

Cvičení 11 Ritzova metoda

$$I[y] = \int_0^1 (y'^2 - y^2 - 2xy) dx$$

$$y(0) = 0$$

$$y(1) = 0$$

EL tce: $y'' + y + x = 0$

$$y = A \sin x + B \cos x - x$$

$$B = 0$$

$$0 = A \sin 1 - 1$$

$$A = \frac{1}{\sin 1}$$

$$y = \frac{1}{\sin 1} \sin x - x$$

$$y_n = x^n(1-x)$$

$$y_1 = x(1-x)$$

$$y_2 = x^2(1-x);$$

$$y_3 = x^3(1-x)$$

úroveň $I[c_1(x(1-x))] = \int_0^1 [(c_1(1-2x))^2 - (c_1(x(1-x)))^2 - 2x^2 c_1(1-x)] dx$

$$= C_1^2 \frac{3}{10} - C_1 \frac{1}{6}$$

$$\frac{\partial I}{\partial C_1} = 0 \Rightarrow C_1 \frac{3}{5} = \frac{1}{6}$$

$$C_1 = \frac{5}{18}$$

$$y_1 = \frac{5}{18} x(1-x)$$

úroveň 2: $y^{(2)} = C_1 y_1 + C_2 y_2$

$$I[C_1 y_1 + C_2 y_2] = \int_0^1 (C_1 y_1' + C_2 y_2')^2 - (C_1 y_1 + C_2 y_2)^2 - 2x(C_1 y_1 + C_2 y_2) dx$$

$$= \frac{1}{210} (63C_1^2 + 7C_1(-5 + 9C_2) + C_2(-21 + 26C_2))$$

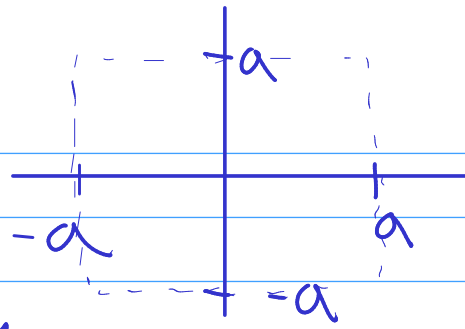
$$I = I(C_1, C_2)$$

$$\frac{\partial I}{\partial C_1} = \frac{\partial I}{\partial C_2} = 0$$

$$C_1 = \frac{71}{369}$$

$$C_2 = \frac{7}{41}$$

$$2D \quad U = U(x, y)$$



$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = -1$$

$$U(-a, y) = U(a, y) \\ = U(x, -a) = U(x, a) = 0$$

$$I[U] = \frac{1}{2} \int_{-a}^a \int_{-a}^a (U_x^2 + U_y^2 - 2U)$$

$$U^{(1)} = (x^2 - a^2)(y^2 - a^2)$$

$$U^{(2)} = (x^2 + y^2)(x^2 - a^2)(y^2 - a^2)$$

$$U^{(h)} = (x^2 + y^2)^{h-1} (x^2 - a^2)(y^2 - a^2)$$

úroveň 1

$$I[c_1 U_1] = \int_{-a}^a \int_{-a}^a \left(2c_1 x (y^2 - a^2) \right)^2 \\ + (2c_1 y (x^2 - a^2))^2 \\ - 2c_1 (x^2 - a^2)(y^2 - a^2) \\ dx dy$$

$$= (2c_1)^2 \frac{128}{45} a^8 \cdot 2$$

$$- 2c_1 \frac{16}{9} a^6$$

$$\frac{\partial I}{\partial c_1} = 0 \Rightarrow c_1 = \frac{5}{16a^2} \approx \frac{0,3125}{a^2}$$

$$I [c_1 y_1 + c_2 y_2] = \dots$$

$$\frac{\partial I}{\partial c_1} = 0, \quad \frac{\partial I}{\partial c_2} = 0$$

$$c_1 \approx 0,2922$$

$$c_2 \approx 0,054$$

$$c_1, c_2, c_3 \Rightarrow \begin{aligned} c_1 &\approx 0,296 \\ c_2 &\approx 0,0316 \\ c_3 &\approx 0,029 \end{aligned}$$