$$K(x) = \frac{3}{4}(1-x^2) \qquad K \in S_{02}$$

$$V(K) = \int_{-1}^{1} K^2(x)\,dx = \frac{9}{16}\int_{-1}^{1} 1 - 2x^2 + x^4 \,dx = \frac{9}{16}\left[x - \frac{2}{3}x^3 + \frac{x^5}{5}\right]_{-1}^{1}$$

$$= \frac{9}{16}\left(1 - \frac{2}{3} + \frac{1}{5} + 1 - \frac{2}{3} + \frac{1}{5}\right) = \frac{9}{16}\,\frac{30-20+6}{15} = \underline{\underline{\frac{3}{5}}}$$

$$\beta_K^{*} = \int_{-1}^{1} x^2 \frac{3}{4}(1-x^2)\,dx = \frac{3}{4}\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_{-1}^{1} = \frac{3}{4}\left(\frac{1}{3} - \frac{1}{5} + \frac{1}{3} - \frac{1}{5}\right)$$

$$= \frac{3}{2}\,\frac{5-3}{15} = \underline{\underline{\frac{1}{5}}}$$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$f_\sigma(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-\frac{x^2}{2\sigma^2}} = \frac{1}{\sigma} f\left(\frac{x}{\sigma}\right)$$

$$V(f_\sigma'') = \int f_\sigma''(x) \cdot f_\sigma''(x)\, dx = (f_\sigma'' * f_\sigma'')(0) = f^{(IV)}_{\sqrt{\sigma^2+\sigma^2}}(0)$$

$$f^{(IV)}(x) = (x^4 - 6x^2 + x + 3) f(x)$$

$$f^{(IV)}_{\sqrt{\sigma^2+\sigma^2}}(x) = \frac{1}{(\sqrt{\sigma^2+\sigma^2})^5} (\dots) f^{(IV)}\left(\frac{x}{\sqrt{\sigma^2+\sigma^2}}\right)$$

$$f_\sigma^{(k)}(x) = \frac{1}{\sigma^{k+1}} f^{(k)}\left(\frac{x}{\sigma}\right)$$

$$f^{(IV)}_{\sqrt{\sigma^2+\sigma^2}}(0) = 3 \cdot \frac{1}{\sigma^5 \cdot \sqrt{2}^5} \cdot \frac{1}{\sqrt{2\pi}} = \frac{3}{8\sqrt{\pi}} \cdot \sigma^{-5}$$

$$D_2 = \frac{3}{32\sigma^5\sqrt{\pi}}, \quad \delta_{02}^5 = \frac{V(k)}{\beta_2^2} = \frac{\frac{3}{5}}{\frac{1}{25}} = 15$$

$$h_{REF}^5 = \frac{5 \cdot 15 \cdot 32 \cdot \sqrt{\pi}}{8n \cdot 2 \cdot 3} \sigma^5 = \frac{40\sqrt{\pi}}{n} \cdot \sigma^5$$

$$h_{opt,0,k}^{2k+1} = \frac{\delta_{0k}^{2k+1}}{2nk D_k}, \quad D_k = \int \left(\frac{f^{(k)}(x)}{k!}\right)^2 dx$$

$$h_{opt,0,k}^{2k+1} = \frac{V(k)(k!)^2}{\beta_k^2 \cdot 2nk \int (f^{(k)}(x))^2 dx}$$

$$\delta_{0k}^{2k+1} = \frac{V(k)}{\beta_k^2}$$