7 cvičení: Čaplyginova úloha

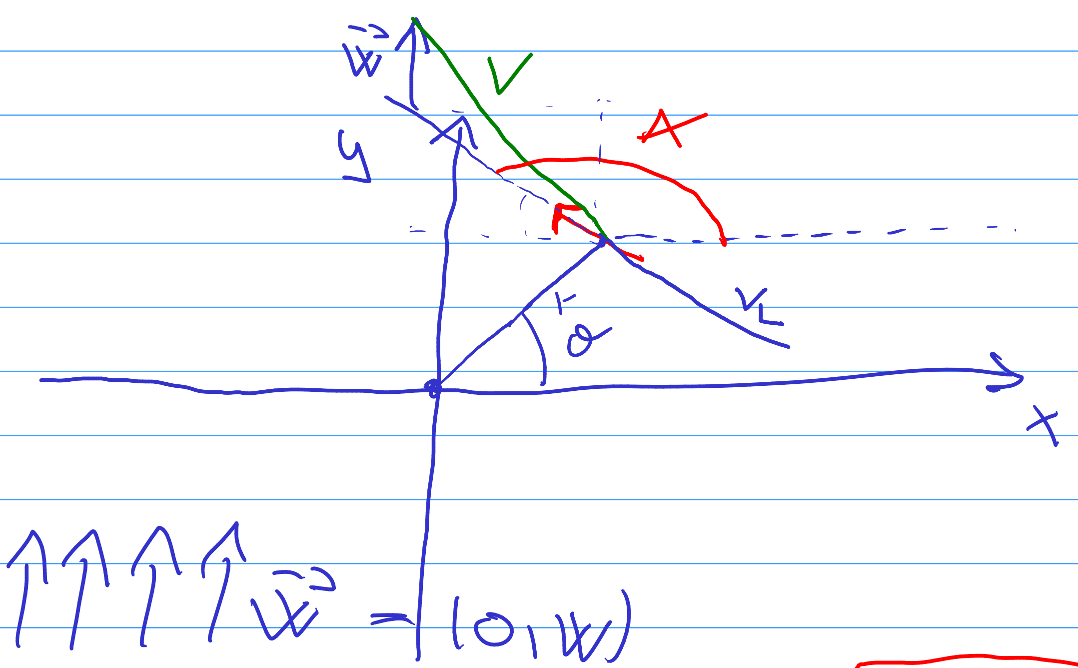
Letadlo letí $|v| = \text{konst}$ v rovině $h_0 = \text{konst}$

silný vítr $\vec{W} = \text{konst}$

Ptáme co má letadlo vzít za trajekt., při aby S (uzavřená traj) byla největší

$x(t), y(x)$ $\quad |V| = \text{konst}$

$$\sqrt{\dot{x}^2 + \dot{y}^2} - v_0 = 0$$



$\vec{w} = (0, W)$

$S = \int L \, dt$; podmínka $\boxed{\sqrt{\dot{x}^2 + \dot{y}^2} = |\vec{v}_L + \vec{w}|}$

$f(x,y) \qquad x^2 + y^2 = 1$

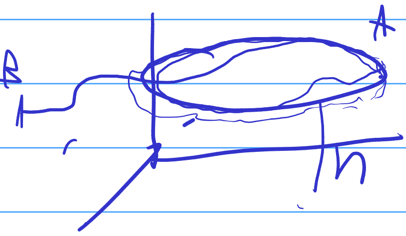
$f(x,y) - \lambda(x^2 + y^2 - 1) \sim$ metoda Lagran. mult

$$S=\int L(x,y,\dot x,\dot y)\,dt+\lambda(t)\,g(x,y)$$

$$\dot x = V_L\cos A$$
$$\dot y = V_L\sin A + w$$

$$S=\underbrace{\int L(x,y,\dot x,\dot y)\,dt}+\lambda_1(t)\left(\dot x - V_L\cos A\right)+\lambda_2(t)\left(\dot y - V_L\sin A - w\right)$$

$$S=\int_S dx\wedge dy$$



stokes theorem $\displaystyle =\int_{\partial S}\omega$

$$d\omega = dx\wedge dy \leftarrow \left.\begin{matrix} x\,dy \\ -y\,dx \\ \underline{\tfrac{1}{2}(x\,dy-y\,dx)}\end{matrix}\right\}\, dx\wedge dy$$

$$S=\int_{\partial S}\frac{1}{2}(x\,dy-y\,dx)=\int\frac{1}{2}(x\dot y-y\dot x)\,dt$$

from $x,y,\boxed{A,\lambda_1,\lambda_2}$

$$\delta x:\quad \frac{1}{2}\dot y-\frac{d}{dt}\left(-\frac{1}{2}y+\lambda_1\right)=0 \qquad (1)$$

$$\delta y:\quad -\frac{1}{2}\dot x-\frac{d}{dt}\left(\frac{1}{2}x+\lambda_2\right)=0 \qquad (2)$$

$$\delta A:\quad V_L\left(\lambda_1\sin A-\lambda_2\cos A\right)=0 \qquad (3)$$

$$\delta\lambda_1:\quad \boxed{\dot x - V_L\cos A = 0} \qquad (4)$$

$$\delta\lambda_2\quad \dot y - V_L\sin A - w = 0 \qquad (5)$$

(1) $\frac{d}{dt}(y - \lambda_1) = 0$

$y - \lambda_1 = c_1$

(2) $\frac{d}{dt}(-x - \lambda_2) = 0$

$x + \lambda_2 = -c_2$

$V_L \cdot ((y + c_1)\sin A + (x + c_2)\cos A) = 0$

$x + c_2 = r\sin A$

$y + c_1 = -r\cos A$

(4+5) $\dot{r}\sin A + r\dot{A}\cos A = V_L \cos A \quad | \sin$

$-\dot{r}\cos A + r\dot{A}\sin A = V_L \sin A + W \quad | \cos$

$\dot{r}(\sin^2 A + \cos^2 A) = V_L \cos A \sin A - V_L \sin A \cos A - W\cos A$

$\dot{r} = -W\cos A \qquad \dot{x} = V_L \cos A$

$\dot{r} = -\frac{W}{V_L}\dot{x} \quad | \int$

$$\boxed{r = -\frac{W}{V_L}x + C}$$

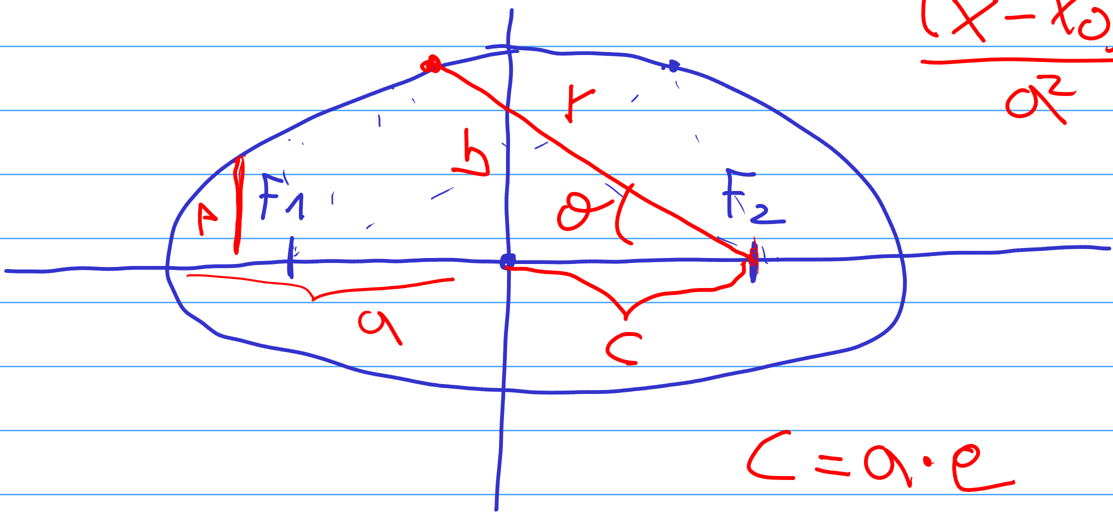
$r = \sqrt{x^2 + y^2}$

$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = C$

$$r = -\frac{W}{V_L} r\cos\theta + C$$

$$r = \frac{C}{1+\frac{W}{V_L}\cos\theta}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$$\frac{(x-x_0)^2}{a^2} + \left(\frac{y^2}{b^2}\right) = 1$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$C = a \cdot e$$

$$\frac{(x-c)^2}{a^2} + \left(\frac{y^2}{b^2}\right) = 1$$

$$r^2\cos^2\theta - 2ae r\cos\theta + a^2e^2 + \frac{r^2\sin^2\theta}{1-e^2}$$

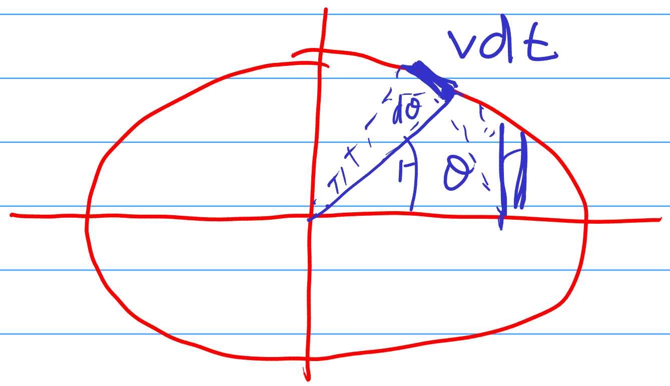
$$= a^2$$

$$r^2\left[(1-\sin^2\theta) + \frac{\sin^2\theta}{1-e^2}\right] - 2ar\cos\theta = a^2(e^2-1)$$

$$r = \frac{a(1-e^2)}{1+a\cos\theta}$$

$$A = \pi \cdot ab$$

$$dA = \frac{r \cdot v\,dt}{2}$$



$$dS = \frac{1}{2} r^2 d\theta = S$$

$$\parallel$$

$$\frac{1}{2} p v_L dt = \frac{1}{2} p v_L T$$

$$p = \frac{rT\sqrt{1-\frac{w^2}{v^2}}}{2\pi}$$