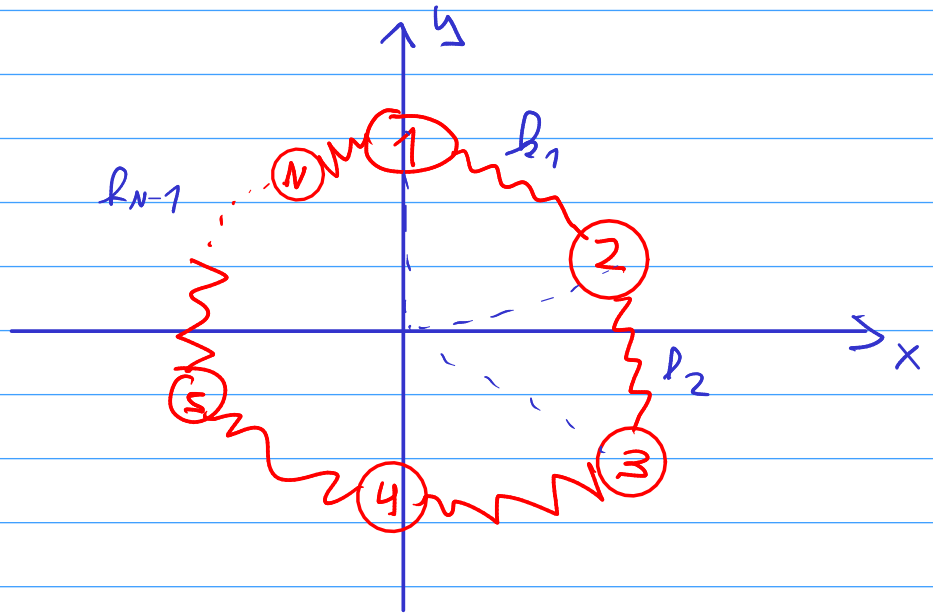
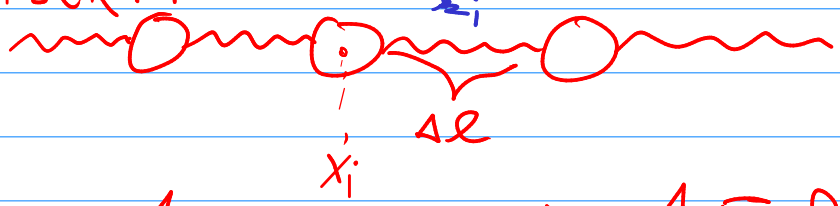
4 cvičení soustava N harmonických oscilátorů



1D řetízek



$$T = \frac{1}{2}\sum m_i \dot{x}_i^2 \; ; \; V = \frac{1}{2}\sum k_i \underbrace{(x_{i+1} - x_i)}_{\Delta l}^2$$

Souřadnice

① x_1, y_1 ; $U_1 = (x_1, y_1)$

② x_2, y_2 ; $U_2 = (x_2, y_2)$

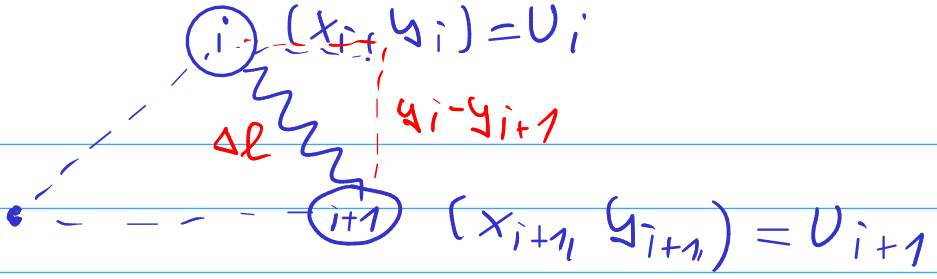
⋮

Ⓝ x_N, y_N ; $U_N = (x_N, y_N)$

$$\vec{U} = (U_1 \dots, U_N) \; , \; U_i$$

$$T = \frac{1}{2}\sum_{i=1}^{N} m_i (\dot{x}_i^2 + \dot{y}_i^2) = \frac{1}{2}\sum m_i \dot{U}_i^2$$

$$V = \frac{1}{2}\sum k_i (\Delta l_i)^2$$



$$\Delta\ell = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2}$$

$$= \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}$$

$$= \sqrt{(v_{i+1} - v_i)^2}$$

$$(x_{i+1}, y_{i+1}) - (x_i, y_i)$$

$$T = \frac{1}{2}\sum_{i=1}^{N} m_i (\dot{x}_i^2 + \dot{y}_i^2) = \frac{1}{2}\sum_{i=1}^{N} m_i \dot{v}_i^2$$

$$V = \frac{1}{2}\sum_{i=1}^{N-1} k_i (v_{i+1} - v_i)^2 + \frac{1}{2} k_N (v_N - v_1)^2$$

$$(v_{i+1})^2 - 2 v_{i+1} v_i + v_i^2$$

$$\vec{v} = (v_1, \ldots, v_N)$$

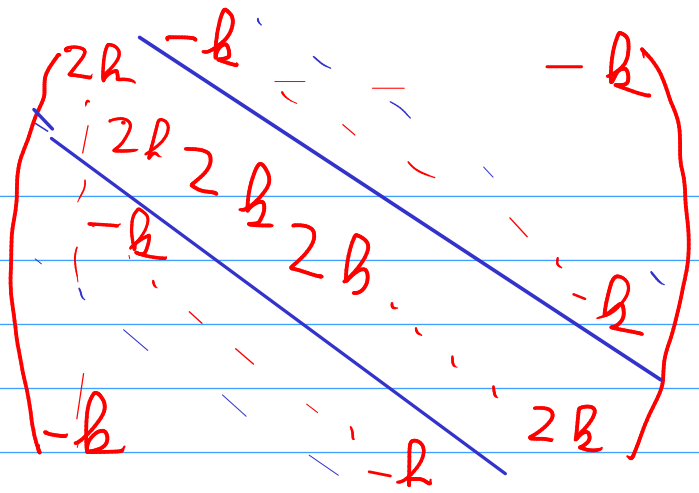
$$T = \frac{1}{2}\, \dot{\vec{v}}\, M\, \dot{\vec{v}}^T$$

$$= \frac{1}{2} (\dot{v}_1 \ \ldots \ \dot{v}_N) \begin{pmatrix} m_1 & & \\ & \ddots & \\ & & m_N \end{pmatrix} \begin{pmatrix} \dot{v}_1 \\ \vdots \\ \dot{v}_N \end{pmatrix}$$

$$V = \frac{1}{2}\, \vec{v}\, K\, \vec{v}^T$$

$$\frac{1}{2} (v_1 \ldots v_N) \begin{pmatrix} k_1 + k_N & -k_1 & & -k_N \\ -k_1 & k_2 + k_3 & & \\ & & \ddots & \\ -k_N & & & k_{N-1} + k_N \end{pmatrix} \begin{pmatrix} v_1 \\ \vdots \\ v_N \end{pmatrix}$$

$k_i = k$



$$= k \begin{pmatrix} 2 & -1 & & & -1 \\ -1 & 2 & & & \\ & & \ddots & & \\ & & & 2 & -1 \\ -1 & & & -1 & 2 \end{pmatrix}$$

$$L = T - V = \frac{1}{2} \dot{v}_i M^i_j \dot{v}^j - \frac{1}{2} v_i k^i_j v^j$$

$$\frac{\partial L}{\partial v_k} = \frac{d}{dt} \frac{\partial L}{\partial \dot{v}_k}$$

$$\frac{\partial L}{\partial v_k} = -\frac{1}{2} \delta_{ik} k^i_j v^j - \frac{1}{2} v_i k^i_j \delta^j_k$$

$$= -\frac{1}{2} \underline{\underline{k^k_j}} v^j - \frac{1}{2} v_i \underline{\underline{k^i_k}} = -\vec{v} k$$

$$= v^i k_{ik}$$

$$k = k^T \Leftrightarrow k_{ij} = k_{ji}$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{v}_k} = \ddot{v} M$$

$$\ddot{u} M = -K u$$

$$m_i = m \qquad b_i = b$$

$$\ddot{u}\, m\, \underline{1} = -b \begin{pmatrix} -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \end{pmatrix} u \,/ m$$

$$\omega^2 = \frac{b}{m}$$

$$\ddot{u} = -\omega^2 A u$$

$A \rightarrow$ vl. vektory
vl. hodnoty

trik z QM.

S ; $AS = SA$ $\quad \langle v_1, \dots, v_N \rangle$

vl. hodnoty $S \Rightarrow$ vl. vektory $S =$ vl. vektory $A \rightarrow$ vl. hodnoty A

$$A v_A = \lambda_A v_A$$

$$S \tilde{v}_B = \lambda_B \tilde{v}_B$$

$$AS = SA$$

$$AS v_A = S A v_A$$

$$A(S v_A) = \lambda_A (S v_A)$$