Variační počet a jeho aplikace F4260

Základní informace:

- domácí úkoly
  - nepovinné př. pro vás (1. hod / týden)
  - povinné 1 př. před -nést na cvičení.
- ukončení kolokvium
  1 složitý příklad
  dle domluvy

Literatura:

- Gelfand, Fomin Calculus of variations
- Tycovi skripta a cvičení z teor. mech.
- Krupková, Swaczyna Variační počet

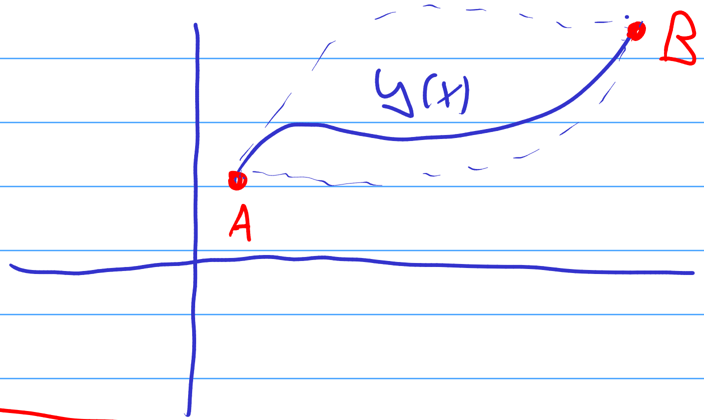
online na:

https://www.physics.muni.cz/~janam/download/O-Krupkova-var-pocet.pdf

I. Cvičení - Eulerova metoda

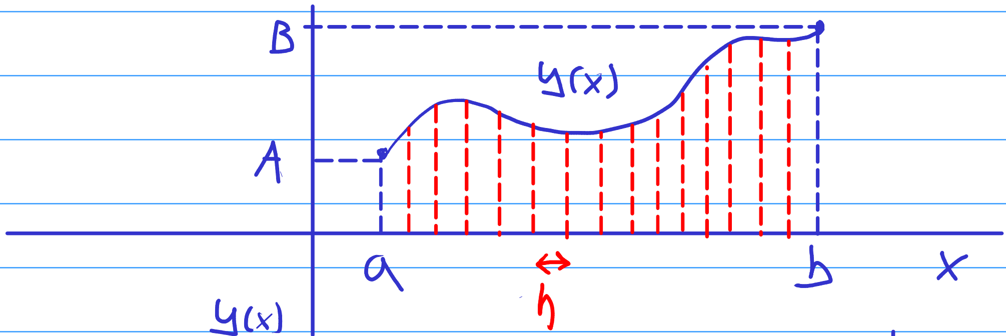
- podmínka stacionárních bodů funkcionálu

$$I[y] = \int_a^b dx \, L\left(x, y(x), \frac{dy(x)}{dx}\right); \quad \begin{matrix} y(a) = A \\ y(b) = B \end{matrix}$$



$$\boxed{\frac{\partial L}{\partial y} - \frac{d}{dx}\frac{\partial L}{\partial y'} = 0}$$

$y(x)$ - křivka



$$h = \frac{b-a}{n}$$

$$x \to \{x_k\}_{k=0}^{n}$$

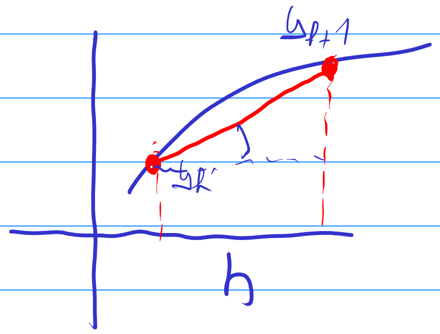
$$x_0 = a$$
$$x_1 = a + h$$
$$x_k = a + kh$$
$$x_n = b$$

$$\int_a^b dx\, L\left(x, y(x), \frac{dy(x)}{dx}\right), \qquad \ell = 0, \dots, n$$

$$x \to \{x_\ell\} \qquad x_\ell$$

$$y \to \{y(x_\ell)\}_{\ell=0}^{n} \qquad y_\ell$$

$$\frac{dy(x)}{dx} \to \frac{y_{\ell+1} - y_\ell}{h}$$



$$\int_a^b dx \to \sum_{\ell=0}^{n} h$$

$$\int_a^b \overset{h}{dx}\, L\left(x, y(x), \frac{dy(x)}{dx}\right) \overset{\approx}{\to} \sum_{\ell=0}^{n} h\, L\left(x_\ell, y_\ell, \frac{y_{\ell+1} - y_\ell}{h}\right)$$

$$= I_h[y] = I_h(y_0, \dots, y_n)$$

$$\frac{\partial I_h}{\partial y_\ell} =$$

$$\sum_{k=0}^{n} h\, L\left(\overset{1}{x_k}, \overset{2}{y_k}, \overset{3}{\frac{y_{k+1}-y_k}{h}}\right)$$

$$\frac{\partial I_h}{\partial y_l} = \sum_{k=0}^{n} h \frac{\partial}{\partial y_l} L\left(x_k, y_k, \frac{y_{k+1}-y_k}{h}\right)$$

$$= \cancel{\sum_{k=0}^{n}} \Bigg(h \frac{\partial L\left(x_k, y_k, \frac{y_{k+1}-y_k}{h}\right)}{\partial y_l} \cdot \cancel{\delta_{kl}} + \qquad z$$

$$h \frac{\partial L\left(x_k, y_k, \frac{y_{k+1}-y_k}{h}\right)}{\partial z} \cdot \left[\frac{\delta_{(k+1)l}}{h} - \frac{\delta_{kl}}{h} \right] \Bigg)$$

$$\frac{\partial I}{\partial y_l} = h \frac{\partial L\left(x_l, y_l, \frac{y_{l+1}-y_l}{h}\right)}{\partial y_l}$$

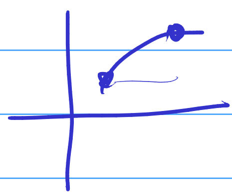
$\delta_{k+1\,l}$

dosaď $k+1=l$

$k = l-1$

$$\frac{\partial L\left(x_{l-1}, y_{l-1}, \frac{y_l-y_{l-1}}{h}\right)}{\partial z} - \frac{\partial L\left(x_l, y_l, \frac{y_{l+1}-y_l}{h}\right)}{\partial z}$$

$$\delta_{kl} \begin{cases} 1 & \text{pokud } k=l \\ 0 & \text{jinak} \end{cases}$$



$$\frac{\partial I}{\partial y_e} = h \frac{\partial L\left(x_e, y_e, \frac{y_{e+1}-y_e}{h}\right)}{\partial y_e}$$

$\partial_{e+1} e$
des $e+1 =$
$k = e -$

$$\frac{\partial L\left(x_{e-1}, y_{e-1}, \frac{y_e - y_{e-1}}{h}\right)}{\partial z} - \frac{\partial L\left(x_e, y_e, \frac{y_{e+1}-y_e}{h}\right)}{\partial z}$$

$$= 0$$

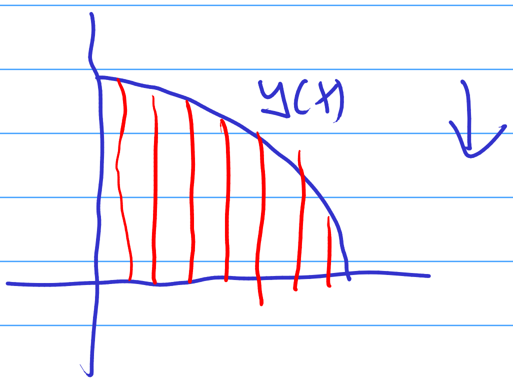
$$\lim_{h \to \infty} \frac{\partial I}{\partial y_e} \frac{1}{h} = \frac{\partial L\left(x, y, \frac{dy}{dx}\right)}{\partial y}$$

$$- \lim \left[\frac{\partial L\left(x_e, y_e, \frac{y_{e+1}-y_e}{h}\right)}{h\, \partial z} - \frac{\partial L\left(x_{e-1}, y_{e-1}, \frac{y_e - y_{e-1}}{h}\right)}{h\, \partial z} \right]$$

$$\text{"} \frac{L_e - L_{e-1}}{h} \text{"}$$

$$= \frac{d}{dx} \frac{\partial L\left(x, y, \frac{dy}{dx}\right)}{\partial z}$$

Zkuste: nepovinný DÚ



$$L = T - V = \frac{1}{2} m v^2 - mgh$$

$$= \frac{1}{2} m \dot{y}^2 - mgy$$

$y \to y_k$

1) $\dot{y} \to \frac{y_{k+1} - y_k}{h}$

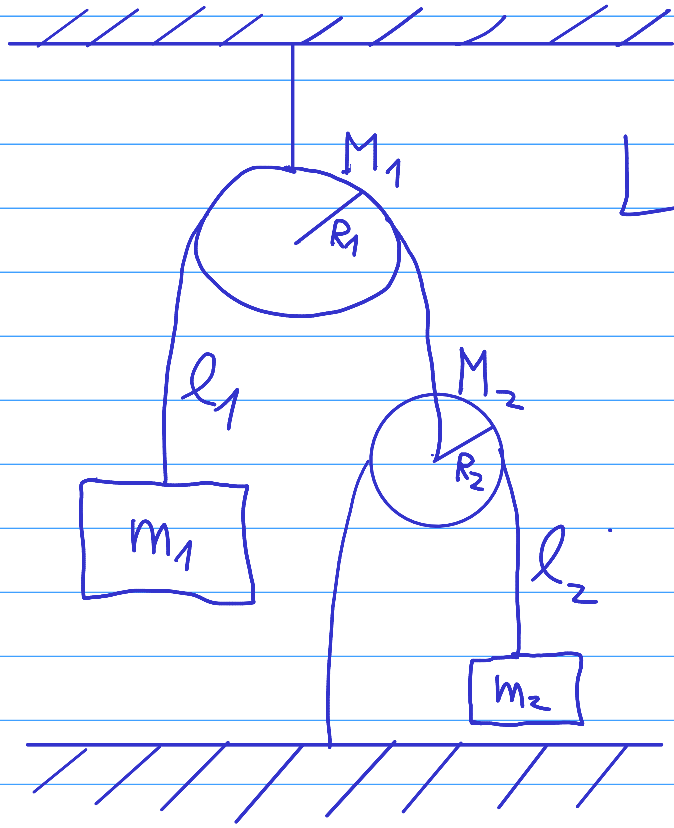
2) $\int L \to \sum L_h$

3) $\frac{\partial I}{\partial y_k}$; vyuzit ∂ a Σ

4) vrátit do $y_k \to y$

II. Cvičení – ukázkové příklady, Kvadratické lagrangiány

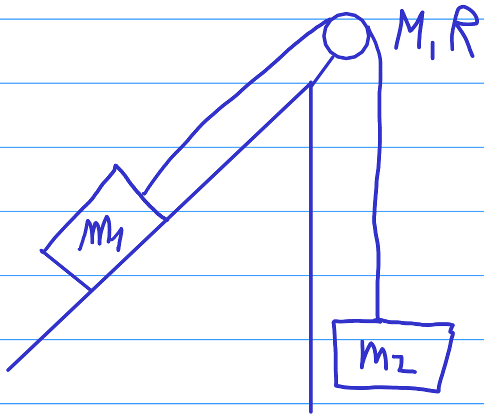
1



$$L = T - V$$

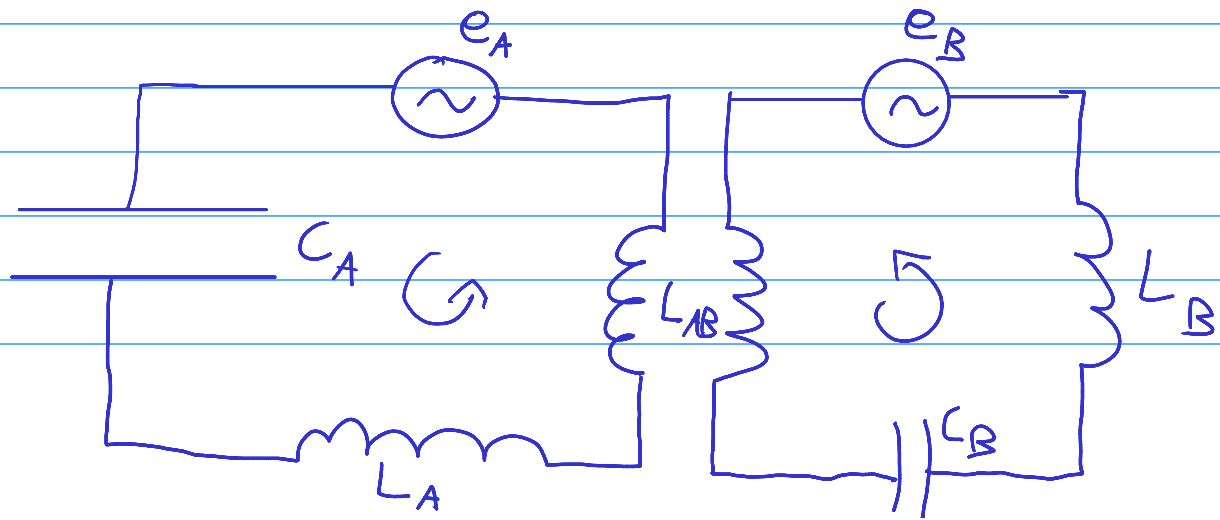
$\downarrow \vec{g}$

2

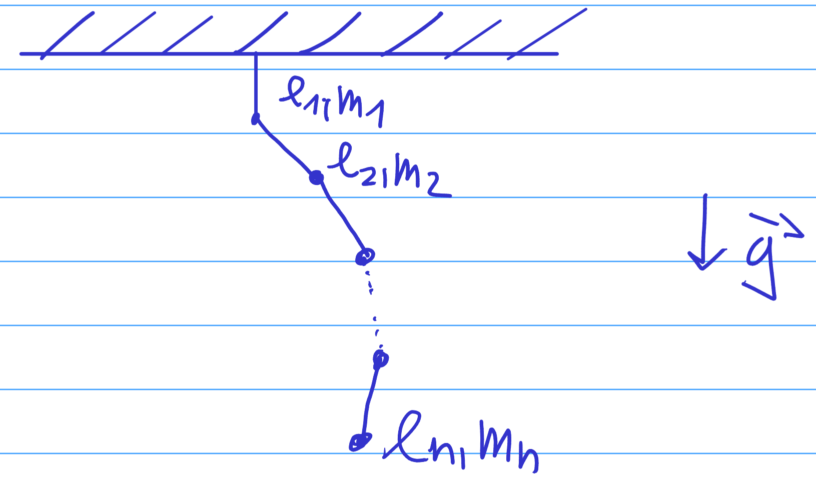


$\downarrow \vec{g}$

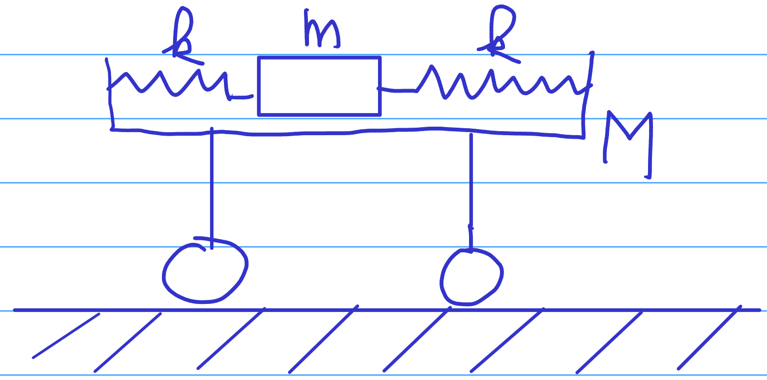
3



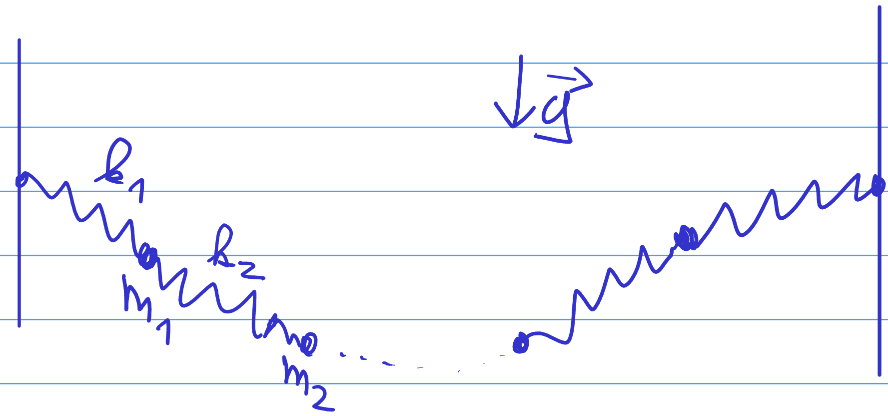
4



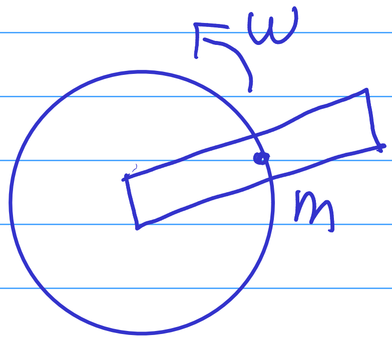
5



6



7



8