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$$h_{opt,0,k} = \left(\frac{V(k)\,\sigma^2\,(k!)^2}{2kn\,A_k\,\beta_k^2} \right)^{\frac{1}{2k+1}} \qquad A_k = \int_0^1 (m^{(k)}(x))^2\,dx$$

$A_k = V(m^{(k)})$

$$\beta_k = \int_{-1}^{1} x^k k(x)\,dx$$

$$V(k) = \int_{-1}^{1} k^2(x)\,dx$$

$\boxed{k=4}$ $\boxed{m(x) = \sin^2 \pi x \,,\ \sigma^2 = \frac{1}{4},\ n = 100}$

$$k(x) = \frac{15}{32}(x^2-1)(7x^2-3)$$

$$A_4 = V(m^{(4)}) = \int_0^1 (m^{(4)}(x))^2\,dx$$

$$= \int_0^1 (\sin^2 \pi x)^{(4)}\,dx = \ldots = \underline{\underline{32\pi^8}}$$

$$V(k) = \int_{-1}^{1} \left[\frac{15}{32}(x^2-1)(7x^2-3)\right]^2 dx = \ldots = \frac{5}{4}$$

$$\beta_4 = \int_{-1}^{1} x^4 \frac{15}{32}(x^2-1)(7x^2-3)\,dx = \ldots = -0{,}0476$$

$$h_{opt,0,4} = \left(\frac{\frac{5}{4}\cdot\frac{1}{4}\,(4\cdot 3\cdot 2)^2}{2\cdot 4\cdot 100\cdot 32\pi^8\cdot(-0{,}0476)^2} \right)^{\frac{1}{9}}$$