|x\| = \sqrt{\langle x, x \rangle}$$

$$\|x+y\| \leq \|x\| + \|y\|$$ ✓

$$\|x+y\|^2 = \langle x+y, x+y \rangle =$$

$$= \langle x,x \rangle + \langle x,y \rangle + \langle y,x \rangle + \langle y,y \rangle$$

$$= \|x\|^2 + 2\underbrace{\langle x,y \rangle}_{\leq \|x\|\,\|y\|} + \|y\|^2 \leq$$

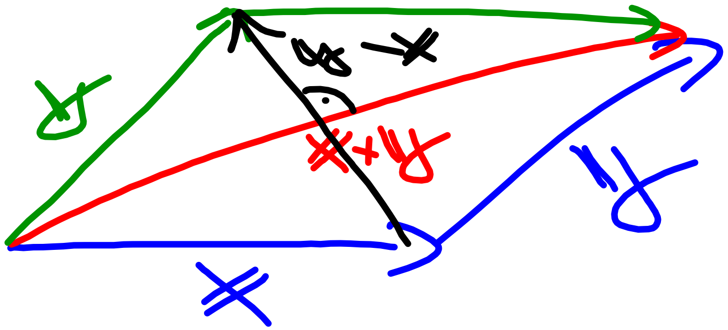
$$\leq \|x\|^2 + 2\|x\|\,\|y\| + \|y\|^2 = (\|x\| + \|y\|)^2$$

$$\|x+y\|^2 = \|x\|^2 + \|y\|^2 + 2\langle x, y\rangle \quad \checkmark$$

$$\cos \sphericalangle(x,y) = \frac{\langle x, y\rangle}{\|x\| \cdot \|y\|}$$

$$\|x-y\|^2 = \|x\|^2 + \|y\|^2 - 2\langle x, y\rangle \quad \checkmark$$

$$\|x+y\|^2 + \|x-y\|^2 = 2\left(\|x\|^2 + \|y\|^2\right)$$



$$\|y\| = \|x\|$$

$$\langle y-x,\ y+x\rangle = \underbrace{\langle y,y\rangle}_{\|y\|} - \cancel{\langle x,y\rangle} + \cancel{\langle y,x\rangle} - \underbrace{\langle x,x\rangle}_{\|x\|} = 0$$

$$\alpha = (u_1, \dots, u_k)$$

$$G(\alpha) = \begin{pmatrix} \langle u_1, u_1 \rangle & & \underset{=0}{\langle u_i, u_j \rangle} \\ & \ddots & \\ & & \langle u_k, u_k \rangle \end{pmatrix}$$

$$u_i \perp u_j \quad \underline{\underline{i \neq j}}$$

$$I_k = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$$

$u_1, \dots, u_k \in V$ OG

$u_i \neq 0, \implies u_1, \dots, u_k$ LN

$c_1 u_1 + \dots + c_k u_k = 0 \quad | \cdot u_i$

$c_1 \langle u_1, u_i \rangle + \dots + c_k \langle u_k, u_i \rangle = 0$

$\underset{\substack{\| \\ 0}}{c_i} \underbrace{\langle u_i, u_i \rangle}_{\neq 0} = 0$

$c_1 = c_2 = \dots = c_k = 0.$

břе 19-13:14

$\langle -, - \rangle : V \times V \to \mathbb{R}$

pozitivně definitní forma na V

$\exists$ báze B na V

matice $\langle , \rangle$ je v bázi B I_n

$I_n = G(B) = (\langle v_i, v_j \rangle)$ $\quad B = (v_1, \dots, v_n)$

$$x = \sum_{i=1}^{m} c_i u_i \quad / \; u_j \cdot$$

$$\langle x, u_j \rangle = c_j \langle u_j, u_j \rangle$$

$$\langle u_j, u_j \rangle = 1$$

$$c_j = \langle x, u_j \rangle$$

$$x = \sum_{i=1}^{n} \langle x, u_i \rangle u_i$$

$$y = \sum_{j=1}^{n} \langle y, u_j \rangle \cdot u_j$$

$$\langle x, y \rangle = \left\langle \sum_i \langle x, u_i \rangle u_i, \sum_j \langle y, u_j \rangle u_j \right\rangle$$

$$= \sum_i \sum_j \langle x, u_i \rangle \langle y, u_j \rangle \langle u_i, u_j \rangle$$

$$= \sum_{i=1}^{n} \langle x, u_i \rangle \cdot \langle y, u_i \rangle$$

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 3 \end{pmatrix}$$

$$|A_1| = |2| = 2 > 0 \qquad |A_2| = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4 - 1 = 3 > 0$$

$$|A_3| = (-1)^4(-1)(1\cdot 0 - 2(-1)) - (-1)^6 \cdot 3 \cdot 3 = -2 + 9 = 7 > 0$$

$$\langle x, y \rangle = x^T \cdot A \cdot y$$

$$G(\varepsilon) = (\langle e_i, e_j \rangle) = A$$

$$e_i^T \cdot A \cdot e_j = a_{ij} \qquad e_2 = (0,1,0)$$

$$e_1 = (1,0,0) = v_1 \quad 1$$

$$v_2 = e_2 - \frac{\langle e_2, v_1 \rangle}{\langle v_1, v_1 \rangle = 2} \cdot v_1 = e_2 - \frac{1}{2}(1,0,0) =$$

$$= \left(-\frac{1}{2}, 1, 0\right)$$

$$(0,1,0) \cdot \begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 1$$

$$v_1 = e_1 = (1, 0, 0)^T$$

$$v_2 = \left(-\frac{1}{2}, 1, 0\right)$$

$$v_3 = e_3 - \frac{\overbrace{\langle e_3, v_1\rangle}^{= -1}}{\underbrace{\langle v_1, v_1\rangle}_{=2}} \cdot v_1 - \frac{\overbrace{\langle e_3, v_2\rangle}^{= \frac{1}{2}}}{\underbrace{\langle v_2, v_2\rangle}_{= \frac{3}{2}}} \cdot v_2$$

$$(-1, 0, 3)\begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{pmatrix} = +\frac{1}{2}$$

$$v_2^T \begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 3 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{pmatrix} = \left(-\frac{1}{2}, 1, 0\right)\begin{pmatrix} 0 \\ \frac{3}{2} \\ \frac{1}{2} \end{pmatrix} = \frac{3}{2}$$

$$v_3 = e_3 + \frac{1}{2} v_1 - \frac{1}{3} v_2 =$$

$$(0,0,1) + \left(\frac{1}{2},0,0\right) - \left(-\frac{1}{6}, \frac{1}{3}, 0\right)$$

$$= \left(\frac{1}{2}+\frac{1}{6}, -\frac{1}{3}, 1\right) = \left(\frac{2}{3}, -\frac{1}{3}, 1\right)$$