Microwave plasma interferometry

Contents

1 Theory

It is well known, that the plasma description can be approached in different ways (conductor, dielectric, magnetic fluid etc.). The appropriate approach is determined by the properties of the discharge (e.g. pressure or degree of thermodynamic equilibrium) and the type of interaction we want to study. Concerning the interaction of plasma and electromagnetic waves, two approaches are widely used - conductive plasma model at low frequencies (plasma conductivity is defined) and dielectric plasma model conductivity at high frequencies introducing the plasma permittivity [1]. The principal value for decision is the plasma frequency, which is the lower limit for wave propagation inside plasma. It is determined by the electron density.

$$\omega_{\mathrm{pl}} = \frac{n_{\mathrm{e}} e^2}{m_{\mathrm{e}} \epsilon_0} \tag{1}$$

where n_{e} denotes the density of free electrons, e denotes the elementary charge, m_{e} denotes the mass of the electron and ϵ_0 denotes the permittivity of vacuum.

1.1 Dielectric model of plasma

Since interferometry is based on the wave propagation through the studied medium, it is necessary to use probing signal at frequencies high enough, so that the plasma is not transparent. Therefore the plasma dielectric model will be used. While direct derivation of plasma permittivity from Boltzmann kinetic equation (BKE) is rather long, we will at least hint the final part - plasma permittivity definition with plasma conductivity known (which is a bit more intuitive parameter in plasma) Taking the differential Maxwell equation $\nabla \times B = \mu_0 (J + \epsilon_0 \frac{\partial E}{\partial t})$ (Amper law) in time-harmonic fields (angular frequency ω) and Ohm's law in form $J = \sigma . E$, where σ is the aforementioned plasma conductivity (already derived from BKE). Using the properties of derivation we can rewrite:

$$\nabla \times B = -\mathrm{i}\omega \mu_0 \epsilon_0 (1 + \frac{\mathrm{i}\sigma}{\omega \epsilon_0}) E, \tag{2}$$

where we rename the expression in parentheses on the right side with ϵ_{r} [2], e.g. the relative plasma permittivity. Had we made the rigorous derivation, we would have noticed that it has a form of a complex tensor. In non-magnetic plasmas however it reduces to complex scalar. While the exact equation for plasma permittivity depends on the distribution function (and furthermore contribution from the heavy particles), Maxwell distribution is a good approximation most of the time. The equation for relative permittivity in non-magnetic plasma at collision frequency for electron-neutral momentum transfer ν_{m} ("collision frequency" in further text) is:

Její přesný tvar závisí na rozdělovací funkci elektronů (a příspěvku těžkých částic), většinou lze ale použít tvar pro Maxwellovské rozdělení rychlostí. Relativní permitivita nemagnetického plazmatu při srážkové frekvenci pro přenos hybnosti elektron-neutrál ν_{m} (dále jen "srážková frekvence") je pak:

$$\epsilon_{\mathrm{r}} = 1 - \frac{n_{\mathrm{e}} e^2}{m_{\mathrm{e}} \epsilon_0 \omega} \frac{(\omega - \mathrm{i}\nu_{\mathrm{m}})}{(\omega^2 + \nu_{\mathrm{m}}^2)} \tag{3}$$

1.2 Complex refractive index

The behaviour of electromagnetic waves in plasma is quite similar to the case of common dielectrics. The oddity of the plasma case is the value of the relative permittivity (real part), which is lower than one, resulting in greater wavelengths than in vacuum. For the description of the wave propagation, however, permittivity is not the most convenient parameter. The index of refraction N is much more intuitive - its real part is directly proportional to the phase velocity (and therefore the phase shift $\mathrm{d}\phi = k_0(1-n(z))\mathrm{d}z$, where $k_0 = \omega/c$ a $\mathrm{d}z$ is an infinitesimal step of the path) while its imaginary part is directly proportional to losses in medium (and therefore drop in the transmitted power $\frac{\mathrm{d}P}{P} = -\frac{2\omega\kappa(z)}{c}\mathrm{d}z$, where P is denotes the power). For clarity, let us assume that the difference of the index of refraction from unity can be fully attributed to the electron effect (further effects, including temperature induced change of the refractive index in gas can be neglected). The integrated equations for power loss and phase shift in homogeneous medium (n a κ independent of z) are:

$$\Delta\phi = k_0(1-n)\Delta z; \quad P = P_0 e^{-\frac{2\omega\kappa}{c}\Delta z} \tag{4}$$

The permittivity $\epsilon_\mathrm{r} = \epsilon_1 + \mathrm{i}\epsilon_2$ and index of refraction $N = n + \mathrm{i}\kappa$ are (in non-magnetic media) related b quadratic relation $\epsilon_\mathrm{r} = N^2$. Following the rules of complex calculus, we may express the components of refractive index:

$$n = \sqrt{\frac{\epsilon_1 + \sqrt{\epsilon_1^2 + \epsilon_2^2}}{2}}; \quad \kappa = \sqrt{\frac{-\epsilon_1 + \sqrt{\epsilon_1^2 + \epsilon_2^2}}{2}} \tag{5}$$

Relating equations (4) and (3) via (5) directly connects interferometric measurement (phase shift and power loss) with plasma parameters - electron density and collision frequency.

2 Experimental set-up

The high-voltage source with current regulation is used to excite discharge in the fluorescent tube (18 mm inner diameter). It is a typical glow discharge in argon mixed with mercury vapours at low pressure (normally the pressure inside the fluorescent tube is around 400 Pa), where the electron density is lower than in atmospheric plasmas.

The classical configuration of an interferometric experiment (e.g. Mach-Zehnder [5]) assumes splitting the initial signal into two tracks (arms) - reference and probing. These are ultimately brought together, the transmitted signals interfere with each other and the result signal is detected. As it is usually not possible to guarantee that changing the conditions in the probing arm will affect only the phase (the amplitude may also change), one detector is considered insufficient and the two-detector quadrature detection method is used [6]. The measured signals can be easily turned into phase and amplitude information.

This experiment, however, is a bit different (Fig. 1) - instead of classical configuration, the Vector Network Analyzer (VNA) **miniVNA Tiny** by **mRS** is used. Simply put, it is a kind of interferometer, where the reference arm is integrated in the device and only the probing arm is attached to external ports. One port serves as the source of the signal with defined amplitude and the transmitted signal enters the second, receiving, port. With the computer interface, various outputs can be shown. Either the phase and power are displayed directly, or the scattering matrix parameters are used. These complex parameters represent the transmission of power between ports.

The discharge tube (the fluorescent tube) passes through a waveguide section (WR340) through the centre of narrower sides at the 35° angle. Thus the whole part encompassed inside the waveguide is interacting with the wave

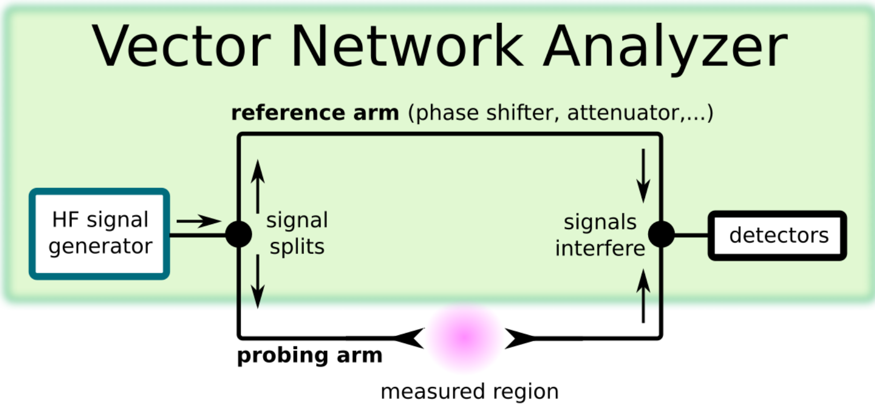

Figure 1: Classical interferometer from the perspective of Network Analyzer - aside from the probing arm, everything is integrated in the device.

3 Measurement

The first part of the measurement consists of setting up the measurement and the discharge tube. If the VNA is not calibrated, we have to perform the calibration ourselves, using defined loads (short-circuit, open and 50Ω). As soon as the calibration is finished, the probing arm - waveguide with the discharge tube - is connected to VNA via coaxial cables with SMA connectors. Using the native interface, we set the measurement parameters (frequency, power, observed data, etc.) while the computer output provides all exports and graphics. The discharge operation is trivial - once the power supply is ON, the current is set to desired value.

The main goal of the experiment is the determination of the electron density and the collision frequency in the discharge, however, the successful measurement of especially the collision frequency can prove strenuous, and therefore it is necessary to make a few precautions. The measurement frequency itself is very important - it has to be high enough to propagate inside the waveguide, but on the other hand increasing frequency means plasma permittivity closer to unity and the method sensitivity drops (smaller phase shifts). Therefore frequencies from the waveguide frequency band are used.

Second thing to consider is the approximation of equations (5) so that the desired plasma parameters can be explicitly expressed without significant systematic error. In this configuration it is safe to assume, that losses in the waveguide are caused exclusively by the attenuation in plasma (e.g. no channelling outside, no losses in the walls, etc.) Provided that also the collision frequency is low ($\omega \gg \nu_\mathrm{m}$) the real and imaginary part of the refractive index can be approximated as follows.

3.1 Plasma density

The first step is the approximation of the real part of refractive index - the contribution of imaginary part ϵ_2 is in this case minimal and can be completely neglected. If the term ν_m^2 in the denominator is also left out (insignificant in sum with ω^2), we get:

$$\epsilon_\mathrm{r} = 1 - \frac{n_\mathrm{e} e^2}{m_\mathrm{e} \epsilon_0 \omega^2} \tag{6}$$

combining with the first equation in (4) the phase shift $\Delta\phi$ is:

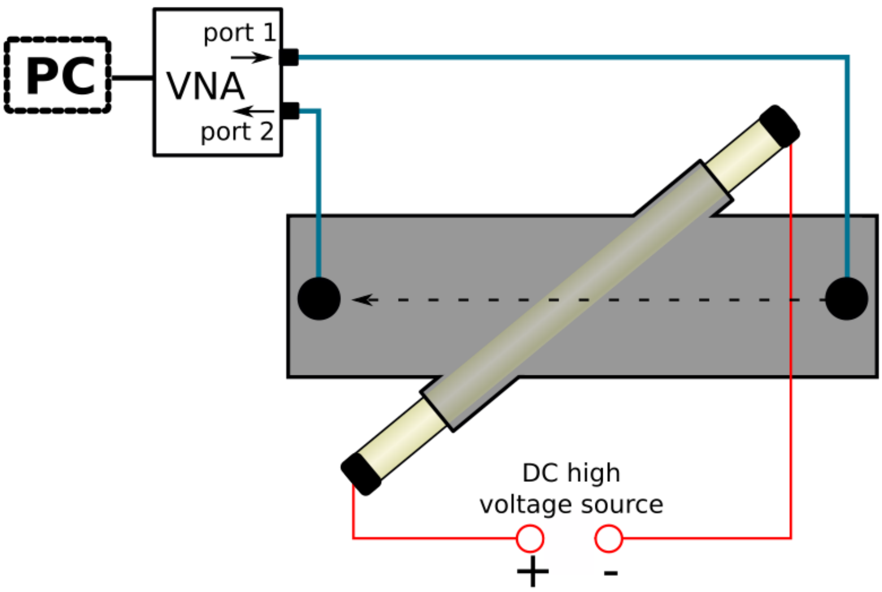

Figure 2: Experimental set-up from the top view. The most important part of the probing arm is the waveguide, through which passes the discharge tube. The rest of the probing arm consists of coaxial cables and connectors. The output from VNA is displayed on the computer, where also the export of the data can be done.

$$\Delta\phi = k_0 \left(1 - \sqrt{1 - \frac{n_{\mathrm{e}} e^2}{m_{\mathrm{e}} \epsilon_0 \omega^2}} \right) \Delta \mathrm{z} \tag{7}$$

and finally after some rearranging, the calculation of electron density is expressed - from the difference of measured phases at the second port with and without plasma presence.

$$n_{\mathrm{e}} = \frac{1 - \left(1 - \frac{\Delta\phi}{k_0 \Delta z} \right)^2 m_{\mathrm{e}} \epsilon_0 \omega^2}{e^2} \tag{8}$$

3.2 Collision frequency

Imaginary part of the refractive index is a bit more difficult to approximate properly and also expects prior knowledge of the electron density therefore it is performed as the latter one. Using the Taylor series, it is possible to approximate:

$$\kappa = \frac{\mid \epsilon_2 \mid}{2\sqrt{2} \mid \epsilon_1 \mid} \tag{9}$$

where ϵ_1 and ϵ_2 can be substituted with (3). If quadratic terms of collision frequency are neglected and also the left side is replaced (4) with $\kappa = \frac{c \ln \frac{P_0}{P}}{2\omega}$, we can ultimately express the collision frequency:

$$\nu_{\mathrm{m}} = \frac{c \ln \frac{P_0}{P}}{2\omega \Delta z} \frac{2\sqrt{2} \mid \epsilon_1 \mid}{\frac{n_{\mathrm{e}} e^2}{m_{\mathrm{e}} \epsilon_0 \omega^3}} \tag{10}$$

Unlike the phase, correct determination of plasma induced losses requires measurement of both the power transmitted and reflected. The losses in plasma are then equal to the difference between VNA set power and the sum of reflected and transmitted power.

3.3 Plasma thickness

For both calculations the plasma thickness has to be estimated. It may be more difficult than it seems, because in reality there is no plasma slab. Even if the plasma fills up the discharge tube homogeneously. Even if the plasma fills the discharge tube homogeneously, it is at best a diagonally positioned cylinder that fills only part of the waveguide cross-section (theoretically the wave could "evade" the plasma altogether).

Though this "homogeneous plasma" slab assumption may not be valid, the theory is still applicable at the designated waveguide mode. The cylinder only needs to be approximated with a slab of equivalent volume.

Of course the correct approach would be the numerical modelling of the experiment, yet the process of model construction, testing and application exceed the scope of this course. Thus the modelling results (fits of electron density and collision frequency) are presented on Fig. 3 - use it to compare with your results and as an test of approximations used for our calculations.

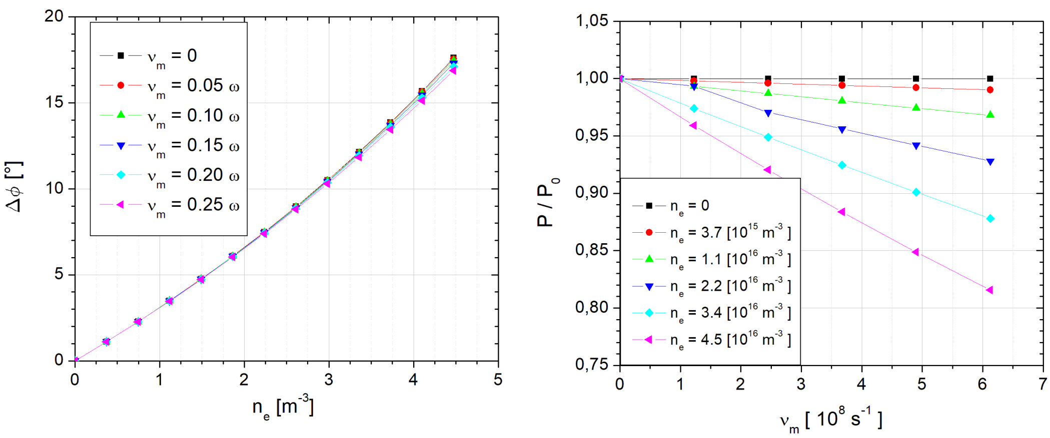

Figure 3: Fit of the numerical model. Unlike the measurement, the model solves the inverse problem - given the plasma parameters, the phasse shift and power loss are computed. Model was calculated for the frequency 2.45 GHz, meaning the angualr frequency was $\omega = 2\pi \times 2.45s^{-}1$.

4 Tasks

1. Prepare the interferometric experiment with the network analyzer. perform the calibration if needed.
2. Measure the phase shifts and amplitude changes induced by plasma for suitable measurement frequency.
3. Determine the electron density and the collision frequency (using the geometric approach, the WR340 standard width is approx. 86 mm, the discharge tube diameter is 18 mm and the angle between the discharge tube and the waveguide is 35°) and compare these results with the numerical model fit.

4. Investigate the plasma parameters evolution with discharge current.
5*. Perform the measurement at different frequenc(y/ies).

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