

12. SOIL TEMPERATURE AND HEAT FLOW

IMPORTANCE OF SOIL TEMPERATURE

Soil temperature, its value at any moment and the manner of its variation in time and space, is a factor of primary importance in determining the rates and directions of soil physical processes and of energy and mass exchange with the atmosphere. Temperature governs evaporation and aeration as well as the types and rates of chemical reactions that take place in the soil. Finally, soil temperature strongly influences biological processes, such as seed germination, seedling emergence and growth, root development, and microbial activity.

Soil temperature varies in response to changes in the radiant, thermal, and latent energy exchange processes that take place primarily through the soil surface. The effects of these phenomena are propagated into the soil profile by a complex series of transport processes, the rates of which are affected by time-variable and space-variable soil properties. Hence the quantitative formulation and prediction of the soil thermal regime can be a formidable task. Even beyond passive prediction, the possibility of actively controlling or modifying the thermal regime requires a thorough knowledge of the processes at play and of the environmental and soil parameters determining their rates. The pertinent soil parameters include the specific heat capacity, thermal conductivity and thermal diffusivity (all of which are strongly affected by bulk density and wetness) as well as the internal sources and sinks of heat operating at any time.

Many reviews of soil temperature and heat flow have been published over the years by (among others): de Vries (1975b), Campbell (1977), Taylor and Jackson (1986), Fuchs (1986), Hanks (1992), Evett (2002), and McInnes (2002).

MODES OF ENERGY TRANSFER

We begin with basic physics. There are three principal modes of energy transfer: radiation, convection, and conduction. By *radiation*, we refer to the emission of energy in the form of electromagnetic waves from all bodies above 0 K. According to the *Stefan–Boltzmann law*, the total energy emitted by a body, J_t , integrated over all wavelengths, is proportional to the fourth power of the absolute temperature T of the body's surface. This law is usually formulated

$$J_t = \epsilon \sigma T^4 \quad (12.1)$$

where σ is a constant and ϵ is the *emissivity coefficient*, which equals unity for a perfect emitter (generally called a *black body*). The absolute temperature also determines the wavelength distribution of the emitted energy. *Wien's law* states that the wavelength of maximal radiation intensity λ_m is inversely proportional to the absolute temperature:

$$\lambda_m = 2900/T \quad (12.2)$$

where λ_m is in micrometers. The radiative intensity as a function of wavelength and temperature is given by *Planck's law*:

$$E_\lambda = C_1/\lambda^5 [\exp(C_2/\lambda T) - 1] \quad (12.3)$$

where E_λ is energy flux emitted in a given wavelength range and C_1 , C_2 are constants.

Since the temperature of the soil surface averages about 300 K (though it can range from below 273 K, the freezing point, to 330 K or even higher), the radiation emitted by the soil surface has its peak intensity at a wavelength of about 10 μm and its wavelength distribution over the range of 3–50 μm . This is in the realm of infrared, or heat, radiation.

A very different spectrum is emitted by the sun, which acts as a black body at an effective surface temperature of about 6000 K. The sun's radiation includes the visible light range of 0.3–0.7 μm as well as some infrared radiation of greater wavelength (up to about 3 μm) and some ultraviolet radiation ($\lambda < 0.3 \mu\text{m}$). Since there is very little overlap between the two spectra, it is customary to distinguish between them by calling the incoming solar spectrum *short-wave radiation* and the spectrum emitted by the earth *long-wave radiation*.

The second mode of energy transfer, called *convection*, involves the movement of a heat-carrying mass, as in the cases of ocean currents and atmospheric winds. An example more pertinent to soil physics would be the infiltration of warm wastewater into an initially cold soil.

Conduction, the third mode of energy transfer, is the propagation of heat within a body by internal molecular motion. Because temperature is an expression of the kinetic energy of a body's molecules, the existence of a temperature difference within a body will normally cause the transfer of kinetic energy by the collisions of rapidly moving molecules from the warmer region of the body to their neighbors in the colder region. The process of heat conduction is thus analogous to diffusion; and in the same way that diffusion tends in time to equilibrate a mixture's composition throughout, heat conduction tends to equilibrate a body's internal temperature.

In addition to the three modes of energy transfer described, there is a composite phenomenon that one may recognize as a fourth mode, namely, *latent*

heat transfer. A prime example is the process of distillation, which includes the heat-absorbing stage of evaporation, followed by the convective or diffusive movement of the vapor, and ending with the heat-releasing stage of condensation. A similar catenary process can also occur in transition back and forth from ice to liquid water in soils subject to freezing and thawing.

ENERGY BALANCE FOR A BARE SOIL

A detailed elucidation of the energy and water balances of vegetated fields is given in Chapter 20. At this stage we outline the energy regime of a bare (unvegetated) soil. We begin with the radiation balance of a bare surface, which can be written thus (van Bavel and Hillel, 1976):

$$J_n = (J_s + J_a)(1 - \alpha) + J_{li} - J_{lo} \quad (12.4)$$

Here J_n is the *net radiation*, that is, the sum of all incoming minus outgoing radiant energy fluxes, J_s is the incoming flux of short-wave radiation directly from the sun, J_a is the short-wave diffuse radiation from the atmosphere (sky), J_{li} is the incoming long-wave radiation flux from the sky, J_{lo} is the outgoing long-wave radiation emitted by the soil, and α is the *albedo*, or *reflectivity coefficient*, which is the fraction of incoming short-wave radiation reflected by the soil surface rather than absorbed by it. We shall disregard in the present context all terms that do not pertain to the soil, namely, J_s , J_a , and J_{li} .

The albedo α is an important characteristic of soil surfaces, and it can vary widely in the range of 0.1–0.4, depending on the soil's basic color (whether dark or light colored), the surface's roughness, and the inclination of the incident radiation relative to the surface (Sellers, 1965). In the short run, the albedo also depends on the changing wetness of the exposed soil (Jackson et al., 1974). The drier the soil, the smoother its surface; the brighter its color, the higher its albedo. To a certain extent, the albedo can be modified by various surface treatments, such as tillage and mulching.

Apart from the reflected short-wave radiation, governed by the albedo, we have another soil-dependent process, namely, the emission of long-wave radiation. In accordance with Eq. (12.1), the emitted flux J_{le} depends on surface temperature but is also affected by emissivity ϵ . This parameter, in turn, depends on soil wetness and generally varies between 0.9 and 1.0.

The net radiation received by the soil surface is transformed into heat, which warms the soil and air and vaporizes water. We can thus write the surface energy balance as follows:

$$J_n = S + A + LE \quad (12.5)$$

where S is the soil heat flux (the rate at which heat is transferred from the surface downward into the soil profile), A is the “sensible” heat flux transmitted from the surface to the air above, and LE is the evaporative heat flux, a product of the evaporative rate E and the latent heat per unit quantity of water evaporated, L . The total surface energy balance [combining Eqs. (12.4) and (12.5)] is:

$$(J_s + J_a)(1 - \alpha) + J_{li} - J_{lo} - S - A - LE = 0 \quad (12.6)$$

Conventionally, all components of the energy balance are taken as positive if directed toward the surface and negative otherwise.

Sample Problem

Consider the energy balance of a bare-surface soil, assuming the following conditions: The daytime (12-hr) average global (sun and sky) radiation is $3.35 \times 10^4 \text{ J/m}^2 \text{ min}$ ($0.8 \text{ cal/cm}^2 \text{ min}$). The albedo is 0.15. The average soil surface temperature during the diurnal period is 27°C . Advection in daytime balances the outflow of sensible heat during the night, so diurnal net sensible heat exchange with the atmosphere is negligible. Evaporation is 2 mm/day . The emissivity is 0.9, and the atmosphere returns 60% of the long-wave radiation emitted by the ground. Estimate the daytime soil heat transfer term. Is it positive or negative?

Following Eq. (12.6), the diurnal energy balance can be written

$$J_s(1 - \alpha) - J_l - S - A - LE = 0$$

where J_s is incoming global short-wave radiation, α is albedo, J_l is net long-wave emitted radiation, S is heat flow into the soil, A is sensible heat transfer to the air, and LE is latent heat loss. To obtain S , we rearrange this equation to read

$$S = J_s(1 - \alpha) - J_l - A - LE$$

For the net short-wave radiation, we have

$$\begin{aligned} J_s(1 - \alpha) &= (3.35 \times 10^4 \text{ J/m}^2 \text{ min})(720 \text{ min/day})(1 - 0.15) \\ &= 2.05 \times 10^7 \text{ J/m}^2 \text{ daytime} \end{aligned}$$

Net outgoing long-wave radiation (using the *Stefan-Boltzmann law*) is:

$$J_l = 0.4 (\epsilon \sigma T^4)$$

Note: The total energy radiated per unit surface of a “black body” in unit time is proportional to the fourth power of the thermodynamic temperature. The constant of proportionality has the value of $5.67 \times 10^{-8} \text{ J sec}^{-1} \text{ m}^{-2} \text{ K}^{-4}$.

$$\begin{aligned} J_l &= 0.4 \times 0.9 \times (5.67 \times 10^{-8} \text{ J sec}^{-1} \text{ m}^{-2} \text{ K}^{-4})(8.64 \times 10^4 \text{ sec/diurnal})(273 + 27)^4 \text{ K} \\ &= 1.43 \times 10^7 \text{ J/m}^2 \end{aligned}$$

Note: The factor 0.4 is used because, as stated, the atmosphere returns 60% of the outgoing J_l . Also, the emission of longwave radiation takes place throughout the 24-hr day.

The net sensible heat transfer A is negligible. For the latent heat loss term, we have

$$\begin{aligned} LE &= 2.43 \times 10^6 \text{ J/kg} \times 2 \text{ kg/m}^2 \text{ day} = 4.86 \times 10^6 \text{ J/m}^2 \text{ per daytime} \\ &= 580 \text{ cal/g} \times 0.2 \text{ g/cm}^2 \text{ day} = 116 \text{ cal/cm}^2 \text{ day} \end{aligned}$$

Note: Evaporation = $2 \text{ mm/day} = 2 \text{ L/m}^2 \text{ day} = 2 \text{ kg/m}^3$ (water density = 1000 kg/m^3).

Finally, we can sum up all of these quantities to obtain the soil heat flow:

$$\begin{aligned} S &= 2.05 \times 10^7 - 1.43 \times 10^7 - 0 - 0.49 \times 10^7 \\ &= 1.3 \times 10^6 \text{ J/m}^2 \text{ per diurnal} (\sim 32.1 \text{ cal/cm}^2 \text{ day}) \end{aligned}$$

Ergo, the soil is gaining heat. If this amount of heat is absorbed in the top 0.2 m of the soil with a specific heat capacity of 2000 J/kg and a bulk density of 1600 kg/m^3 , it will raise the temperature by 2 degrees.

CONDUCTION OF HEAT IN SOIL

The conduction of heat in solids was analyzed as long ago as 1822 by Fourier, whose name is given to the linear equation that has been used ever since to describe heat conduction. This equation is mathematically analogous

to the diffusion equation (Fick's law) as well as to Ohm's law for the conduction of electricity and Darcy's law for water flow in soil.

The first law of heat conduction, known as *Fourier's law*, states that the flux of heat in a homogeneous body is in the direction of, and proportional to, the temperature gradient:

$$q_h = -\kappa \nabla T \quad (12.7)$$

Here q_h is the thermal flux (i.e., the amount of heat conducted across a unit cross-sectional area in unit time), κ (Greek letter kappa) is thermal conductivity, and ∇T is the spatial gradient of temperature T . In one-dimensional form, this law is written

$$q_h = -\kappa_x (dT/dx) \quad \text{or} \quad q_h = -\kappa_z (dT/dz) \quad (12.8)$$

Here dT/dx is the temperature gradient in any direction, designated x , and dT/dz is, specifically, the gradient in the vertical direction representing soil depth ($z = 0$ being the soil surface). The subscripts attached to the thermal conductivity term are meant to account for the possibility that this parameter may have different values in different directions (i.e., that it may be nonisotropic). The negative sign in these equations is due to the fact that heat flows from a higher to a lower temperature (i.e., in the direction of, and in proportion to, a *negative* temperature gradient).

Equation (12.7) is sufficient to describe heat conduction under steady-state conditions, that is to say, where the temperature at each point in the conducting medium is invariant and the flux is constant in time and space. To account for nonsteady (transient) conditions, we need a second law analogous to Fick's second law of diffusion as embodied in Eq. (9.13). To obtain the second law of heat conduction, we invoke the *principle of energy conservation* in the form of the *continuity equation*, stating that, in the absence of internal sources or sinks of heat, the time rate of change in heat content of a volume element must equal the change of flux with distance:

$$\rho c_m (\partial T / \partial t) = -\nabla \cdot q_h \quad (12.9)$$

where ρ is mass density and c_m *specific heat capacity per unit mass* (defined as the change in heat content of a unit mass of the body per unit change in temperature). The product ρc_m (often designated C) is the *specific heat capacity per unit volume*, and $\partial T / \partial t$ is the time rate of temperature change. Note that the symbol ρ represents the total mass per unit volume, including the water in the case of a moist soil. The symbol ∇ (del) is the shorthand representation of the three-dimensional gradient. Equation (12.9) can thus be restated as

$$\rho c_m (\partial T / \partial t) = -(\partial q_x / \partial x + \partial q_y / \partial y + \partial q_z / \partial z)$$

where x, y, z are the orthogonal direction coordinates.

Combining Eqs. (12.9) and (12.7), we obtain the desired *second law of heat conduction*:

$$\rho c_m (\partial T / \partial t) = -\nabla \cdot (\kappa \nabla T) \quad (12.10)$$

which, in one-dimensional form, is

$$\rho c_m (\partial T / \partial t) = (\partial / \partial x) [\kappa (\partial T / \partial x)] \quad (12.11)$$

Sometimes we may need to account for the possible occurrence of heat sources or sinks in the realm where heat flow takes place. Heat sources include such phenomena as organic matter decomposition, wetting of initially dry soil material, and condensation of water vapor. Heat sinks are generally associated with evaporation. Lumping all these sources and sinks into a single term S , we can rewrite the last equation as

$$\rho c_m (\partial T / \partial t) = (\partial / \partial x) [\kappa (\partial T / \partial x)] \pm S(x, t) \quad (12.12)$$

in which the source–sink term is shown as a function of both space and time.

The ratio of the thermal conductivity κ to the volumetric heat capacity C ($= \rho c_m$) is called the *thermal diffusivity*, designated D_T . Thus,

$$D_T = \kappa / C \quad (12.13)$$

Substituting D_T for κ , we can rewrite Eq. (12.8) and (12.11):

$$q_h = -D_T C (\partial T / \partial x) \quad (12.14)$$

In the special case where D_T can be taken as constant (not a function of distance), we can write

$$\partial T / \partial t = D_T (d^2 T / dx^2) \quad (12.15)$$

To solve the foregoing equations so as to obtain a description of how temperature varies in space and time, we need to know, by measurement or calculation, the pertinent values of the three parameters just defined, namely, the volumetric heat capacity C , thermal conductivity κ , and thermal diffusivity D_T . Together, they are called the *thermal properties of soils*.

VOLUMETRIC HEAT CAPACITY OF SOILS

A soil's volumetric heat capacity C is defined as the change of a unit volume's heat content per unit change in temperature. It is expressed as calories per cubic centimeter per degree or joules per cubic meter per degree. Thus, C depends on the composition of the soil's solid phase (mineral and organic components), on bulk density, and on soil wetness (Table 12.1).

TABLE 12.1 Densities and Volumetric Heat Capacities of Soil Constituents (at 10°C) and of Ice (at 0°C)

Constituent	Density ρ		Heat capacity C	
	(g/cm ³)	(kg/m ³)	(cal/cm ³ K)	(J/m ³ K)
Quartz	2.66	2.66×10^3	0.48	2.0×10^6
Other minerals (average)	2.65	2.65×10^3	0.48	2.0×10^6
Organic matter	1.3	1.3×10^3	0.6	2.5×10^6
Water (liquid)	1.0	1.0×10^3	1.0	4.2×10^6
Ice	0.92	0.92×10^3	0.45	1.9×10^6
Air	0.00125	1.25	0.003	1.25×10^3

The value of C can be estimated by summing the heat capacities of the various constituents, weighted according to their volume fractions. As given by de Vries (1975a), it is

$$C = \Sigma(f_{si}C_{si} + f_wC_w + f_aC_a) \quad (12.16)$$

Here, f denotes the volume fraction of each phase: solid (subscripted s), water (w), and air (a).

The solid phase includes a number of components subscripted i , such as various minerals and organic matter, and the symbol Σ indicates the summation of the products of their respective volume fractions and heat capacities. The C value for water, air, and each component of the solid phase is the product of the particular density and the specific heat per unit mass (i.e., $C_w = \rho_w c_{mw}$, $C_a = \rho_a c_{ma}$, $C_{si} = \rho_{si} c_{mi}$).

Most of the minerals composing soils have nearly the same values of density (about 2.65 g/cm^3 , or $2.65 \times 10^3 \text{ kg/m}^3$) and of heat capacity ($0.48 \text{ cal/cm}^3 \text{ K}$, or $2.0 \times 10^6 \text{ J/m}^3 \text{ K}$). Since it is difficult to separate the different kinds of organic matter present in soils, it is tempting to lump them all into a single constituent (with an average density of about 1.3 g/cm^3 , or $1.3 \times 10^3 \text{ kg/m}^3$, and an average heat capacity of about $0.6 \text{ cal/cm}^3 \text{ K}$, or $2.5 \times 10^6 \text{ J/m}^3 \text{ K}$).

Although the density of water is less than half that of mineral matter (about 1 g/cm^3 , or $1.0 \times 10^3 \text{ kg/m}^3$), its specific heat is more than twice as large ($1 \text{ cal/cm}^3 \text{ K}$, or $4.2 \times 10^6 \text{ J/m}^3 \text{ K}$). Finally, since the density of air is only about 1/1000 that of water, its contribution to the specific heat of the composite soil can generally be neglected.

Thus, Eq. (12.16) can be simplified as follows:

$$C = f_m C_m + f_o C_o + f_w C_w \quad (12.17)$$

where subscripts m , o , w refer to mineral matter, organic matter, and water, respectively. Note that $f_m + f_o + f_w = 1 - f_a$ and the total porosity $f = f_a + f_w$. The reader will recall that in preceding chapters we designated the volume fraction of water f_w as θ . Knowing the approximate average values of C_m , C_o and C_w , we can further simplify Eq. (12.17) to give

$$C = 0.48f_m + 0.60f_o + f_w \quad (12.18)$$

The use of Eq. (12.18) must be qualified in the case of frozen or partially frozen soils, since the properties of ice differ from those of liquid water ($\rho = 0.92 \text{ g/cm}^3$, or $0.92 \times 10^3 \text{ kg/m}^3$, and $C = 0.45 \text{ cal/cm}^3 \text{ K}$, or $1.9 \times 10^6 \text{ J/m}^3 \text{ K}$). In typical mineral soils, the volume fraction of solids is in the range of 0.45–0.65, and C values range from about $1 \text{ MJ/m}^3 \text{ K}$ (less than $0.25 \text{ cal/cm}^3 \text{ K}$) in the dry state to about $3 \text{ MJ/m}^3 \text{ K}$, or $0.75 \text{ cal/cm}^3 \text{ K}$, in the water-saturated state.

The measurement of heat capacity and specific heat is described by Kluitenberg (2002).

THERMAL CONDUCTIVITY AND DIFFUSIVITY

Thermal conductivity κ is defined as the quantity of heat transferred through a unit area of the conducting body in unit time under a unit temperature gradient. As shown in Table 12.2, the thermal conductivities of specific

TABLE 12.2 Thermal Conductivities of Soil Constituents (at 10°C) and of Ice (at 0°C)

Constituent	mcal/cm sec K	(W/m K)
Quartz	21	8.8
Other minerals (average)	7	2.9
Organic matter	0.6	0.25
Water (liquid)	1.37	0.57
Ice	5.2	2.2
Air	0.06	0.025

soil constituents differ widely (see also Table 12.3). Hence the space-averaged (macroscopic) thermal conductivity of a soil depends on its mineral composition and organic matter content as well as on the volume fractions of water and air.

Since the thermal conductivity of air is very much smaller than that of water or solid matter, a high air content (or low water content) corresponds to a low thermal conductivity. Moreover, since the proportions of water and air vary continuously, κ is also time variable. Soil composition is seldom uniform in depth; hence κ is generally a function of depth as well as of time. It also varies with temperature, but under normal conditions this variation is ignored. Unlike heat capacity, thermal conductivity is sensitive not only to the mineral composition of a soil but also to the sizes, shapes, and arrangements of soil particles. In the normal range of soil wetness experienced in the field, C may undergo a threefold or fourfold change, whereas the corresponding change in κ may be a hundredfold or more.

TABLE 12.3 Average Thermal Properties of Soils and Snow^a

Soil type	Porosity f	Volumetric wetness θ	Thermal conductivity (10^{-3} cal/cm sec °C)	Volumetric heat capacity C_v (cal/cm sec °C)	Damping depth (diurnal) d (cm)
Sand	0.4	0.0	0.7	0.3	8.0
	0.4	0.2	4.2	0.5	15.2
	0.4	0.4	5.2	0.7	14.3
Clay	0.4	0.0	0.6	0.3	7.4
	0.4	0.2	2.8	0.5	12.4
	0.4	0.4	3.8	0.7	12.2
Peat	0.8	0.0	0.14	0.35	3.3
	0.8	0.4	0.7	0.75	5.1
	0.8	0.8	1.2	1.15	5.4
Snow	0.95	0.05	0.15	0.05	9.1
	0.8	0.2	0.32	0.2	6.6
	0.5	0.5	1.7	0.5	9.7

^a After van Wijk and de Vries (1963).

The relationship between the overall thermal conductivity of a soil and the specific conductivities and volume fractions of the soil's constituents is very intricate. Two relatively simple alternative cases can be envisaged: a dry soil and a water-saturated soil with the same internal structure. In both cases we have a two-phase system in which the particles are dispersed in a continuous fluid (air or water) with a volume fraction f_0 and thermal conductivity κ_0 . The particles then occupy a volume fraction $f_1 = 1 - f_0$ and have a thermal conductivity κ_1 . A composite thermal conductivity for the medium can be defined as follows: Consider a representative cube of soil with side L , large in comparison with the diameters of the particles and pores. Assume that the upper face is at a temperature T_1 and the bottom face is at a lower temperature, T_2 . A constant heat flux q_h will then pass through the cube, proportional to the temperature gradient, with κ_c as the proportionality factor for the composite medium:

$$q_h = -\kappa_c (dT/dx) = \kappa_c (T_1 - T_2)/l$$

Since the cube is a mixture of two phases, the composite thermal conductivity κ_c will be intermediate between κ_0 and κ_1 . According to de Vries (1975a),

$$\kappa_c = (f_0\kappa_0 + kf_1\kappa_1)/(f_0 + k) \quad (12.19)$$

where the factor k is the ratio of the average temperature gradient in the particles to the corresponding gradient in the continuous fluid:

$$k = (dT/dz)_2/(dT/dz)_1$$

The value of k depends not only on the ratio κ_1/κ_0 , but also on the particle sizes, shapes, and mode of packing.

If there are several types of particles with different shapes or conductivities, Eq. (12.20) can be generalized:

$$\kappa_c = \frac{\sum_{i=1}^n k_i f_i \kappa_i}{\sum_{i=1}^n k_i f_i} \quad (12.20)$$

Here n is the number of particle classes within which all particles have about the same shape and conductivity. The thermal conductivity of soils of widely differing compositions can be estimated by Eq. (12.20).

The following form of Eq. (12.20), for an unsaturated soil, was used by van Bavel and Hillel (1975, 1976):

$$\kappa_c = (f_w\kappa_w + k_s f_s \kappa_s + k_a f_a \kappa_a)/(f_w + k_s f_s + k_a f_a)$$

where κ_w , κ_a , and κ_s are the specific thermal conductivities of the soil constituents (water, air, and an average value for the solids, respectively). The factor k_s represents the ratio between the space average of the temperature gradient in the solidphase and that in the water phase. It depends on grain shapes as well as on mineral composition and organic matter content. The k_a factor represents the corresponding ratio for the thermal gradient in the air and water phases.

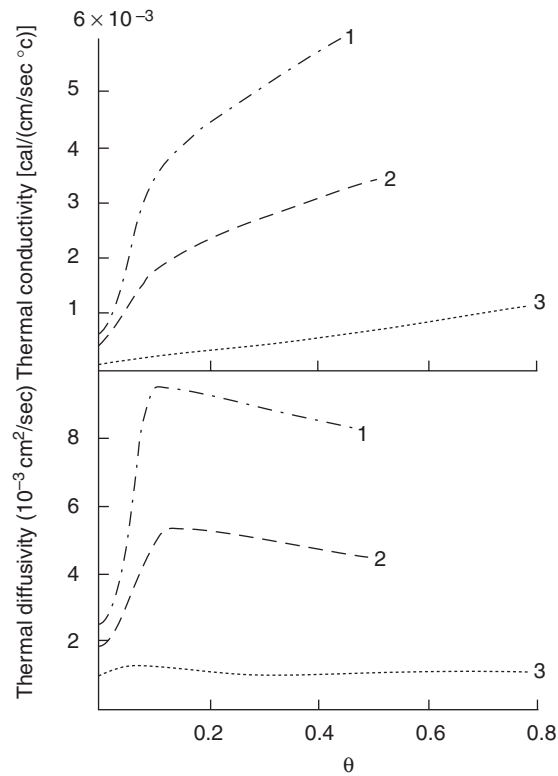


Fig. 12.1. Thermal conductivity and thermal diffusivity as functions of volume wetness (volume fraction of water) for (1) sand (bulk density 1460 kg/m^3 , volume fraction of solids 0.55); (2) loam (bulk density 1330 kg/m^3 , volume fraction of solids 0.5); and (3) peat (volume fraction of solids 0.2). (After de Vries, 1975.)

The dependence of thermal conductivity and diffusivity on soil wetness is illustrated in Fig. 12.1. The influence of latent heat transfer by the diffusion of water vapor in the air-filled pores is proportional to the temperature gradient in these pores. It can be taken into account (van Bavel and Hillel, 1976; Hillel, 1977) by adding to the thermal conductivity of air an apparent conductivity due to evaporation, transport, and condensation of water vapor (the so-called *vapor enhancement factor*). This value is strongly temperature dependent and rises rapidly with increasing temperature.

Methods of measuring thermal conductivity were summarized by Jackson and Taylor (1986) and more recently by Bristow (2002).

The thermal diffusivity D_h , instead of the conductivity κ , is sometimes desired (Horton, 2002). It can be defined as the change in temperature produced in a unit volume by the quantity of heat flowing through the volume in unit time under a unit temperature gradient. An alternative definition, easier to perceive, is that the thermal diffusivity is the ratio of the conductivity to the product of the specific heat and density:

$$D_h = \kappa / c_s \rho = \kappa C_v \quad (12.21)$$

where C_v is the volumetric heat capacity. As shown in the preceding section, the specific heat and density of both solids and water must be considered when calculating C_v :

$$C_v = \rho_s(c_s + c_w w) \quad (12.22)$$

where ρ_s is the density of dry soil, c_s is the specific heat of dry soil, c_w is the specific heat of water, and w is the ratio of the mass of water to the mass of dry soil. The thermal diffusivity can be measured directly, as described by Jackson and Taylor (1986).

SIMULTANEOUS TRANSPORT OF HEAT AND MOISTURE

The flows of water and of thermal energy under nonisothermal conditions in the soil are interactive phenomena: The one entails the other. Temperature gradients affect the moisture-potential field and induce both liquid and vapor movement. Reciprocally, moisture gradients move water, which carries heat. The simultaneous occurrence of temperature gradients and moisture gradients in the soil therefore brings about the combined transport of heat and moisture. This combined transport can generally be ignored in the extreme cases of a saturated or nearly saturated soil and of a nearly dry soil. In the former, the influence of temperature gradients on liquid water flow is generally small in comparison with the influence of gravity or pressure gradients; in the latter, the movement of heat can entail no significant movement of either liquid water or vapor. Thus, we are left with the problem of how to deal with the wide range of intermediate situations in which transport of liquid water and of vapor can be both significant and mutually influenced.

Two separate approaches to the combined transfer of heat and moisture have been attempted: (1) a *mechanistic approach*, based on a physical model of the soil system, and (2) a *thermodynamic approach*, based on the phenomenology of irreversible processes in terms of coupled forces and fluxes. Though starting from different points of view, the two approaches have been shown to be related and, properly formulated, can be cast into an equivalent mold (Groenevelt and Bolt, 1969; Jury, 1973).

The mechanistic approach was originally formulated by Philip and de Vries (1957). Their model was based on the concept of viscous flow of liquid water under the influence of gravity and of capillary and adsorptive forces and on the concept of vapor movement by diffusion. Local “microscopic-scale” thermodynamic equilibrium between liquid and vapor was assumed to exist at all times and at each point within the soil. The general differential equation describing moisture movement in a porous system under combined temperature and moisture gradients for unidimensional vertical flow is, accordingly,

$$\partial\theta/\partial t = \nabla \cdot (D_T \nabla T) + \nabla \cdot (D_w \nabla \theta) - \partial K/\partial z \quad (12.23)$$

where q is volumetric wetness, t is time, T is absolute temperature, D_T is the water diffusivity under a temperature gradient (the sum of the liquid and vapor diffusivities), D_w is the water diffusivity under a moisture gradient, K is the hydraulic conductivity, and z is the vertical space coordinate. The last term

on the right-hand side is due to the gravity gradient and becomes positive if z is taken to be increasing downward.

The heat transfer equation is, similarly,

$$C_v \partial T / \partial t = \nabla \cdot (\kappa \nabla T) - L \nabla \cdot (D_{w,vap} \nabla \theta) \quad (12.24)$$

Here C_v is volumetric heat capacity, κ is apparent thermal conductivity of the soil, L is latent heat of vaporization of water, and $D_{w,vap}$ is diffusivity for heat conveyed by water movement (mostly vapor). The preceding equations are both of the diffusion type, involving θ - and T -dependent diffusivities as well as gradients of both θ and T .

Taken together, Eqs. (12.23) and (12.24) describe the coupled transport of moisture and heat in soils. The mechanistic nature of the theory and of the coefficients involved was explained by de Vries (1975b). The assumption of local thermodynamic equilibrium links the vapor pressure p_v to the matric potential ψ by the following relation: $p_v = p_{vs} h = p_{vs} \exp(Mg\psi/RT)$, where p_{vs} is the saturated vapor pressure at the particular temperature T , h is relative humidity, M is molar mass, g is the acceleration of gravity, and R is the universal gas constant. The diffusivities for water and heat by vapor transport are obtained by use of this relationship. However, the difficulty encountered in making the theory operational is in measuring the diffusivities. A more fundamental problem is that, since the two mechanisms of flow represented in each equation do interact, they are not, strictly speaking, simply additive.

To consider the approach based on *irreversible thermodynamics*, we must first understand in principle the difference between this relatively new branch of science and the older, “classical” thermodynamics, which deals with reversible processes and equilibrium states. Classical thermodynamics can predict whether, and in what direction (but not at what rate), a spontaneous process will occur in a system not at equilibrium. However, in a natural system any number of different forces might be operating simultaneously to produce mutually interacting fluxes in a combination of irreversible processes. For instance, a concentration gradient causes diffusion, a pressure gradient causes convection, and a temperature gradient results in the transfer of heat, with each of these fluxes affecting the others. If the system is not too far from equilibrium, the fluxes are taken to be related linearly to the forces causing them.

In application to simultaneous water and heat flow, as an example, the approach based on the thermodynamics of irreversible processes formulates a pair of *phenomenological equations* in which the fluxes of moisture q_w and heat q_h are expressed as linear functions of the moisture potential (e.g., pressure) gradient dP/dz and the temperature gradient dT/dz :

$$\begin{aligned} q_w &= -L_{ww}(1/T) (dP/dz) - L_{wh}(1/T^2)(dT/dz) \\ q_h &= -L_{hw}(1/T) (dP/dz) - L_{hh}(1/T^2)(dT/dz) \end{aligned} \quad (12.25)$$

The four phenomenological coefficients occurring in these equations (L_{ww} , L_{wh} , L_{hw} , L_{hh} , relating water flow to the water potential gradient, water flow to the thermal potential gradient, heat flow to the water potential gradient, and heat flow to the thermal potential gradient, respectively) are unknown functions of P (or θ) and T . According to *Onsager's theorem* (Katchalsky and Curran, 1965), the cross-coupling coefficients L_{wh} and L_{hw} are equal when the

fluxes and forces are properly formulated. Thus, the number of coefficients that must be measured is reduced.

An apparent advantage of the irreversible thermodynamics approach is that it makes no *a priori* assumptions regarding the mechanisms of the transport phenomena formulated. Hence it would seem to be less restrictive than a physical theory, whose validity is constrained at the outset by its mechanistic assumptions. The disadvantage of the approach, however, is precisely in its failure to provide insight into the nature and internal workings of the processes considered.

THERMAL REGIME OF SOIL PROFILES

In nature, soil temperature varies continuously in response to the ever-changing meteorological regime acting on the soil-atmosphere interface. That regime is governed by a regular periodic succession of days and nights and of summers and winters. Yet the regular diurnal and annual cycles are perturbed by such irregular episodic phenomena as cloudiness, cold waves, warm waves, rainstorms or snowstorms, and periods of drought. Added to these external influences are the soil's own changing properties (i.e., temporal changes in reflectivity, heat capacity, and thermal conductivity as the soil alternately wets and dries, and the variation of all these properties with depth), as well as the influences of geographic location, vegetative cover, and — finally — human management. All these labile factors complicate the effort to define the thermal regime of soil profiles.

The simplest mathematical representation of nature's fluctuating thermal regime is to assume that at all depths in the soil the temperature oscillates as a pure harmonic (sinusoidal) function of time around an average value. Since nature's actual variations are not so orderly, this may be a rather crude approximation. Nonetheless, it is an instructive exercise in itself, and when used in conjunction with field data it may lead to a better understanding, and perhaps even provide a basis for the prediction, of a soil's thermal regime.

To begin, let us assume that although soil temperature varies differently at different depths in the soil, the average temperature is the same for all depths. We next choose a starting time ($t = 0$) such that the surface is at the average temperature. The temperature at the surface can then be expressed as a function of time (Fig. 12.2):

$$T(0,t) = T_{\text{ave}} + A_0 \sin \omega t \quad (12.26)$$

where $T(0,t)$ is the temperature at $z = 0$ (the soil surface) as a function of time t , T_{ave} is the average temperature of the surface (as well as of the profile), and A_0 is the amplitude of the surface-temperature fluctuation (the range from maximum, or from minimum, to the average temperature). Finally, ω is the radial frequency, which is 2π times the actual frequency. In the case of diurnal variation, the period is 86,400 sec (24 hr), so $\omega = 2\pi/86,400 = 7.27 \times 10^{-5}/\text{sec}$. Note that the argument of the sine function is expressed in radians rather than in degrees.

The last equation is the boundary condition for $z = 0$. For the sake of convenience, let us assume that at infinite depth ($z = \infty$) the temperature is constant

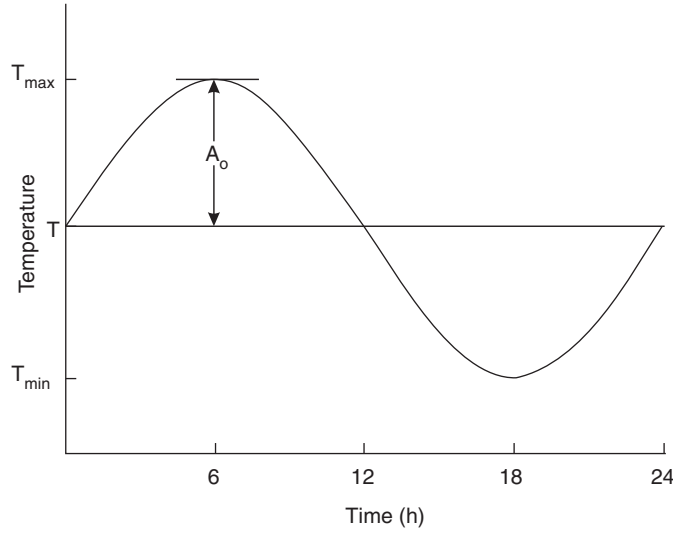


Fig. 12.2. Idealized daily fluctuation of surface soil temperature, according to the equation $T = T_{ave} + A_o \sin(\omega t/p)$, where T is temperature, T_{ave} is average temperature, A_o is amplitude, t is time, and p is the period of the oscillation (in this case, p refers to the diurnal 24 hr).

and equal to T_{ave} . Under these circumstances, the temperature at any depth z can also be represented as a sine function of time, as shown in Fig 12.3:

$$T(z, t) = T_{ave} + A_z \sin[\omega t + \phi(z)] \quad (12.27)$$

in which A_z is the amplitude at depth z . Both A_z and $\phi(z)$ are functions of z but not of t . They can be determined by substituting the solution of Eq. (12.26) in the differential equation $\partial T / \partial t = \kappa(\partial^2 T / \partial z^2)$. This leads to the solution

$$T(z, t) = T_{ave} + A_0 [\sin(\omega t - z/d)] / e^{-z/d} \quad (12.28)$$

The constant d is a characteristic depth, called the *damping depth*, at which the temperature amplitude equals $1/e$ ($1/2.718 = 0.37$) of the amplitude at the soil surface A_0 . The damping depth is related to the thermal properties of the soil and the frequency of the temperature fluctuation:

$$d = (2\kappa / C\omega)^{1/2} = (2D_h / \omega)^{1/2} \quad (12.29)$$

It is seen that at any depth the amplitude of the temperature fluctuation A_z is smaller than A_0 by a factor $e^{z/d}$ and that there is a phase shift (a time delay of the temperature peak) equal to $-z/d$. The decrease of amplitude and increase of phase lag with depth are typical phenomena in the propagation of a periodic temperature wave in the soil.

The physical reason for the damping and retarding of the temperature waves with depth is that a certain amount of heat is absorbed or released along the path of heat propagation when the temperature of the conducting soil increases or decreases, respectively. The damping depth is related inversely to the frequency, as can be seen from Eq. (12.29). Hence it depends directly on the period of the temperature fluctuation considered. The damping depth is

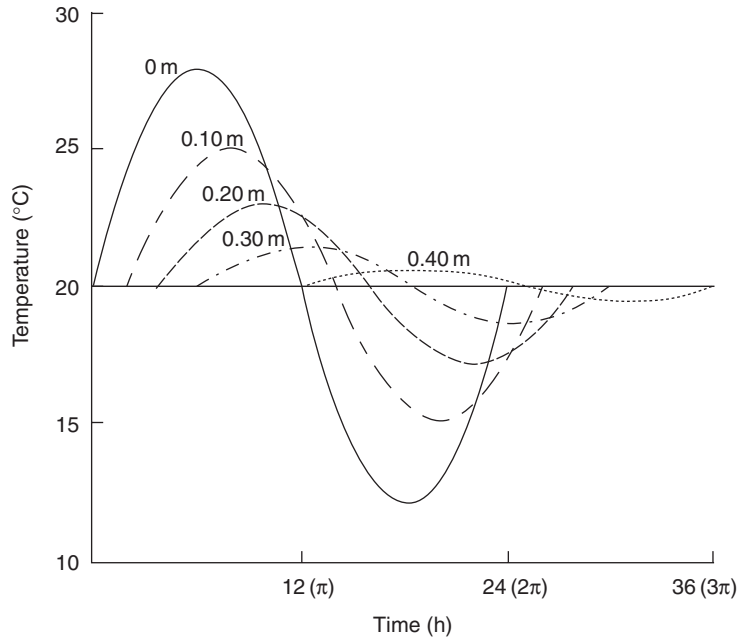


Fig. 12.3. Idealized variation of soil temperature with time for various depths. Note that at each succeeding depth the peak temperature is damped and shifted progressively in time. Thus, the peak at a depth of 0.4 m lags about 12 hr behind the temperature peak at the surface and is only about 1/16 of the latter. In this hypothetical case, a uniform soil was assumed, with a thermal conductivity of 1.68 J/m sec deg (or $4 \times 10^{-3} \text{ cal/cm sec deg}$) and a volumetric heat capacity of $2.1 \times 10^6 \text{ J/m}^3 \text{ deg}$ ($0.5 \text{ cal/cm}^3 \text{ deg}$).

$(365)^{1/2} = 19$ times larger for the annual variation than for the diurnal variation in the same soil. For example, van Wijk and de Vries (1963) calculated the damping depth for a soil with $\kappa = 0.96 \text{ J/m sec deg}$ (equal to $2.3 \times 10^{-3} \text{ cal/cm sec deg}$) and obtained $d = 0.12 \text{ m}$ for the diurnal temperature fluctuation and $d = 2.29 \text{ m}$ for the annual fluctuation. Whereas the amplitude at depth $z = d$ is 0.37 as great as the amplitude at the surface, it is only about 0.05 of the surface amplitude at $z = 3d$ (0.36 m for the diurnal variation in the case of the soil used by these authors). When an arbitrary zero point t_0 is introduced into the time scale, Eq. (12.28) becomes

$$T(z, t) = T_{\text{ave}} + A_0[\sin(\omega t + \phi_0 - z/d)]/e^{z/d} \quad (12.30)$$

The constant ϕ_0 is called the *phase constant*.

The annual variation of soil temperature down to considerable depth causes deviations from the simplistic assumption that the daily average temperature is the same for all depths in the profile. The combined effect of the annual and diurnal variations of soil temperature can be expressed by

$$T(z, t) = T_{\text{ave}, y} + A_y[\sin(\omega_y t + \phi_y - z/d_y)]/e^{z/d_y} + A_d[\sin(\omega_d t + \phi_d - z/d_d)]/e^{z/d_d} \quad (12.31)$$

wherein the subscripted indices y and d refer to the yearly and daily temperature waves, respectively. Thus $T_{ave,y}$ is the annual mean temperature. The daily cycles are now seen to be short-term perturbations superimposed on the annual cycle. Vagaries of weather (e.g., spells of cloudiness or rain) can cause considerable deviations from simple harmonic fluctuations, particularly for the daily cycles. Longer-term climatic irregularities can also affect the annual cycle, of course. Also, since the annual temperature wave penetrates much more deeply than the daily wave, the assumptions of soil homogeneity in depth and of the time constancy of soil thermal properties are clearly unrealistic (Gao et al., 2003).

An alternative theoretical approach is possible, one with fewer constraining assumptions. It is based on numerical, rather than analytical, methods for solving the differential equations of heat conduction. Computer-based mathematical simulation models now allow soil thermal properties to vary in time and space (e.g., in response to periodic changes in soil wetness) so as to account for alternating surface saturation and desiccation and for profile layering. They also allow various climatic inputs to follow more realistic and irregular patterns. The surface amplitude of temperature need no longer be taken to be an independent variable, but one that depends on the surface energy balance and thus is affected by both soil properties and above-soil conditions. Examples of the numerical approach can be found in the published works of van Bavel and Hillel (1975, 1976), Hillel (1977), and Evett et al. (1994).

Other developments of practical importance include techniques for monitoring the soil thermal regime more precisely than was possible previously. One such technique is the *infrared radiation thermometer* for scanning or remote sensing of surface temperature for both fallow and vegetated soils without disturbance of the measured surface. Knowledge of the surface temperature and its variation in time is important in assessing energy exchange between soil and atmosphere as well as in determining boundary conditions for within-soil heat transfer.

An additional technique is the use of *heat flux plates*. These are flat and thin plates or disks of constant thermal conductivity, which allow precise measurement of the temperature difference between their two sides so as to yield the heat flux through them. When embedded horizontally in the soil at regular depth intervals, a series of such heat flux plates can provide a continuous record of heat transfer throughout the profile. There are problems, however. The presence of heat flux plates can distort the flow of heat in the surrounding medium if their thermal conductivity is very different from that of the soil. The experimental error can be minimized by constructing plates of maximal thermal conductivity and minimal thickness and by calibrating them in a medium with a thermal conductivity near to that of the soil in which they are to be placed. Another problem is that such plates preclude vapor flow, which can sometimes be an important component of heat transfer. The use of heat flux plates is described by Fuchs (1986) and by Sauer (2002).

The soil-temperature profile as it might vary from season to season in a frost-free region is illustrated in Fig. 12.4. The diurnal variation of temperature and the directions of heat flow within a soil profile are illustrated in Fig. 12.5.

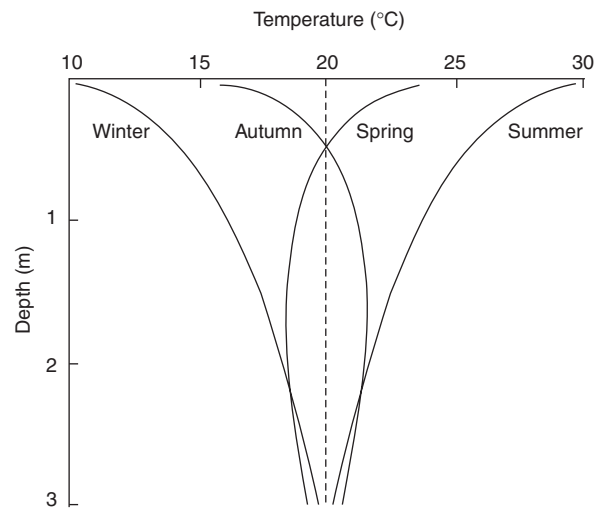


Fig. 12.4. Soil-temperature profile as it varies from season to season in a frost-free region.

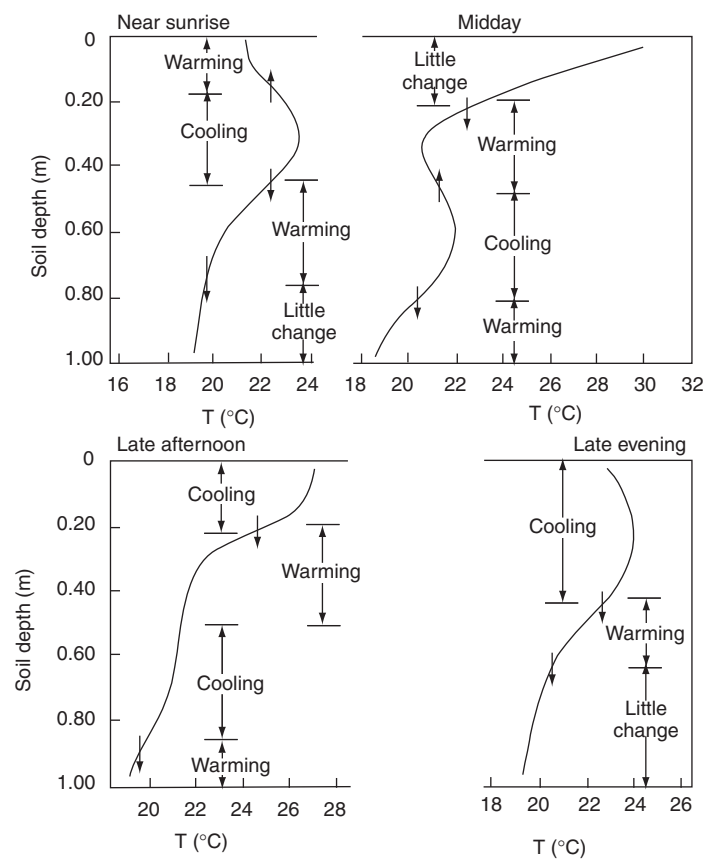


Fig. 12.5. Typical variation of temperature with depth at different times of day in summer. (From Sellers, 1965; data by Carson, 1961.)

Sample Problem

The daily maximum soil-surface temperature is 40°C and the minimum is 10°C. Assume that the diurnal temperature wave is symmetrical, that the mean temperature is equal throughout the profile (with surface temperature equal to the mean value at 6 A.M. and 6 P.M.), and that the “damping depth” is 0.10 m. Calculate the temperatures at noon and midnight for depths 0.05, 0.10, and 0.20 m.

Since the temperature range is 30°C and the mean (T_{ave}) 25°C, and the amplitude at the surface A_0 , the maximum value above the mean is 15°.

Use Eq. (12.28) to calculate the temperature T at any depth z and time t :

$$T(z,t) = T_{ave} + A_0[\sin(\omega t - z/d)]/e^{z/d}$$

where ω is the radial frequency ($2\pi/24$ hr) and d is the “damping depth” at which the temperature amplitude is $1/e$ ($= 0.37$) of A_0 . Note: Radial angle is expressed in radians, not in degrees (i.e., $\sin \pi/2 = 1$, $\sin 3\pi/2 = -1$).

At the soil surface (depth zero):

Noontime temperature (6 hr after $T = T_{ave}$)

$$T(0,6) = 25 + 15 \times [\sin(\pi/2 - 0)]/e^0 = 25 + 15 = 40^\circ\text{C}$$

Midnight temperature (18 hr after $T = T_{ave}$)

$$T(0,18) = 25 + 15 \times [\sin(3\pi/2 - 0)]/e^0 = 25 - 15 = 10^\circ\text{C}$$

At depth 0.05 m:

Noontime temperature:

$$\begin{aligned} T(0.05,6) &= 25 + 15 \times [\sin(\pi/2 - 0.05/0.1)]/e^{0.05/0.1} \\ &= 25 + 15 \times \sin(1.07)/1.65 = 33^\circ\text{C} \end{aligned}$$

Midnight temperature:

$$\begin{aligned} T(0.05,18) &= 25 + 15 \times [\sin(3\pi/2 - 0.05/0.1)]/e^{0.05/0.1} \\ &= 25 + 15 \times \sin(4.21)/1.65 = 17^\circ\text{C} \end{aligned}$$

At depth 0.1 m (the damping depth):

Noontime temperature:

$$\begin{aligned} T(0.1,6) &= 25 + 15 \times [\sin(\pi/2 - 0.1/0.1)]/e^{0.1/0.1} \\ &= 25 + 15 \times \sin(0.57)/2.72 = 28^\circ\text{C} \end{aligned}$$

Midnight temperature:

$$\begin{aligned} T(0.1,18) &= 25 + 15 \times [\sin(3\pi/2 - 0.1/0.1)]/e^{0.1/0.1} \\ &= 25 + 15 \times \sin(3.71)/2.72 = 22^\circ\text{C} \end{aligned}$$

At depth 20 cm:

Noontime temperature:

$$\begin{aligned} T(0.2,6) &= 25 + 15 \times [\sin(\pi/2 - 0.2/0.1)]/e^{0.2/0.1} \\ &= 25 + 15 \times \sin(-0.43)/7.4 = 24.15^\circ\text{C} \end{aligned}$$

Midnight temperature:

$$\begin{aligned}T(0.2, 18) &= 25 + 15 \times [\sin(3\pi/2 - 0.2/0.1)]/e^{0.2/0.1} \\ &= 25 + 15 \times \sin(2.71)/7.4 = 25.85^\circ\text{C}\end{aligned}$$

Note: At a depth of 0.2 m the phase shift is so pronounced that at midnight the temperature is higher than at noon. A useful exercise for students is to plot the sinusoidal course of temperature at each depth, to observe how the phase shift (time lag of maximum and minimum values) increases and the amplitude decreases with depth.