a) $A \simeq A$

$A = (I_n)^{-1} \cdot A \cdot I_n$

b) $A \simeq B \Rightarrow B \simeq A$

$B = P^{-1} A P$, P inv.

$Q = P^{-1}$, $Q^{-1} = P$ $\quad BQ = QA \,/ Q^{-1}$

$B = QAQ^{-1} \,/ Q$ $\quad A = Q^{-1} B Q$

dub 9-15:27

$A \simeq B,\ B \simeq C \Longrightarrow A \simeq C$

$B = P^{-1} A P,\ C = Q^{-1} B Q,$

P, Q nog.

$(PQ)^{-1} = Q^{-1} P^{-1}$

$C = (Q^{-1} P^{-1}) A (P Q) =$

$R = PQ$

$= R^{-1} A R \quad , R$ ~~je~~ nog.

dub 9-15:45

$(i) \Rightarrow (ii)$ $\exists \varphi : V \to V$

$A = (\varphi)_\alpha$, $B = (\varphi)_\beta$ $P_{\alpha\beta}^{-1} = P_{\beta\alpha}$

$B = P_{\beta\alpha} (\varphi)_\alpha P_{\alpha\beta} =$

$= (P_{\alpha\beta})^{-1} A P_{\alpha\beta} \Rightarrow A \equiv B$

$(ii) \Rightarrow (i)$ $A \cong B \overset{?}{\Rightarrow} \exists \varphi : V \to V$

$A = (\varphi)_\alpha$, $B = (\varphi)_\beta$

dub 9-15:48

$B = P^{-1} A P$, P reg. matice

$(\varphi(x))_\alpha = A(x)_\alpha$, α báze V lib. pevně.

$(\varphi)_\alpha = A$ $\quad\beta$? $\quad P = P_{\alpha\beta}$

$B = P^{-1}(\varphi)_\alpha P = (\varphi)_\beta$ $\alpha P_{\alpha\beta} = \beta$

$= P_{\beta\alpha}(\varphi)_\alpha P_{\alpha\beta}$ $=$

$\boxed{\beta = \alpha \cdot P}$

$A = (a_{ij})$, $B = (b_{kl})$

$C = A \cdot B = (c_{il})$ $1 \le i, l \le m$

$D = B \cdot A = (d_{kj})$ $1 \le k, j \le m$

$h(C) = \sum_{i=1}^{m} c_{ii} = \sum_{i=1}^{m} \left(\sum_{j=1}^{m} a_{ij} b_{ji} \right)$

$h(C) = \sum_{j=1}^{m} d_{jj} = \sum_{j=1}^{m} \left(\sum_{i=1}^{m} b_{ji} \cdot a_{ij} \right)$

dub 9-15:58

$$A \equiv B,\quad A = P^{-1} B P,$$

$$|A| = |P^{-1} \cdot B \cdot P| \overset{\text{Prog.}}{=}$$

$$= |P^{-1}| \cdot |B| \cdot |P| = |P| \cdot |P^{-1}| \cdot |B| =$$

$$= |\underbrace{P \cdot P^{-1}}_{I_n}| \cdot |B| = 1\,|B| = |B|$$

$$h(A) = h(P^{-1} B P) = h(\underbrace{P P^{-1}}_{I_n} B)$$

$$= h(B)$$

$$B = P^{-1} A \cdot P$$

$$B v = \lambda v, \quad v \neq 0$$

$$\lambda v = P^{-1} A \underbrace{P v}_{= w \neq 0} \qquad | P$$

$$P \lambda v = A w$$

$$\lambda w = A w$$

dub 9-16:14

(i) $\Longrightarrow$ (ii) φ je diag. $\overset{?}{\Longrightarrow} \exists$ báze

$\alpha = (v_1, \ldots, v_n)$, $\forall v_i$ je vlastní!

$$(\varphi)_\alpha = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}$$

$$(\varphi(v_i))_\alpha = \lambda_i (v_i)_\alpha = (\lambda_i v_i)_\alpha$$

$(-)_\alpha$ bijekce $\Longrightarrow$ $\varphi(v_i) = \lambda_i v_i$

(ii) $\Rightarrow$ (iii) $\alpha = (\mathbb{v}_1, \dots, \mathbb{v}_n)$, $\forall \mathbb{v}_i, \mathbb{v}_j$ je různé

$$\mathbb{x} = \sum_{i=1}^{n} c_i \mathbb{v}_i \in \mathbb{V} \qquad [\mathbb{v}_1] \oplus \dots \oplus [\mathbb{v}_n]$$

$$\{\mathbb{o}\} = [\mathbb{v}_j] \cap \big([\mathbb{v}_1] + \dots \not\oplus_j \dots \oplus [\mathbb{v}_n]\big)$$

$$\mathbb{x} = c_j \mathbb{v}_j = \sum_{i \neq j} c_i \mathbb{v}_i$$

$\Downarrow$

$\mathbb{x} = \mathbb{o}$ $\qquad \mathbb{o} = \sum_{i=1}^{n} c_i \mathbb{v}_i \Rightarrow c_1 = c_2 \dots = c_n = 0$

(iii) $\Rightarrow$ (i) $\quad \dim \mathbb{V} = n$

$$\mathbb{V} = [\mathbf{v}_1] \oplus \dots \oplus [\mathbf{v}_n]$$

i.n.

$\alpha = (\mathbf{v}_1, \dots, \mathbf{v}_n)$ je báze $\mathbb{V}$

$$(\varphi)_\alpha \qquad \varphi(\mathbf{v}_i) = \lambda_i \mathbf{v}_i$$

$$(\varphi(\mathbf{v}_i))_\alpha = (\lambda_i \mathbf{v}_i)_\alpha = \begin{pmatrix} 0 \\ \vdots \\ \lambda_i \\ \vdots \\ 0 \end{pmatrix}$$

$$= \lambda_i (\mathbf{v}_i)_\alpha =$$

$x_i \neq x_j$ pro $i \neq j$

$LN^{?}$ $v_1, \ldots, v_k$ $\quad \varphi(v_i) = x_i v_i$

$x_1, \ldots, x_k$

$v_1, \ldots, v_k$ je LZ $\Big\}$ j nejmenší index

j splňuje $v_j = \sum_{i=1}^{j-1} c_i v_i$, $j \geq 2$

$v_1, \ldots, v_{j-1}$ je $\underline{\underline{LN}}$

$$v_j = \sum_{i=1}^{j-1} c_i v_i$$

$$\varphi(v_j) = \sum_{i=1}^{j-1} c_i \varphi(v_i) =$$

$$= \sum_{i=1}^{j-1} c_i \lambda_i v_i$$

$$\lambda_j v_j = \boxed{\sum_{i=1}^{j-1} c_i \lambda_j v_i} = \varphi(v_j)$$

$$\sum_{i=1}^{j-1} c_i \lambda_i v_i = \sum_{i=1}^{j-1} c_i \lambda_j v_i$$

$$\sum_{i=1}^{j-1} (\lambda_i - \lambda_j) c_i v_i = 0$$

$$c_i(\lambda_i - \lambda_j) \neq 0$$

$$= 0$$

$$\forall i$$

$$\Downarrow$$

$$c_i = 0 \; \forall i$$

$$\Rightarrow v_j = 0$$

SPOR

$\lambda_i \neq \lambda_j$

$\lambda_1, \ldots, \lambda_n$

$v_1, \ldots, v_n$ LN ⇒ báze

w vlastní vektor přísl. k vl. číslu λ_j

$\varphi(w) = \lambda_j w$? $w \in [v_j]$

$\Leftrightarrow w = c v_j$

$$\mathbf{w} = \sum_{i=1}^{n} c_i \mathbf{v}_i$$ nějaký nenulový

$$\Longrightarrow \mathbf{w} - c_j \mathbf{v}_j \notin \text{Lin.}$$

Rom $\mathbf{v}_1, \dots, \mathbf{v}_j, \dots, \mathbf{v}_n$

$$\mathbf{w} = c_j \mathbf{v}_j$$ SPOREM

$$\mathbf{w} \neq c_j \mathbf{v}_j \qquad \varphi(\mathbf{w} - c_j \mathbf{v}_j) =$$

$$\mathbf{w} - c_j \mathbf{v}_j \neq \mathbf{0} \qquad = \lambda_j \mathbf{w} - \lambda_j c_j \mathbf{v}_j =$$

$$= \lambda_j (\mathbf{w} - c_j \mathbf{v}_j) \Longrightarrow$$

$$\underbrace{\mathbf{v}_1, \dots, \mathbf{w} - c_j \mathbf{v}_j, \dots, \mathbf{v}_n \ \mathrm{LN}} \Rightarrow \underline{\underline{\mathrm{SPOR}}}$$

dub 9-16:54

$\lambda \in K$ je vl. číslo matice A

$\iff |A - \lambda I_n| = 0$

$A \cdot v = \lambda I_n v, \quad v \in K^n, v \neq 0$

$\iff (A - \lambda I_n) v = 0, \quad v \neq 0$

$\iff \det(A - \lambda I_n) = 0$

dub 9-16:59

$$A \cong B \Longrightarrow ch_B(x) = ch_A(x)$$

$$B = P^{-1} A \cdot P \qquad P^{-1}(xI_n)P = xI_n$$

$$ch_B(x) = |B - xI_n| =$$

$$= |P^{-1}AP - P^{-1}(xI_n)P| =$$

$$= |P^{-1}(A - xI_n)P| = |P^{-1}| \cdot$$

$$\cdot |A - xI_n| \cdot |P| = |A - xI_n| = ch_A(x).$$

dub 9-17:05

Teoremă: $p(x) = a_n x^n + \ldots + a_1 x + a_0$, $a_n \neq 0$, $n \geq 1$. Pag

x_0 e rădăcină $p(x) \iff$

$$p(x) = (x - x_0)\, q(x).$$

Dz. $\Leftarrow$: $p(x_0) = \underbrace{(x_0 - x_0)}_{0}\, q(x_0) = 0$

$\Rightarrow p(x_0) = 0$

$$h(x) = h(x) - h(x_0) =$$

$$= a_n(\underline{x^n - x_0^n}) + a_{n-1}(\underline{x^{n-1} - x_0^{n-1}}) +$$

$$+ \dots + a_1(x - x_0) =$$

$$\Big[\quad x^k - x_0^k = (x - x_0)(x^{k-1} + x^{k-2}x_0 \dots + \dots + x_0^{k-1})$$

$k > 1$

$$= (x - x_0)\, q(x)$$

dub 9-17:23

Tvrzení:

Nechť $h(x) = \pm x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$, $a_{n-1}, \dots, a_0 \in \mathbb{Z}$, $n \geq 1$.

Pokud má $h(x)$ rac. kořen, pak je to celé číslo, které dělí abs. člen a_0.

Dk. $x_0 = \frac{c}{d} \in \mathbb{Q}$, c, d nesoudělná.

$$\pm \frac{c^n}{d^n} + a_{n-1}\frac{c^{n-1}}{d^{n-1}} + \dots + a_1 \frac{c}{d} + a_0 = 0 \quad / \cdot d^n$$

dub 9-17:26

$$\pm c^n + a_{n-1} c^{n-1} d + \dots + a_1 c d^{n-1} + a_0 d^n = 0$$

$\Rightarrow$ c dělí $a_0 d^n$ $\Rightarrow$ c dělí a_0

$$\begin{pmatrix} 5 & 2 & -3 \\ 4 & 5 & -4 \\ 6 & 4 & -4 \end{pmatrix}; \quad \begin{vmatrix} 5-\lambda & 2 & -3 \\ 4 & 5-\lambda & -4 \\ 6 & 4 & -4-\lambda \end{vmatrix} =$$

$= \det$ Sarr. pravidla $= -\lambda^3 + 6\lambda^2 - 11\lambda + \underline{\underline{6}}$

$\pm 1, \pm 2, \pm 3, \pm 6$ $\quad \lambda_1 = 2, \lambda_2 = 3$

HORNEROVO SCHÉMA $\quad \lambda_3 = 1$

	-1	6	-11	6
2	$\underline{-1}$	$\underline{4}$	$\underline{-3}$	$\underline{\underline{0}}$

$p(\lambda) = (\lambda - 2) \cdot (-\lambda^2 + 4\lambda - 3)$

$$\begin{pmatrix} 2 & & 0 \\ & 3 & \\ 0 & & 1 \end{pmatrix}.$$