$(I) \Rightarrow (II)$ $\quad |A_k| \neq 0$

$$A \overset{(1^+)}{\equiv} B = \begin{pmatrix} \begin{matrix} b_1^{\neq 0} & & 0 \\ & \ddots & \\ 0 & & b_k^{\neq 0} \end{matrix} & \\ & \ddots \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & & \\ & \ddots & \\ 0 & & 1 \end{pmatrix}; \quad P^T = \begin{pmatrix} \ddots & 0 \\ & \ddots \end{pmatrix}$$

$$B = P^T A P \qquad A \rightsquigarrow A_k$$

$$B_k = P_k^T A_k P_k \qquad P \rightsquigarrow P_k$$

$$B = \begin{pmatrix} b_{11} \cdots b_{1r} & 0 \\ & \end{pmatrix}$$

$b_{ij}, \ 1 \le i, j \le r$

$$D_j(P) = \begin{pmatrix} \vdots \\ 1 \\ 0 \end{pmatrix}$$

$$r_i(P^T) = (\sim 1, 0)$$

$$P^T A P$$

$$B_r = P_r^T A_r P_r$$

$$B = \begin{pmatrix} b_1^{\neq 0} & & \\ & \ddots & \\ & & b_n^{\neq 0} \end{pmatrix}$$

$$\underset{\neq 0}{|B_2|} = \underset{1}{\underbrace{|P_2^T|}} \, |A_2| \, \underset{1}{\underbrace{|P_2|}} = |A_2| \neq 0$$

$$b_1 = |B_1| = |A_1|, \quad b_2 = \frac{|B_2|}{b_1} = \frac{|A_2|}{|A_1|}$$

$$1 \le k \le \ell$$

$$|B_{k+1}| = b_1 \cdot \ldots \cdot b_{k+1}$$

$$|B_k| = b_1 \cdot \ldots \cdot b_k$$

$$b_{k+1} = \frac{|B_{k+1}|}{|B_k|} = \frac{|A_{k+1}|}{|A_k|}$$

$|A_k| \neq 0, \quad 1 \leq k \leq n$

$A \overset{?}{\underset{(1+)}{\equiv}} \operatorname{diag}\left(|A_1|, \frac{|A_2|}{|A_1|}, \dots, \frac{|A_n|}{|A_{n-1}|}\right)$

$A_1 = (a_{11})$ lze (1+)

$a_{11} \neq 0$

$A \equiv \left(\begin{array}{c|c} a_{11} & 0 \\ \hline 0 & C_1 \end{array}\right)$

$\boxed{|A| = a_{11} \cdot |C_1|}$

$P^T A P$!!

$P_2^T A_2 P_2 = \begin{pmatrix} a_{11} & 0 \\ 0 & c_{11}^1 \end{pmatrix}$ $\quad a_{11} \neq 0, \; c_{11}^1 \neq 0$

$|A_2| \neq 0$

bře 4-10:21

$$A \underset{(1\times)}{\equiv} \begin{pmatrix} a_{11} & 0 \\ 0 & C_1 \end{pmatrix} \underset{(1\times)}{\equiv}$$

$$c_{11}^{(1)} \neq 0$$

$$= \begin{pmatrix} a_{11} & & 0 \\ 0 & c_{11}^{(1)} & \\ & & \boxed{C_2} \end{pmatrix} = \dots\dots$$

$$= \begin{pmatrix} a_{11} & & & 0 \\ & c_{11}^{(1)} & & \\ 0 & & c_{11}^{(2)} & \\ & & & c_{11}^{(k)} \\ & & & 0 \end{pmatrix}$$

$$a_{11} = |A_1|$$

$$c_{11}^{(1)} = \frac{|A_2|}{|A_1|}$$

$$c_{11}^{(k)} = \frac{|A_k|}{|A_{k-1}|}$$

$$\langle x + x, x\rangle = \langle x, x\rangle + \langle x, x\rangle$$

$$\boxed{x = o} \quad \underline{0} \quad \cancel{\langle o, o\rangle} = \cancel{\langle o, o\rangle} + \underline{\langle o, o\rangle}$$

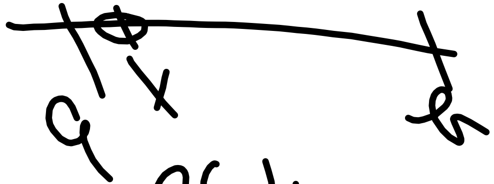
$$\langle x, x\rangle = 0 \Rightarrow x = o$$

$f \neq 0$

$\langle f, f \rangle > 0$

$f^2(x) > 0$

$\langle a, b \rangle$



$f(x) \neq 0$

bře 4-10:49

$$\left|\begin{pmatrix} 3 & 1 & 2 \\ 1 & 1 & 0 \\ 2 & 0 & 7 \end{pmatrix}\right|$$

$|A_1| = 3 > 0$

$|A_2| = 3 - 1 > 0$

$|A_3| = 2 \cdot (-1)^4 \cdot (-2) + 7(-1)^6 \cdot 2 =$

$= -4 + 14 = 10 > 0$

$$\langle x, o \rangle = o$$

$$0 \; \cancel{\langle x, o \rangle} = \langle x, o + o \rangle =$$

$$= \cancel{\langle x, o \rangle} + \langle x, o \rangle$$

$$\exists\, G(u_1, \ldots, u_k)\, c > 0$$

$$\langle u_i, u_j \rangle = \langle u_j, u_i \rangle$$

$$(c_1, \ldots, c_k) \begin{pmatrix} \langle u_1, u_1 \rangle & \ldots & \langle u_1, u_k \rangle \\ & & \vdots \\ \langle u_k, u_1 \rangle & \ldots & \langle u_k, u_k \rangle \end{pmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_k \end{pmatrix} =$$

$$\Big(\langle \underbrace{\sum_{j=1}^{k} c_j u_j}_{\neq}, u_1 \rangle, \ldots, \langle \underbrace{\sum_{j=1}^{k} c_j u_j}_{\neq}, u_k \rangle \Big) \begin{pmatrix} c_1 \\ \vdots \\ c_k \end{pmatrix} =$$

$$\left(\langle \mathbf{x}, u_1 \rangle, \dots, \langle \mathbf{x}, u_k \rangle\right) \begin{pmatrix} c_1 \\ \vdots \\ c_k \end{pmatrix} =$$

$$= \Big(\langle \mathbf{x}, \underbrace{\sum_{j=1}^{k} c_j u_j}_{\mathbf{x}} \rangle \Big) = \underline{\langle \mathbf{x}, \mathbf{x} \rangle \geq 0}$$

$\mathbf{u}_1, \dots, \mathbf{u}_k$ LN

$\mathbf{c} \neq \mathbf{0}$ $\quad \mathbf{c}^T G(\mathbf{u}_1, \dots, \mathbf{u}_k)\, \mathbf{c} > 0$

$$\mathbf{x} = \sum_{j=1}^{k} c_j \cdot \mathbf{u}_j \qquad \langle \mathbf{x}, \mathbf{x} \rangle > 0$$

$\mathbf{x} \neq \mathbf{0}$

bře 4-11:13

$u_1, \dots, u_k$ jsou LZ

$\exists\, \mathbf{c} \neq 0,\ \mathbf{c} = (c_1, \dots, c_k)$

$0 = c_1 u_1 + \dots + c_k u_k = \mathbf{x}$

$\mathbf{c}^T G \mathbf{c} = \langle \mathbf{x}, \mathbf{x} \rangle = \langle 0, 0 \rangle$

$= 0$, tj. $G(u_1, \dots, u_k)$ není poz. def.

$$\begin{pmatrix} 1 & & & \\ & \ddots & & 0 \\ & & 1 & \\ 0 & & & \ddots \end{pmatrix} = \overline{P}^T G(u_1, \dots, u_k) P$$

$$\geq 0$$

$$0$$

$$\underbrace{|P|^2}_{>0} \cdot \underbrace{|G(u_1, \dots, u_k)|}_{\geq 0}$$

$$0$$

$$\langle u,v\rangle \in \mathbb{C}$$

$$\| $$

$$|\langle u,v\rangle| \cdot \underline{\underline{(\cos\alpha + i\sin\alpha)}}$$

$$c = \cos\alpha - i\sin\alpha$$

$$\langle u,v\rangle \cdot c = |\langle u,v\rangle| \cdot 1$$

$$|\langle u,v\rangle| = \langle u, \bar{c}v\rangle = \langle cu, v\rangle$$

$$|\langle u,v\rangle| = |\langle r,u\rangle| \le \|u\|_{\mathbb{R}} \|v\|_{\mathbb{R}}$$

$$= \|u\| \|v\|$$

bře 4-11:27