Statistická fyzika 27.05.2021

BKR

$f(t, \vec{r}, \vec{p})$ ... rozdělovací fce

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla_{\vec{r}} f + \underbrace{\frac{\vec{F}}{m}}_{\text{zrychlení } \vec{a}} \cdot \nabla_p f = \underbrace{\left(\frac{\partial f}{\partial t}\right)_{coll}}_{\text{kolizní člen}} \quad (1)$$

46. Rce kontinuity

(1) ... zintegrujeme přes hybnosti a vynásobíme m

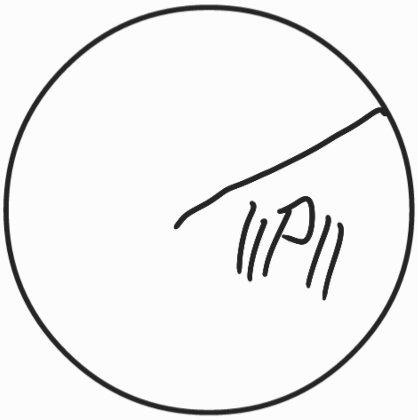
$$\underbrace{m \int d^3p \, \frac{\partial f}{\partial t}}_{\text{I}} + \underbrace{m \int d^3p \, \vec{v} \cdot \nabla_{\vec{r}} f}_{\text{II}} + \underbrace{\int d^3p \, \vec{F} \cdot \nabla_p f}_{\text{III}} = \underbrace{\int d^3p \left(\frac{\partial f}{\partial t}\right)_{coll}}_{\text{IV} \; = 0}$$

$$W = \vec{f} \cdot \int_{\Gamma} d\vec{p}^{\,3} \cdot \nabla_p \vec{f} = \vec{f} \cdot \int_{\partial\Gamma} d\vec{S} \cdot f = 0$$

objemový int.

vnější der.

$$\int_{\Gamma} dw = \int_{\partial\Gamma} w$$



za těchto předpokladů

Stokesův teorém

$\int \propto$ plocha $\propto \|p\|^2$

f konverguje dostatečně rychle

$$\left| \lim_{\|p\| \to \infty} f(\|p\|) \cdot \|p\|^2 = 0 \right|$$

Maxwell-Boltzmann

$$f(\|p\|) \propto e^{-\|p\|^2}$$

$$\sqcup = m \frac{\partial}{\partial t} \int_\Gamma d^3p \, f$$

$$\sqcup = m \nabla_{\vec r} \int_\Gamma d^3p \cdot \vec v \cdot f$$

$$m \frac{\partial}{\partial t} \int_\Gamma d^3p \, f + m \nabla_{\vec r} \int_\Gamma d^3p \cdot \vec v \cdot f = 0 \qquad (2)$$

Definice z kinetické teorie

$$\rho(t,\vec r) = m \cdot n(t,\vec r,\vec p) = m \int_\Gamma d^3p \, \frac{f(\vec r,\vec p,t)}{(2\pi\hbar)^3} \qquad (3)$$

$$\langle \vec v\rangle = \frac{\int d^3p \cdot \vec v \cdot f}{\int d^3p \cdot f} = \frac{\int d^3p \cdot \vec v \cdot f}{(2\pi\hbar)^3 \quad n(\vec r,t)} \qquad (4)$$

(3) a (4) dosadíme do (2)

$$m\frac{\partial}{\partial t}\cancel{(2\pi\hbar)^3}\;\overbrace{n(\vec r,t)}^{\rho(\vec r,t)} = \nabla_{\vec r}\cancel{(2\pi\hbar)^{-1}}\;\overbrace{m\,n(\vec r,t)}^{\rho(\vec r,t)}\,\vec v$$

$$\frac{\partial}{\partial t}\rho(\vec r,t) + \nabla_{\vec r}\left(\rho(\vec r,t)\cdot\vec v\right) = 0 \qquad (3')$$

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47. Tok tepla

1a)

konšt. gradient teploty $\alpha = -\frac{dT}{dy}$

$$T(y) = T_0 - \alpha y \qquad (6)$$

Tok tepla:
$$q_y = \int \frac{d^3p}{(2\pi\hbar)^3}\,\frac{p^2}{2m}\cdot v_y \cdot f \qquad (7)$$

$\vec{f} = \vec{0}$

$\vec{\nabla} p = 0 \Rightarrow p = \text{konst.}$

ve id. plynu $pV = N k_B T$

$$p = \frac{N}{V} k_B T = n k_B T \quad (8)$$

$\frac{N}{V} \equiv n$

v bodě „nula"

$$p = n_0 k_B T_0 = n k_B T(y) = n k_B (T_0 - \alpha y)$$

(9)

bod se souř. y

konc. částic v y

$$\boxed{n = n_0 \frac{T_0}{T_0 - \alpha y}} \quad (10)$$

BKR

$$\cancel{\frac{\partial f}{\partial t}} + \vec{v}\cdot\nabla_{\vec{r}} f + \cancel{\frac{\vec{F}}{m}\nabla_{\vec{p}} f} = \left(\frac{\partial f}{\partial t}\right)_{coll} \qquad (11)$$

předp. stacionární stav

rovnovážné rozdělení

$$\left(\frac{\partial f}{\partial t}\right)_{coll} = \frac{f - f_0}{\tau} \qquad (12)$$

relaxační čas

$$f_0 = n\left(\frac{2\pi\hbar}{mk_BT}\right)^{3/2}\exp\left(-\frac{p^2}{2mk_BT}\right) \overset{z\,(10)}{=}$$

$$\underbrace{n_0\frac{T_0}{(T_0-\alpha y)^{5/2}}\left(\frac{2\pi\hbar^2}{mk_B}\right)^{3/2}}_{K}\exp\left(-\frac{p^2}{2mk_B(T_0-\alpha y)}\right) \qquad (13)$$

BKR (11)

$$\vec{v}\cdot\nabla_{\vec{r}} f = \frac{f - f_0}{\tau} \qquad (11')$$

$$\nabla_{\vec{r}} f \approx \nabla_{\vec{r}} f_0 \qquad (15)$$

všechny změny pouze ve směru z

$$v_z \frac{\partial f_0}{\partial z} = \frac{f - f_0}{\tau} \qquad (16)$$

$$\Rightarrow f = f_0 + \tau \cdot v_z \frac{\partial f_0}{\partial z} \qquad (17)$$

$$\frac{\partial f_0}{\partial z} = K \cdot \left[-\frac{5}{2} \frac{(-\alpha)}{(T_0 - \alpha z)^{7/2}} + \frac{\frac{p^2}{2mk_B} \cdot \frac{(-\alpha)}{(T_0 - \alpha z)^2}}{(T_0 - \alpha z)^{5/2}} \right] \cdot$$

$$\exp(\ldots) =$$

$$= \frac{K \alpha \exp\left(-\frac{p^2}{2mk_B (T_0 - \alpha z)}\right)}{(T_0 - \alpha z)^{7/2} \; 2} \left[5 - \frac{p^2}{mk (T_0 - \alpha z)} \right]$$

$$f = \frac{K \exp\left(-\frac{p^2}{2mk_B (T_0 - \alpha z)}\right)}{(T_0 - \alpha z)^{5/2}} \cdot \left\{ 1 + \frac{\tau v_z \alpha}{2 (T_0 - \alpha z)} \cdot \right.$$

$$\cdot \left[\frac{p^2}{m k_B (T_0 - \alpha z)} - 5\right]\Bigg\} \qquad (18)$$

máme spočítáno nerovnovážné f, můžeme spočítat tok tepla z (17)

$$q_z = \int \frac{d^3p}{(2\pi\hbar)^3}\, \frac{p^2}{2m}\, v_z \cdot f =$$

$$= \underbrace{\frac{K}{2m(2\pi\hbar)^3 (T_0-\alpha z)^{5/2}}}_{\text{const.}} \int d^3p \cdot p^2 \cdot \frac{p_z}{m} \cdot \exp\left(-\frac{p^2}{2mk_B(T_0-\alpha z)}\right)$$

$$\left\{1 + \frac{\tau v_z \alpha}{2(T_0-\alpha z)}\left[\frac{p^2}{mk_B(T_0-\alpha z)} - 5\right]\right\} =$$

$$c \cdot \Bigg[\int d^3p\, p^2 \cdot p_z \cdot \exp\left(-\frac{p^2}{2mk_B(T_0-\alpha z)}\right) +$$

$$+ \int d^3p\, p^2 \cdot \frac{p_z^2}{m} \cdot p^2 \exp\left(-\frac{p^2}{2mk_B(T_0-\alpha z)}\right) \frac{\tau \alpha}{2(T_0-\alpha z)} -$$

$$= \left[\int d^3p \, p^2 \frac{p_y^2}{m} \exp\left(- \frac{p^2}{2 m k_B (T_0 - \alpha y)} \right) \frac{5 T \alpha}{2 (T_0 - \alpha y)} \right].$$

$p_y = p \cos \vartheta$

$$= \int_0^{2\pi} d\varphi \underbrace{\int_0^{\pi} d\vartheta \sin\vartheta \cos\vartheta}_{0} \int_0^{\infty} dp \, p^5 \exp\left(- \frac{p^2}{2 m k_B (T_0 - \alpha y)} \right) \ldots$$

$$2\pi \int_0^{\pi} d\vartheta \frac{\sin 2\vartheta}{2} = -\frac{\pi}{2} \cos(2\vartheta) \Big|_0^{\pi} = 0$$

$$= \int_0^{2\pi} d\varphi \int_0^{\pi} d\vartheta \sin\vartheta \cos^2\vartheta \int_0^{\infty} dp \, p^8 \exp\left(- \frac{p^2}{2 m k_B (T_0 - \alpha y)} \right)$$

$t := \frac{p^2}{\ldots} \rightarrow p = \sqrt{\ldots t}$

$$\cdot \frac{5 T \alpha}{2 m (T_0 - \alpha y)} = \frac{5 T \alpha \pi}{m (T_0 - \alpha y)} \cdot \frac{2}{3} \cdot \left[2 m k_B (T_0 - \alpha y) \right]^{9/2} \cdot \int_0^{\infty} dt \, t^{7/2} \ldots \exp(-t)$$

$$\int_0^{\pi} dx \sin x \cos^2 x = \left| \begin{array}{l} \text{subst.} \\ y = \cos x \\ dy = -\sin x \, dx \end{array} \right| = -\int_1^{-1} dy \, y^2 = \frac{y^3}{3} \Big|_{-1}^{1} = \underline{\underline{\frac{2}{3}}}$$

$$= \frac{5J\alpha\pi}{m(T_0-\alpha g)}\,\frac{2}{3}\left[2mk_B(T_0-\alpha g)\right]^{9/2}\cdot\Gamma\left(\frac{9}{2}\right) =$$

$$= \frac{\cancel{10}^{5}J\alpha\pi}{\cancel{3}}\,C\,(T_0-\alpha g)^{7/2}(2mk_B)^{9/2}\,\frac{\cancel{105}^{35}}{\cancel{16}_{8}}\sqrt{\pi} =$$

$$= \frac{175\,J\alpha\pi}{8}\,n_0 T_0\left(\frac{2\pi\hbar^2}{mk}\right)^{3/2}\frac{(T_0-\alpha g)^{7/2}(2mk_B)^{9/2}}{2m^2(2\pi\hbar)^3(T_0-\alpha g)^{3/2}} =$$

$$= \frac{175}{2}\cdot J\alpha\, n_0 T_0\cdot(mk_B)^3(T_0-\alpha g)^4$$

$$U = \frac{5J\alpha}{2(T_0-\alpha g)m}\underbrace{\int_0^{2\pi}d\varphi}_{2\pi}\underbrace{\int_0^{\pi}d\vartheta\,\sin\vartheta\cos^2\vartheta}_{2/3}\cdot\int_0^{\infty}dp\,p^6\exp\left(-\frac{p^2}{2mk_B(T_0-\alpha g)}\right) =$$

$$= \frac{4}{3}\pi\frac{5J\alpha}{2(T_0-\alpha g)m}\left[2mk_B(T_0-\alpha g)\right]^{7/2}\cdot\underbrace{\int_0^{\infty}dz\,z^{5/2}\exp(-z)}_{\Gamma(7/2)}$$

$$\Gamma(7/2) = \frac{5}{2} \cdot \frac{3}{2} \cdot \frac{1}{2} \sqrt{\pi} = \frac{15}{8} \sqrt{\pi}$$

$$= \frac{2}{3} \cdot \frac{15}{8} \pi^{-3/2} \frac{T\alpha}{(T_0 - \alpha z)/m} C \left[2 m k_B (T_0 - \alpha z)\right]^{7/2} =$$

$$= \quad . \; . \; . \; . \; .$$

Tok tepla:

$$q = -\kappa \cdot \frac{dT}{dz} \qquad \boxed{T_0 - \alpha z \approx T_0}$$

$$\boxed{\kappa = \frac{5}{2} \frac{n_0 k_B^2 T_0 B}{m}}$$

$\boxed{DV}$

$\rho(0) > 0,\ r_0$

$\langle T \rangle = \frac{1}{2} m v^2$

$\sum F_i \cdot r_i \ldots \hat{\ }$

$\langle U \rangle = \frac{G M^2}{r}$

$\int dm \ldots$

$4\pi r^2 \cdot \rho(r)$

$G \frac{m(r)\, m_B}{r}$