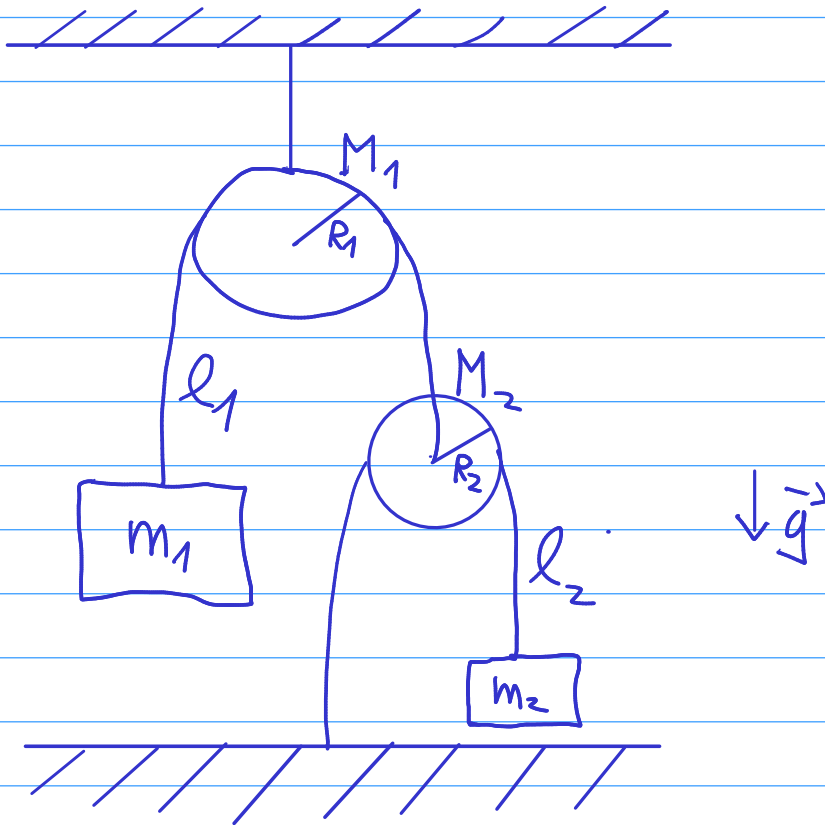
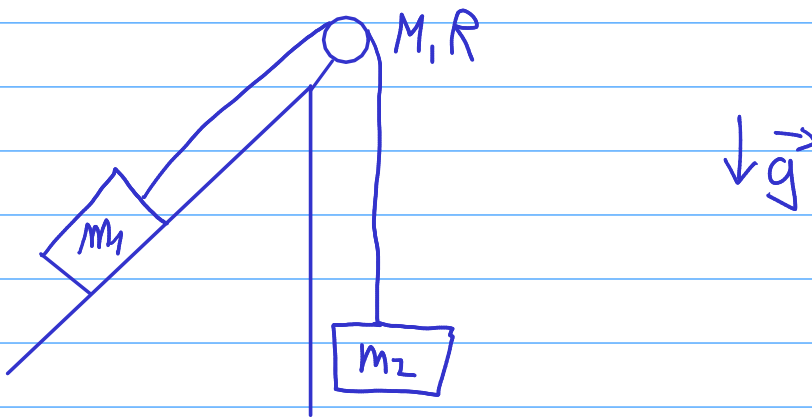


## II. Cvičení - ukázkové příklady, kvadratické lagrangiany

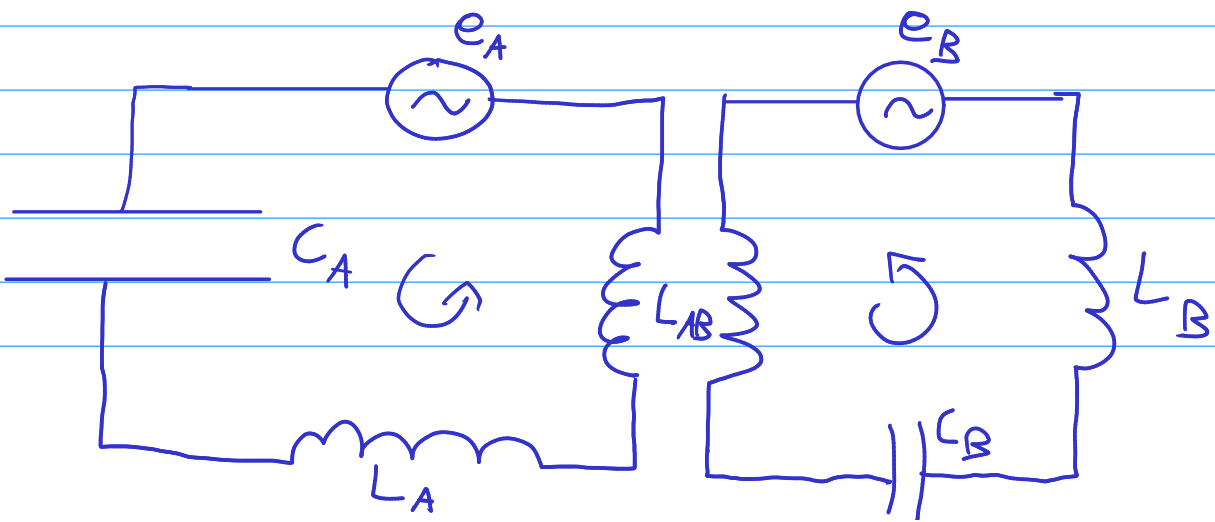
1



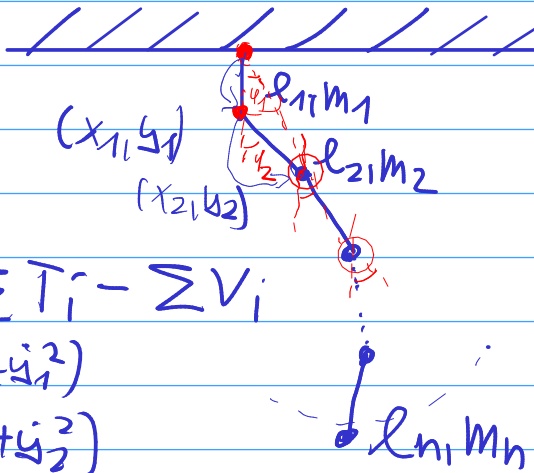
2



3



4



$$x_1 = l_1 \sin \phi_1, \quad y_1 = l_1 \cos \phi_1$$

$$x_2 = x_1 + l_2 \sin \phi_2$$

$$y_2 = y_1 + l_2 \cos \phi_2$$



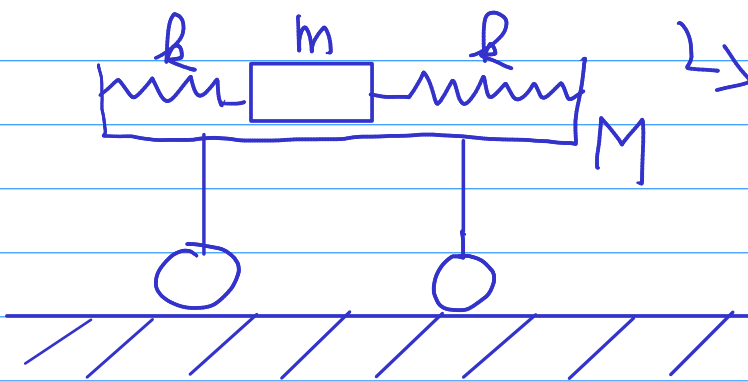
$$x_{i+1} = x_i + l_i \sin \phi_i$$

$$y_{i+1} = y_i + l_i \cos \phi_i$$

$$L = \sum T_i - \sum V_i$$

$$\frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2)$$

5



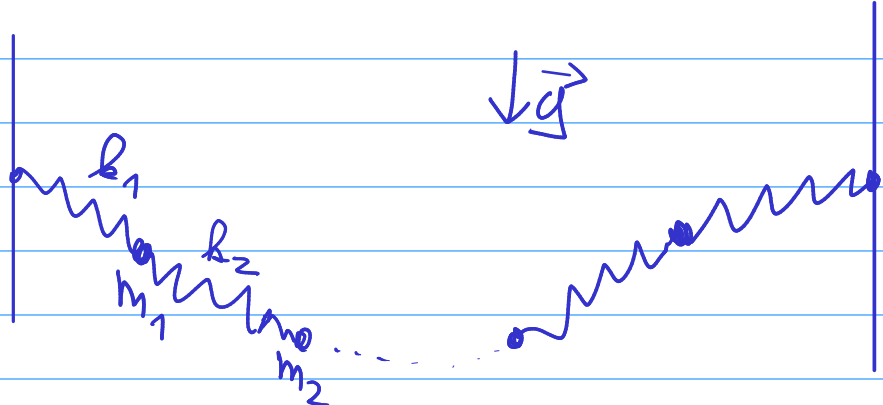
$$\frac{1}{2} l_1^2 \dot{\phi}_1^2 (m_1 + m_2)$$

$$\frac{1}{2} l_2^2 \dot{\phi}_2^2 (m_2 + m_n)$$

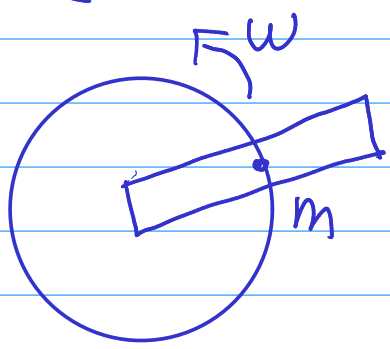
$$\frac{1}{2} l_n^2 \dot{\phi}_n^2 m_n$$

$$\phi_i - \phi_j$$

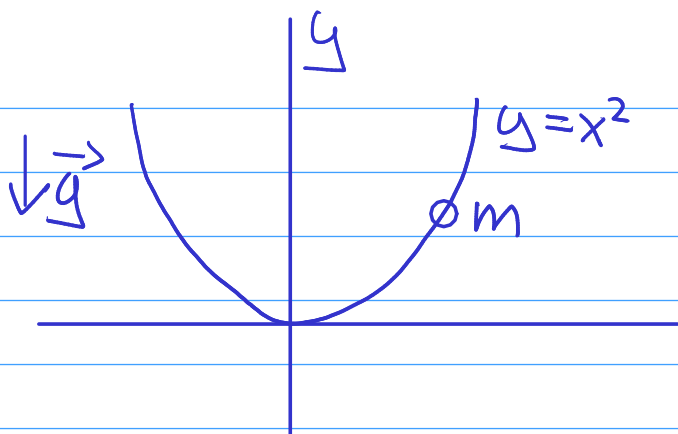
6



7



8

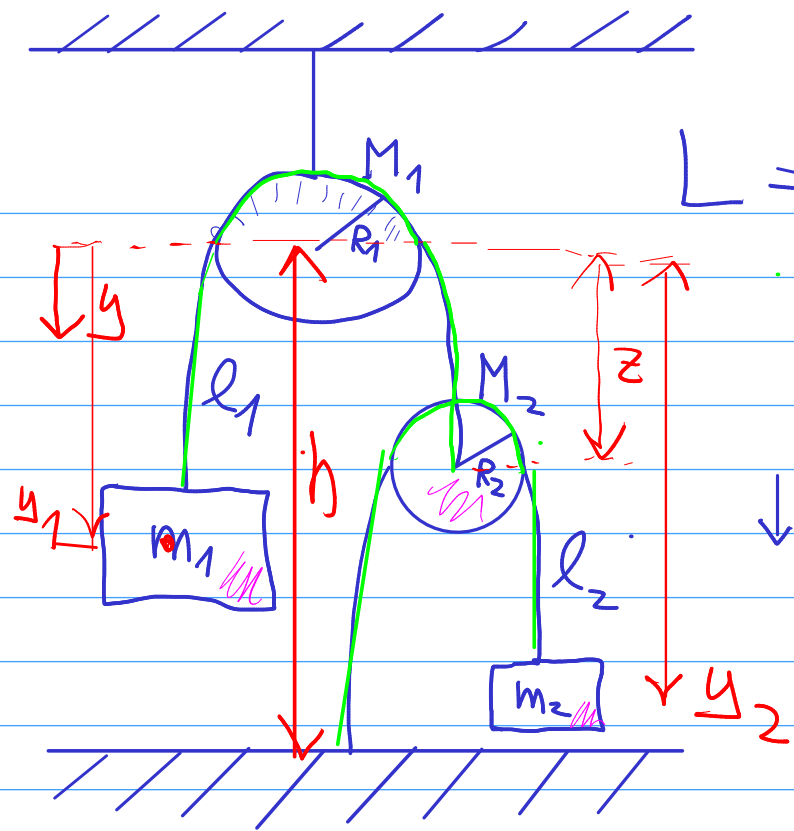


Mihulé cvičení:

Volný pád

$$\frac{m}{h^2} (y_e - y_{e+1} - y_{e-1} + y_e) - mg = 0$$

$$\frac{\left[ \left( \frac{y_e - y_{e-1}}{h} \right) - \left( \frac{y_{e+1} - y_e}{h} \right) \right]}{h}$$



$$L = T - V$$

$$S = \int L(x, y(x), y'(x)) dx$$

- 1) sous. sous
  - 2) pomeine
  - 3) podminky / uoby
- ↓  
? stavne volnost

$$l_1 = y_1 + z + \pi R_1 \quad \left| \frac{d}{dt} \right.$$

$$l_2 = \pi R_2 + (y_2 - z) + h - z \quad \left| \frac{d}{dt} \right.$$

$$0 = \dot{y}_1 + \dot{z}$$

$$0 = 0 + \dot{y}_2 - \dot{z} - \dot{z} \Rightarrow \dot{y}_2 = 2\dot{z}$$

$$\dot{y}_1 = -\dot{z}$$

$$\dot{y}_2 = -2\dot{y}_1$$

$y_1$

$$T = \frac{1}{2} m_1 \dot{y}_1^2 + \frac{1}{2} M_1 \dot{y}_1^2 + \frac{1}{2} m_2 \dot{y}_2^2$$

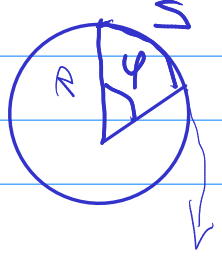
$$+ \frac{1}{2} J_1 \omega_1^2 + \frac{1}{2} J_2 \omega_2^2$$

$$\rightarrow J_i = \frac{1}{2} M_i R_i^2 \quad i=1,2$$

$$\frac{1}{4} M_2 \dot{y}_2^2 = M_2 \dot{y}_1^2$$

$$\frac{1}{4} M_1 \dot{y}_1^2$$

$\omega = \dot{\varphi}$



$$\varphi = \frac{s}{R}$$

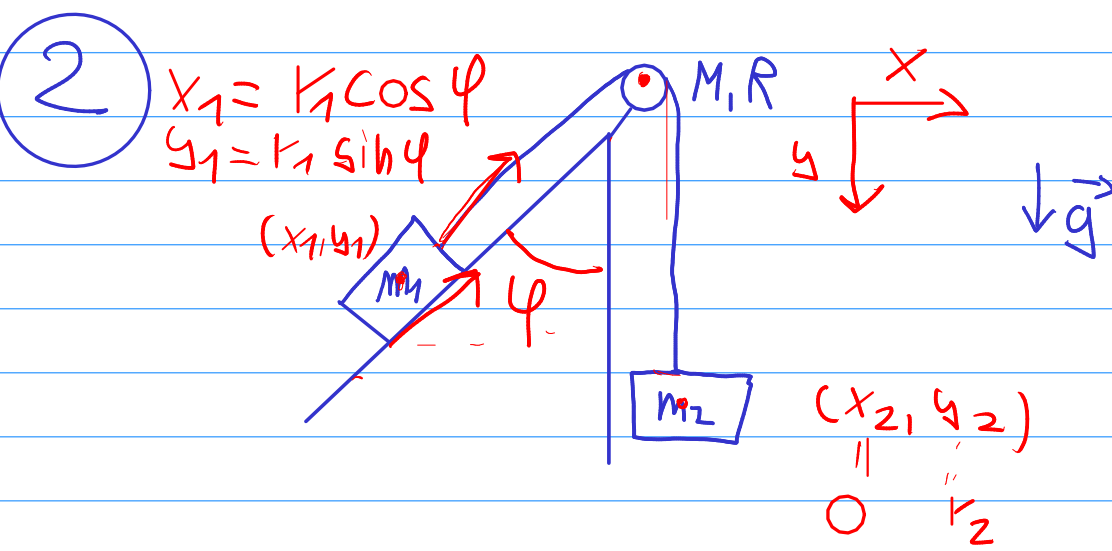
$$\dot{\varphi} = \frac{\dot{s}}{R}$$

$$L = T - V$$

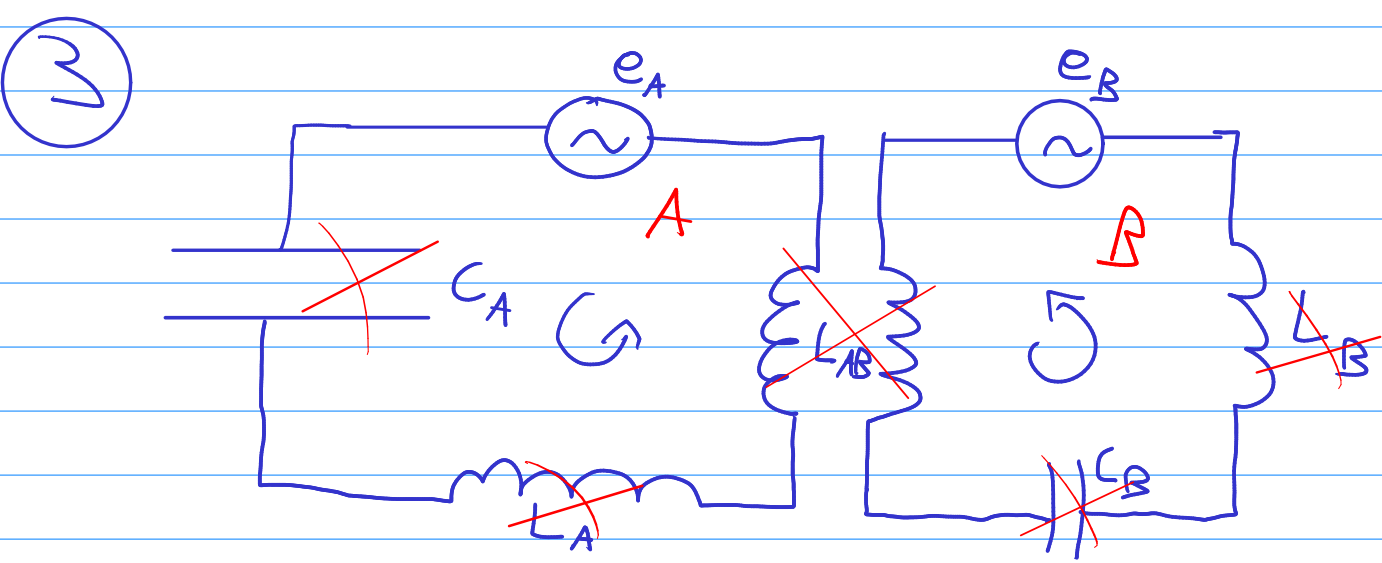
$$\dot{y}_1 = -\dot{z} \Rightarrow y_1 = -z + C$$

$$V = -m_1 g y_1 + g M_2 y_1 + m_2 g 2 y_1$$

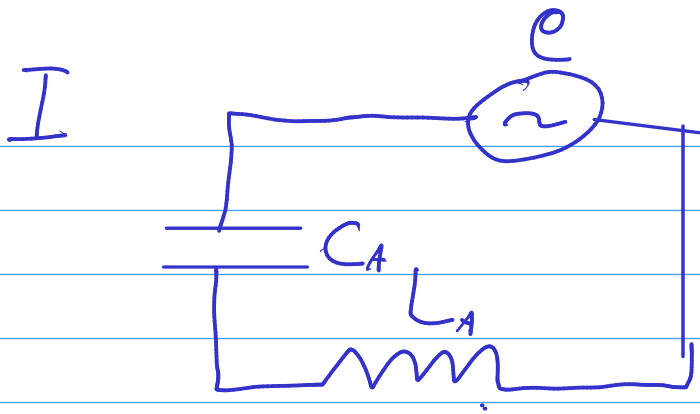
$$\frac{\partial L}{\partial y_1} = \frac{d}{dt} \frac{\partial L}{\partial \dot{y}_1} \Rightarrow \ddot{y}_1 = \frac{m_1 - M_2 - 2m_2}{m_1 + 4m_2 + \frac{1}{2}M_1 + 2M_2} g$$



1stufen voll  $l = r_1 + r_2$   
 $\dot{r}_1 = -\dot{r}_2$



$$L = L_A + L_B = \frac{L \dot{Q}_A^2}{2} - \frac{Q_A}{2C_A} + \frac{L \dot{Q}_B^2}{2} - \frac{Q_B}{2C_B} + \frac{1}{2} L_{AB} \dot{Q}_A \dot{Q}_B$$



$$U \quad \varphi = \frac{Q}{C_A} + L \dot{I}; \quad \dot{I} = -\dot{Q}$$

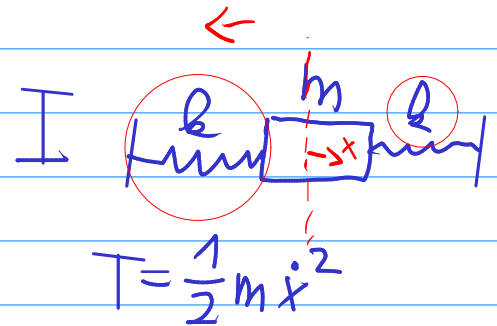
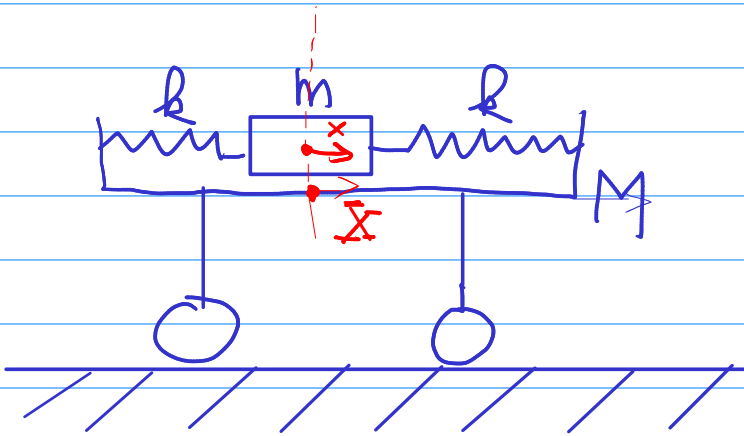
$$\varphi = \frac{Q}{C_A} + L \ddot{Q}$$

$$L = \frac{L \dot{Q}^2}{2} - \frac{Q^2}{2C}$$

$\uparrow$  EL  $\downarrow$ 
 $\downarrow$

$T$ 
 $V$

5



$$V = \frac{1}{2} k x^2 + \frac{1}{2} b x^2$$

$$V = b x^2$$

$$\frac{1}{2} m \dot{x}^2 \quad V =$$

$$\frac{1}{2} M \dot{X}^2$$

$$\vec{F} = -\frac{\partial V}{\partial x} = -b \vec{x}$$