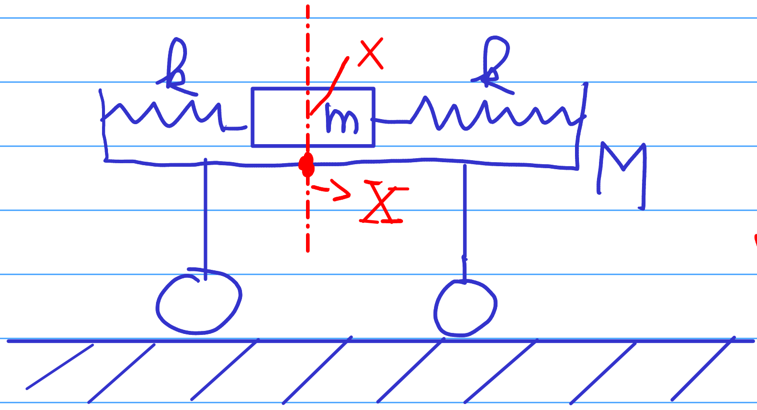
8 cvičení

1) Čaplygin doplnění

2) Problém z 2. cvičení

⑤



malé x - poloha kostky
velké X - poloha středu vozíku
v čase $t=0$
$x = X = 0$

2 stupně volnosti x a X

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2}(m+M)\dot{X}^2$$

$V = k(X-x)^2 \to$ obě pružiny mají stejnou energii takže $\frac{1}{2} + \frac{1}{2} = 1$

$$L = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} M \dot{X}^2 - k(X-x)^2$$

? ∃ symetrie ? $\quad X \to X'(t, x, X)$

$x \to x'(t, x, X)$

$$X = X + \varepsilon_{h_X} \; ; \; x = x + \varepsilon_{h_x} \qquad \partial_X, \partial_x$$

$$-k(X-x)^2$$

$$\frac{\partial L}{\partial X} h_X + \frac{\partial L}{\partial x} h_x = 0$$

$$-2k(X-x)h_X \quad +2k(X-x)h_x = 0$$

$$2k(X-x)(-h_X \quad + h_x) = 0$$

$$h_X = h_x$$

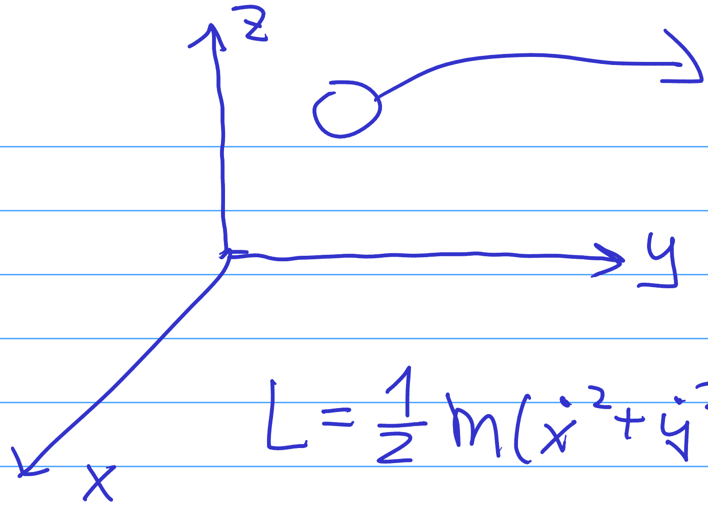
$$\Rightarrow \quad P_X h_X + P_x h_x = \text{konst}$$

$$M\dot{X} + m\dot{x} = \text{konst}$$

$$x + \frac{M}{m} X = x_{SH} \qquad \dot{x}_{SH} = \text{konst}$$

$$X + \frac{m}{M} x \qquad X - x$$

(2)



$$L = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - mgz$$

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}}$$

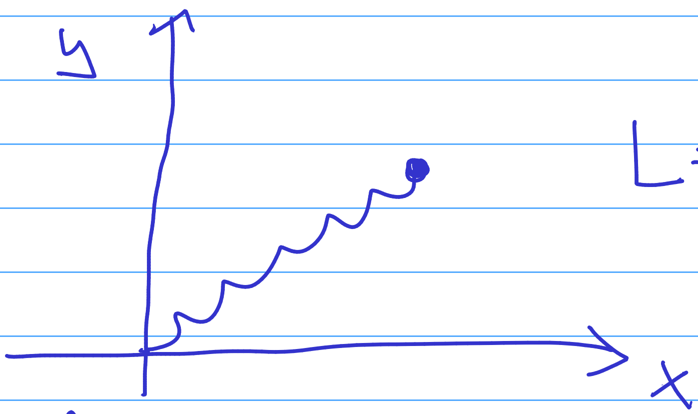
$L \not\ni x$

$L \not\ni y$

Symetrie

- $x \to x + \varepsilon_1$, $y \to y$, $z \to z$ — $m\dot{x} = \text{konst}$
- $x \to x$, $y \to y + \varepsilon_2$, $z \to z$ — $m\dot{y} = \text{konst}$
- $x \to x + \varepsilon_1$, $y \to y + \varepsilon_2$, $z \to z$ — $A(m\dot{x}) + B(m\dot{y}) = \text{konst}$

(4)



$$L = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2) - \frac{1}{2} k (x^2 + y^2)$$

$x, y \to r, \varphi$

$L \not\ni \varphi$

$$\frac{\partial L}{\partial \varphi} = \frac{d}{dt}\frac{\partial L}{\partial \dot{\varphi}}$$

$x \to x + \varepsilon h_x(t, x, y)$

$y \to y + \varepsilon h_y(t, x, y)$

$$\frac{\partial L}{\partial x} h_x + \frac{\partial L}{\partial y} h_y + \frac{\partial L}{\partial \dot{x}} \dot{h}_x + \frac{\partial L}{\partial \dot{y}} \dot{h}_y = 0$$

$$-kxh_x - kyh_y + m\dot{x}\dot{h}_x + m\dot{y}\dot{h}_y = 0$$

$$L = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2) - \frac{1}{2} k(x^2 + y^2).$$

$$-kxh_x - kyh_y = 0$$

$$m\dot{x}\dot{h}_x + m\dot{y}\dot{h}_y = 0$$

$$xh_x = -yh_y$$

$$\dot{x}\dot{h}_x = -\dot{y}\dot{h}_y$$

$$h_x = y$$
$$h_y = -x$$

$$x \to x + \varepsilon y$$
$$y \to y - \varepsilon x$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 1 & \theta \\ -\theta & 1 \end{pmatrix}$$

$$p_x h_x + p_y h_y = m(\dot{x}y - \dot{y}x) = \text{konst}.$$

$$x = r\cos\varphi$$
$$y = r\sin\varphi$$

$$mr^2\dot{\theta} = \text{konst}$$

5) ZZE $-V$ T

$$L = 4x^2 + 8x + \dot{x}^2$$

$$\frac{\partial L}{\partial t} = 0 \qquad t \to t + \varepsilon$$

$$H = T + V = \dot{x}^2 - 4x^2 + 8x = \text{konst}$$

$$\frac{\partial L}{\partial t} \cdot \varepsilon = 0 \qquad \ddot{x} = 8x + 8$$

-> dopočítať = nepovinný DÚ.