$$\hat{m}(x;p,h) = e_1^T \cdot (X^T W X)^{-1} X^T W y$$

kde $X = \begin{pmatrix} 1 & x_1 - x \\ \vdots & \vdots \\ 1 & x_n - x \end{pmatrix}$ $W = \begin{pmatrix} k_h(x_1-x) & & 0 \\ & \ddots & \\ 0 & & k_h(x_n-x) \end{pmatrix}$

$$(a+b+c)^2 = (a+b)^2 + (2a+2b)c + c^2$$
$$a^2+b^2+c^2+2ab+2ac+2bc$$

$$X^T W X = \begin{pmatrix} \sum k_h(x_i-x) & \sum (x_i-x) k_h(x_i-x) \\ \sum (x_i-x) k_h(x_i-x) & \sum (x_i-x)^2 k_h(x_i-x) \end{pmatrix} = n \begin{pmatrix} s_0 & s_1 \\ s_1 & s_2 \end{pmatrix}$$

$$\hat{s}_r(x,h) = \frac{1}{n} \sum_{i=1}^{n} (x_i-x)^r k_h(x_i-x)$$

$$|X^T W X| = n^2(s_0 s_2 - s_1^2) = \sum_{i=1}^{n} k_h(x_i-x) \cdot \sum_{j=1}^{n} (x_j-x)^2 k_h(x_j-x) - \left(\sum_{i=1}^{n} (x_i-x) k_h(x_i-x)\right)^2$$

$$= \sum_{i,j} (x_j-x)^2 k_h(x_i-x) k_h(x_j-x) - \sum_{i,j} (x_i-x)(x_j-x) k_h(x_i-x) k_h(x_j-x)$$

$$= \sum_{i,j} k_h(x_i-x) k_h(x_j-x) \Big[ \underbrace{(x_j-x)^2 - (x_i-x)(x_j-x)}_{(x_j-x)(x_j-x_i)} \Big]$$

$$(X^T W X)^{-1} X^T W y = \frac{1}{n^2(s_0 s_2 - s_1^2)} \cdot n \begin{pmatrix} s_2 & -s_1 \\ -s_1 & s_0 \end{pmatrix} \cdot \begin{pmatrix} k_h(x_1-x) & \dots & k_h(x_n-x) \\ (x_1-x) k_h(x_1-x) & \dots & (x_n-x) k_h(x_n-x) \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$= \frac{1}{n(s_0 s_2 - s_1^2)} \begin{pmatrix} s_2 & -s_1 \\ -s_1 & s_0 \end{pmatrix} \cdot \begin{pmatrix} \sum k_h(x_i-x) \cdot y_i \\ \sum (x_i-x) k_h(x_i-x) y_i \end{pmatrix}$$

$$\hat{m}(x;1,h) = \frac{1}{n(s_0 s_2 - s_1^2)} \cdot \cancel{\sum_{i=1} (x_i-x)^2 k_h(x_i-x) \cdot \sum_{j=1}^{n} k_h} \quad s_2 \cdot \sum k_h(x_i-x) y_i - s_1 \sum (x_i-x) k_h(x_i-x) y_i$$

$$= \frac{1}{n(s_0 s_2 - s_1^2)} \cdot \sum \big(s_2 - s_1(x_i-x)\big) k_h(x_i-x) y_i$$