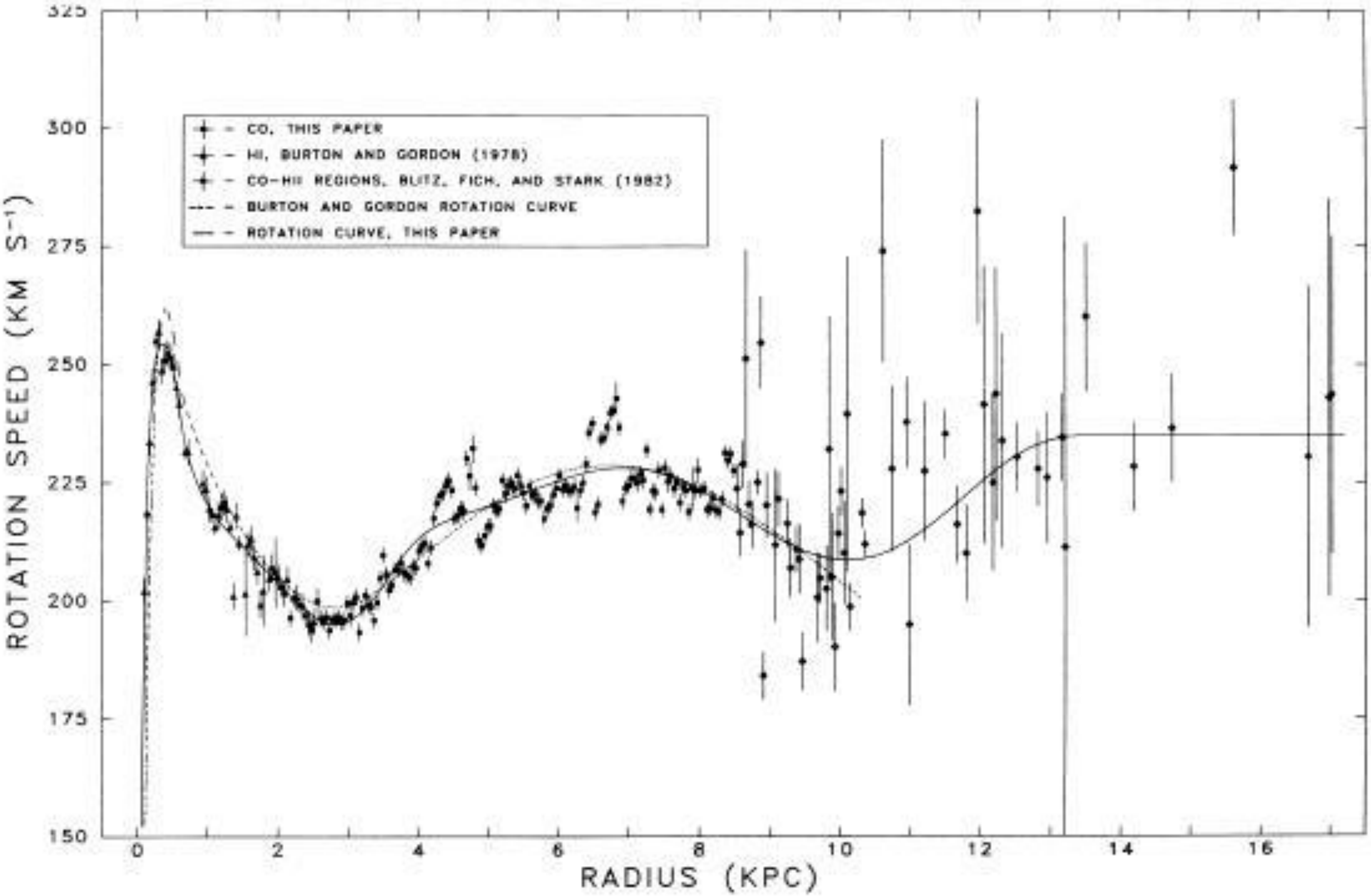
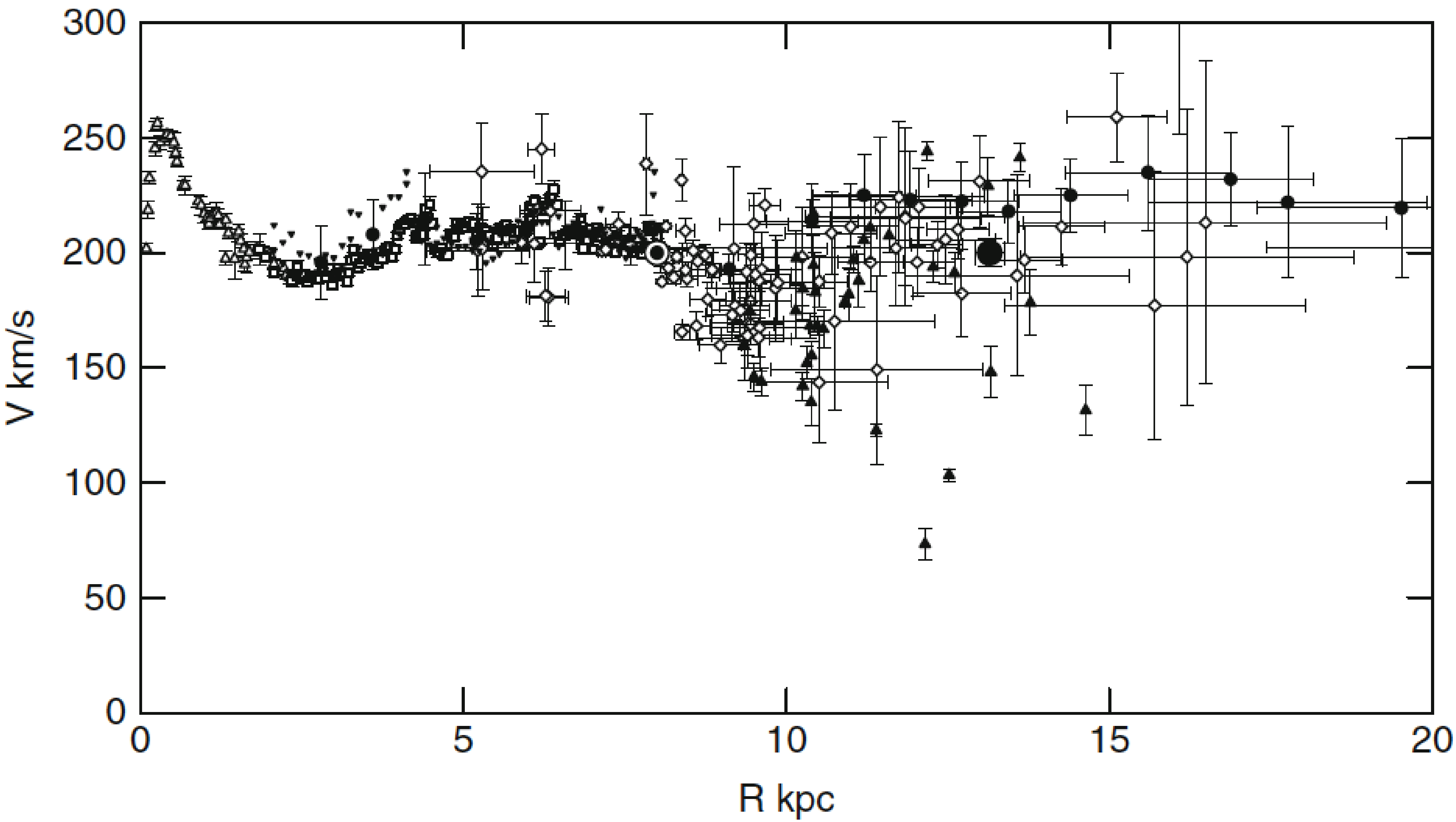
Kinematical membership criteria

- Members follow the motion of the cluster center of gravity
- Internal velocity distribution
- From best to …

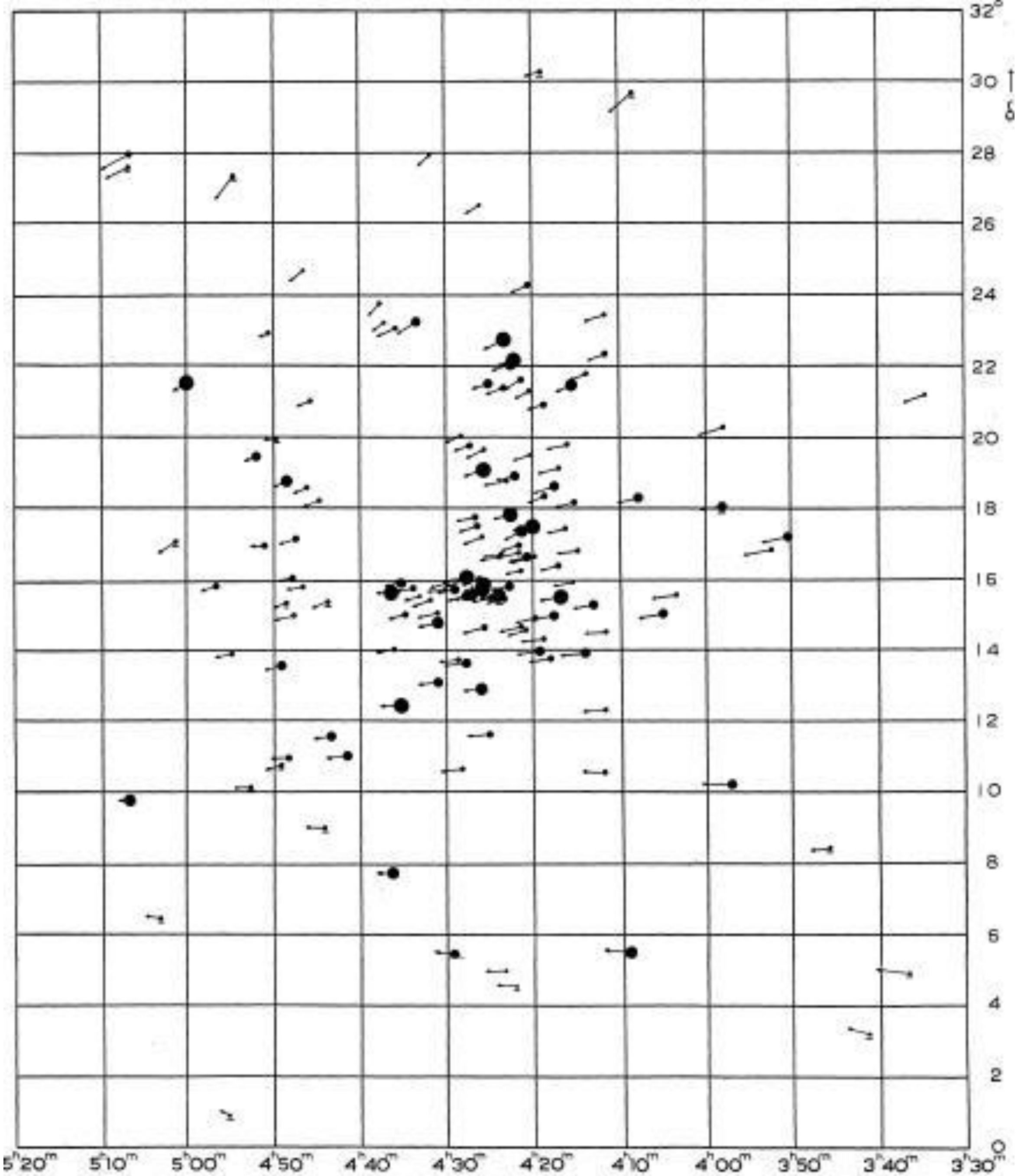
1. Radial velocity and proper motion
2. Radial velocity
3. Proper motion



Clemens, 1985, ApJ, 295, 422



Sofue, 2021, Galaxies, 8, 37



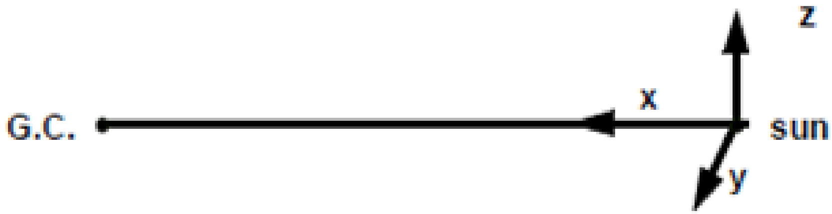
Map of the Hyades brighter than $9^m.0$ visual. The size of the dots is a measure for the magnitude of the stars; the arrows show the annual proper motion (1 mm ≈ 0".030). Five stars in the very outer regions of the cluster are not shown on the map. Underlined dots indicate stars of which no radial velocity is known.

Hyades

Van Bueren, 1952, BAN, 11, 385

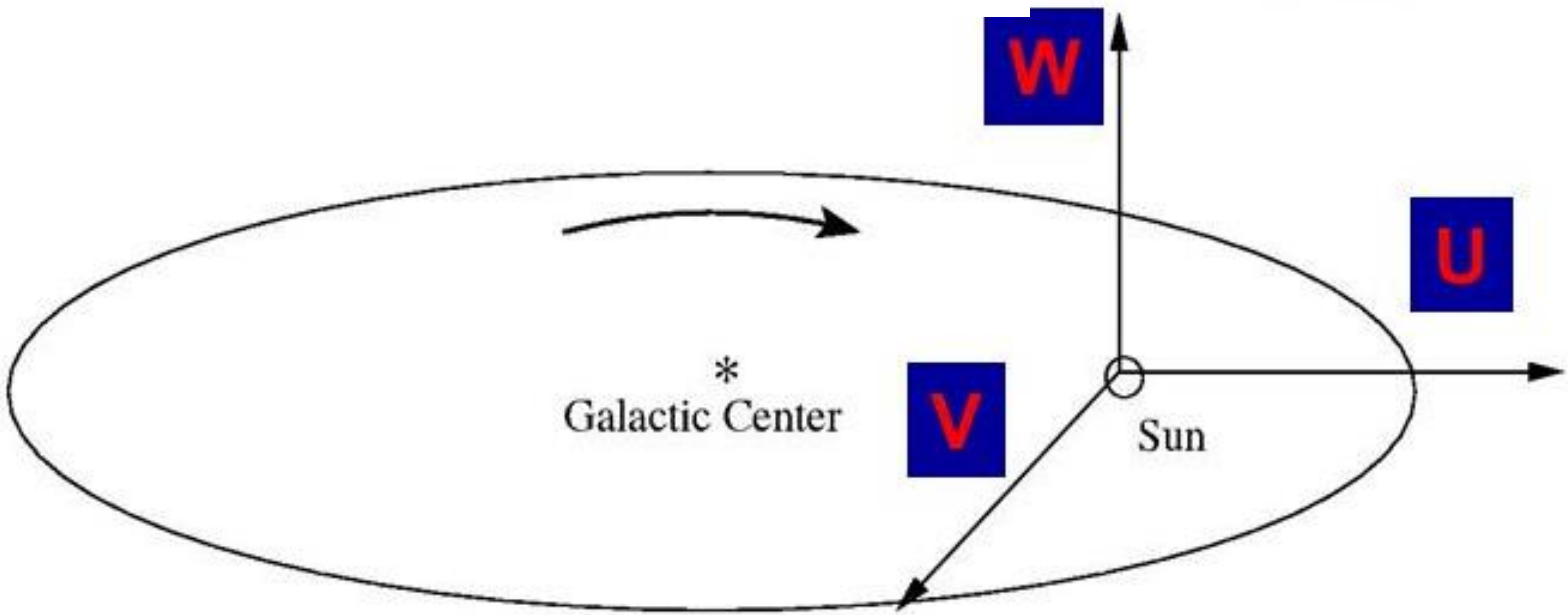
After the correction of the solar motion

Galactic Components



- **coordinates:**
  - x: l = 0 b = 0 (Galactic Centre)
  - y: l = 90 b = 0 (Cygnus) direction of disc rotation
  - z: b = 90 (North Galactic Pole)

Be careful about the sign/direction of X and U

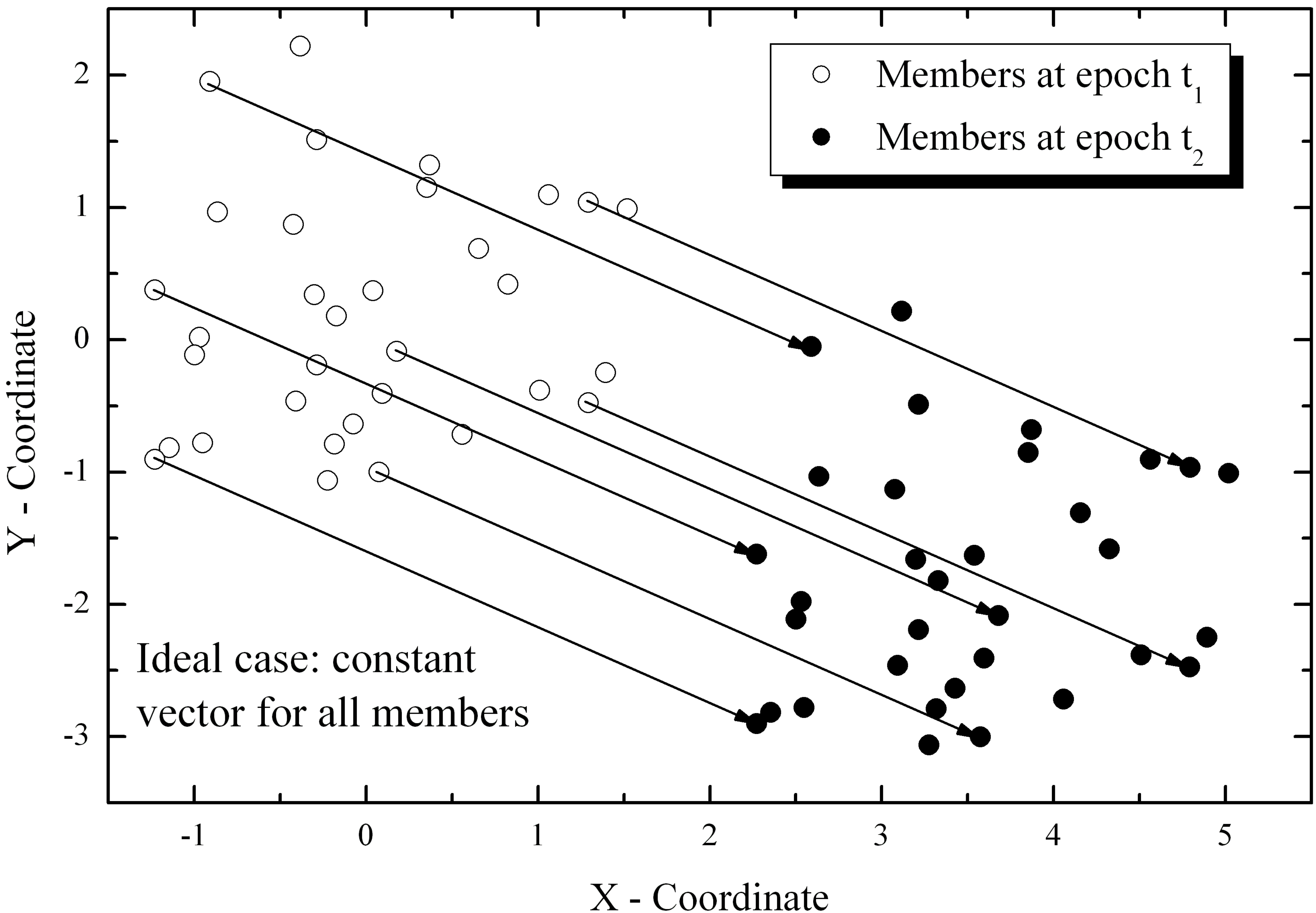


Motion of the Sun

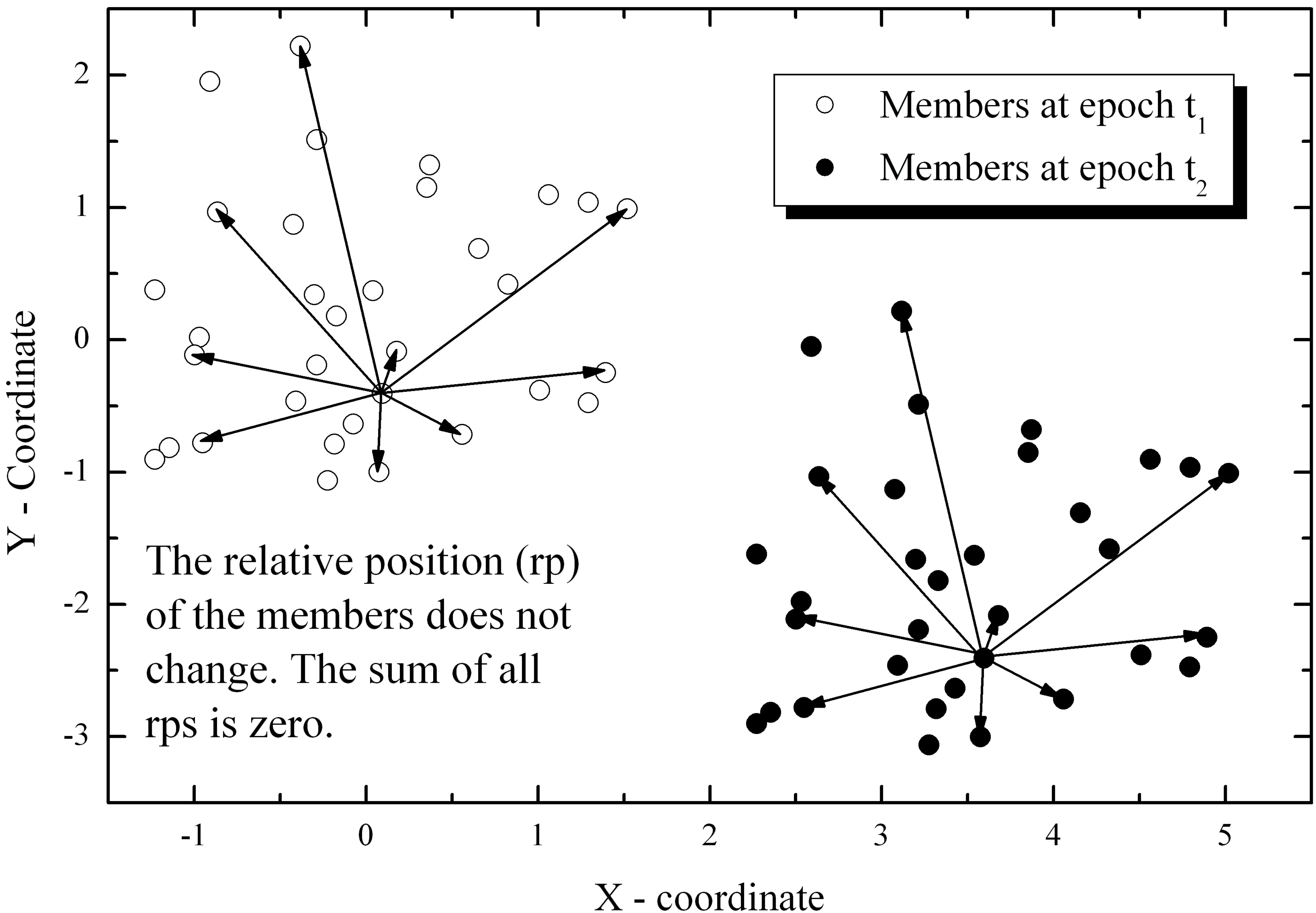
- Peculiar Apex motion: α = 18h28m, $\delta = +30^{o}$; $l = 56.24^{o}$, $b = +22.54^{o}$
- (U,V,W) = (-10, +5, +7) km s^{-1}
- v_{orbit} = 220 km s^{-1}
- Local Standard of Rest (LSR)

Determination of the kinematical membership

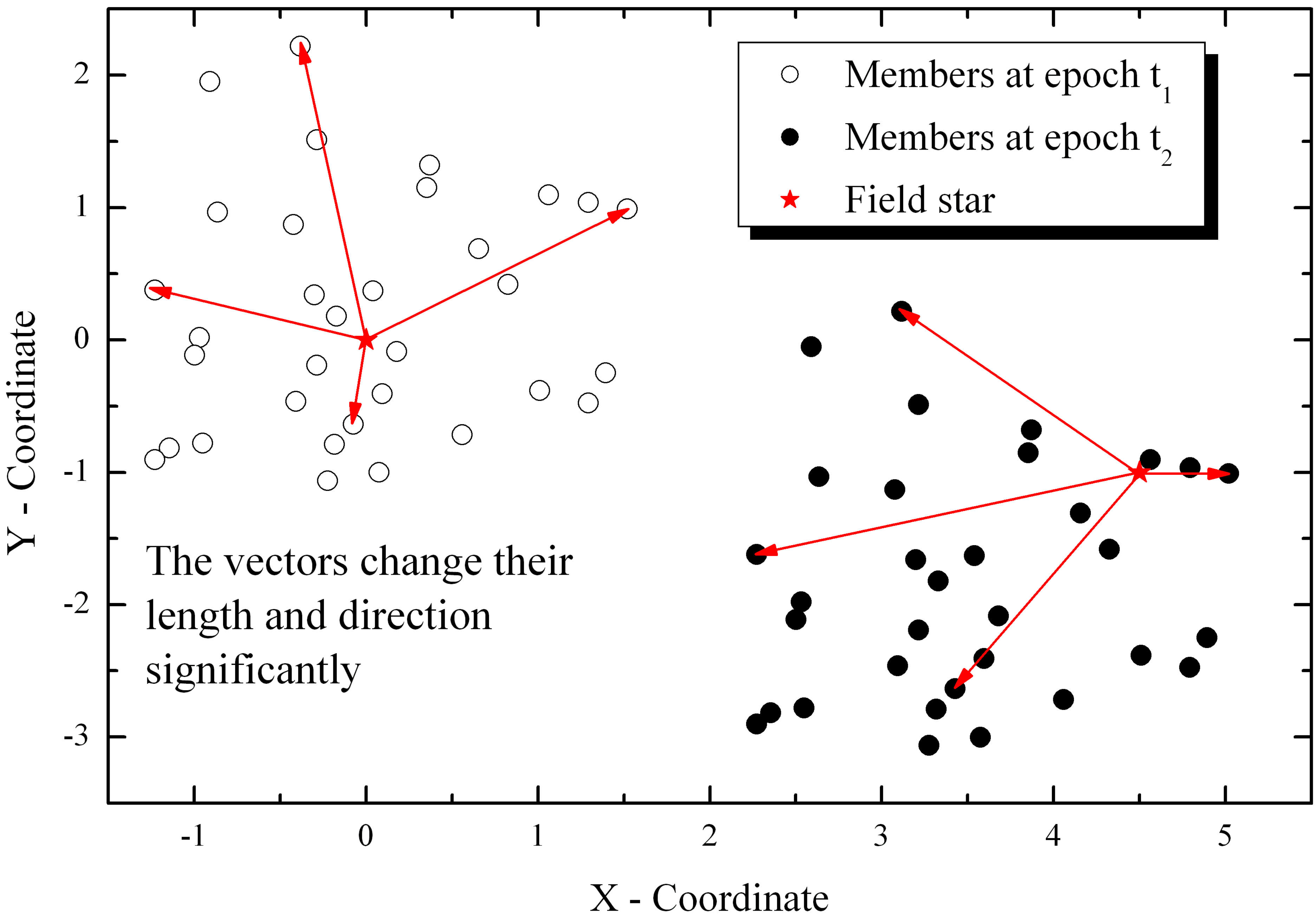
- Three possibilities:
  1. Observation of the position at two difference times (= epochs), with a very large time basis. First photographic plates around 1860, largest time scale about 160 years
  2. Proper motions of stars in the direction of the Declination α and Right Ascension δ
  3. Radial velocity measurements



Members at epoch t_1
Members at epoch t_2
Ideal case: constant
vector for all members
Y - Coordinate
X - Coordinate
2
1
0
-1
-2
-3
-1
0
1
2
3
4
5



Members at epoch t_1
Members at epoch t_2
The relative position (rp) of the members does not change. The sum of all rps is zero.
Y - Coordinate
X - coordinate
2
1
0
-1
-2
-3
-1
0
1
2
3
4
5


Members at epoch t_1
Members at epoch t_2
Field star
The vectors change their length and direction significantly
Y - Coordinate
X - Coordinate
-1
0
1
2
3
4
5
-3
-2
-1
0
1
2

Mathematical method

- Measurement of the position (X, Y) at two different epochs t_1 (´) and t_2 (´´) for each star
- Calculate the absolute distance in X and Y for both epochs and each star individually

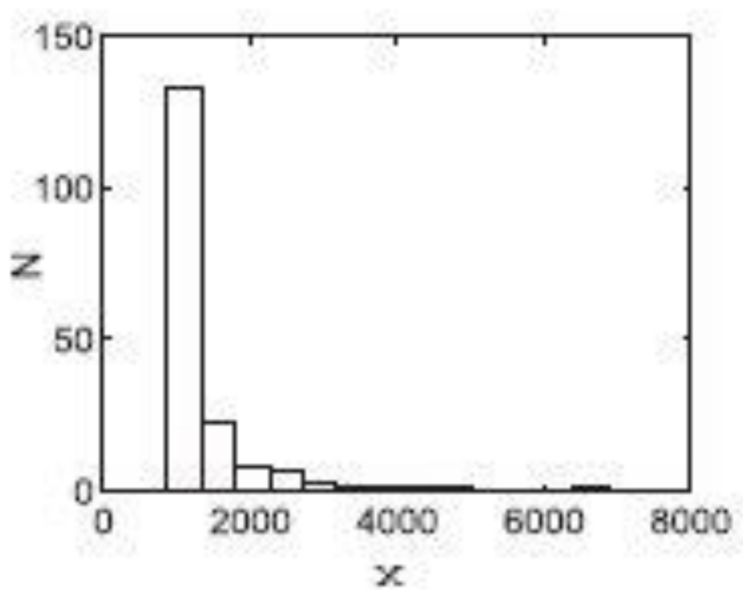
$$S'_{x_i} = \sum_{j=1}^{N} |x'_i - x'_j|, \qquad S'_{y_i} = \sum_{j=1}^{N} |y'_i - y'_j|, \qquad (1)$$

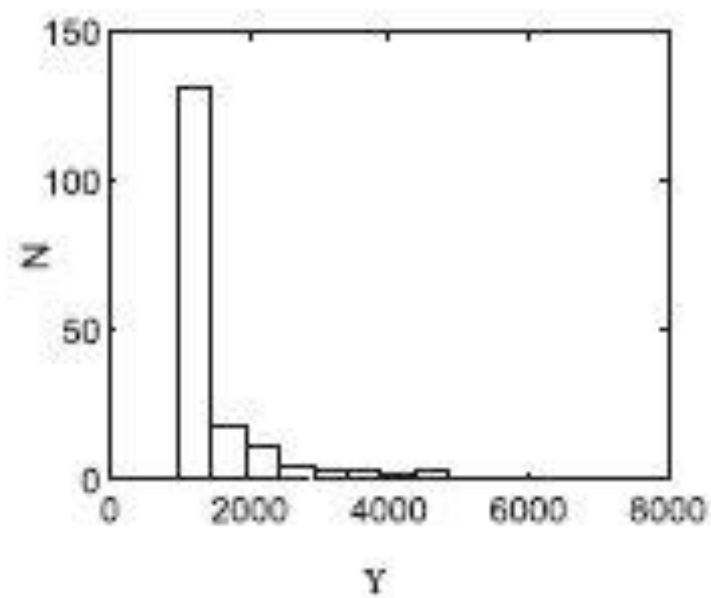
$$S''_{x_i} = \sum_{j=1}^{N} |x''_i - x''_j|, \qquad S''_{y_i} = \sum_{j=1}^{N} |y''_i - y''_j|, \qquad (2)$$

- Determine the differences of the absolute distances

$$\delta S_{x_i} = S'_{x_i} - S''_{x_i}, \qquad \delta S_{y_i} = S'_{y_i} - S''_{y_i}, \qquad i = 1, \ldots, N. \qquad (3)$$

- Plot the histograms of the differences of the absolute distances. The members have to group around the minimum of the distributions (ideal case: minimum = zero).





Example from Javakhishvili et al. (2006, A&A, 447, 915) for Collinder 121

Now we need a mathematical formalism to describe the membership probability from the distributions

- Calculate the absolute distance in X and Y for both epochs and each star individually

$$S'_{x_i} = \sum_{j=1}^{N} (x'_i - x'_j), \qquad S'_{y_i} = \sum_{j=1}^{N} (y'_i - y'_j), \tag{4}$$

$$S''_{x_i} = \sum_{j=1}^{N} (x''_i - x''_j), \qquad S''_{y_i} = \sum_{j=1}^{N} (y''_i - y''_j). \tag{5}$$

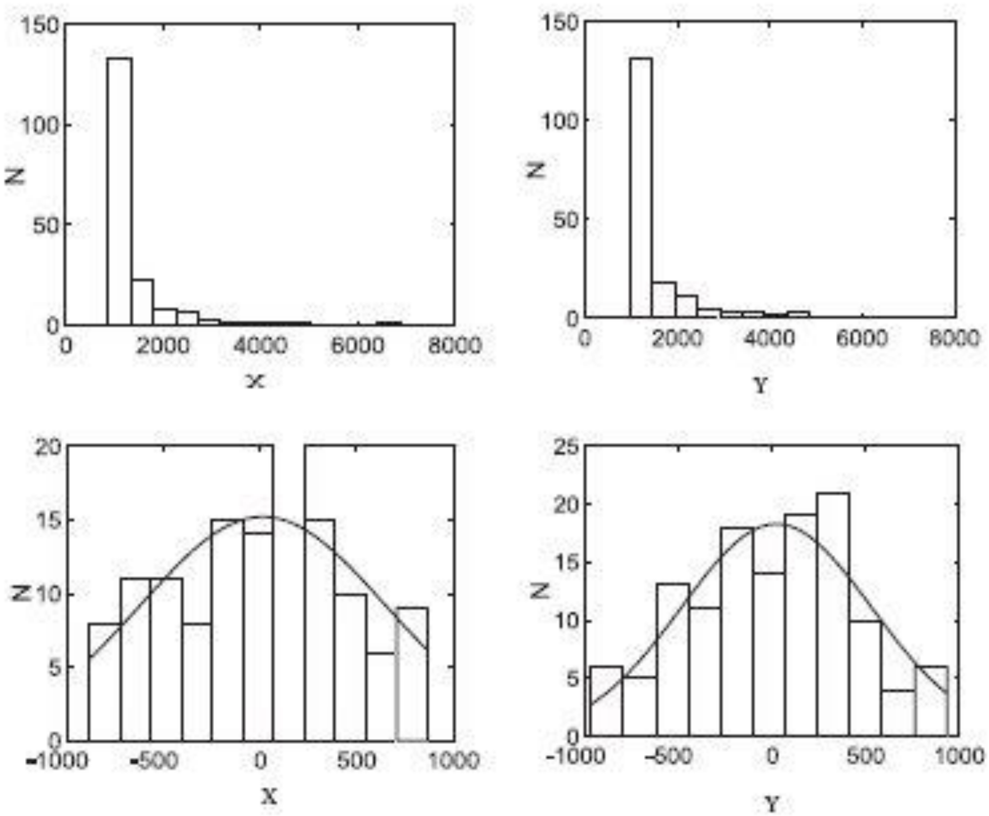
- Plot the histograms of the differences of the absolute distances
- The distributions are fitted with Gaussian functions

$$f(x) = \frac{A_x}{w_x\sqrt{\pi/2}} e^{-2(\frac{x-x_0}{\sigma x})^2}, \qquad f(y) = \frac{A_y}{w_y\sqrt{\pi/2}} e^{-2(\frac{y-y_0}{\sigma_y})^2}, \tag{6}$$

- The probability *p*, if a star is member of the star cluster is defined as

$$p_x = e^{-2\left(\frac{x-x_0}{\sigma_x}\right)^2}, \qquad p_y = e^{-2\left(\frac{y-y_0}{\sigma_y}\right)^2}. \tag{7}$$

$$p = p_x * p_y. \tag{8}$$



Javakhishvili et al., 2006, A&A, 447, 915 for Collinder 121

From these diagrams, the membership probability can be exactly determined

- In the same way, the proper motions in α and δ can be used, the basic equations and the determination of the membership probability is exactly the same

$$\delta\mu_{\alpha_i} = \sum_{j=1}^{N} |\mu_{\alpha_i} - \mu_{\alpha_j}|, \qquad \delta\mu_{\delta_i} = \sum_{j=1}^{N} |\mu_{\delta_i} - \mu_{\delta_j}| \tag{9}$$

$$\tilde{\delta}\mu_{\alpha_i} = \sum_{j=1}^{N} (\mu_{\alpha_i} - \mu_{\alpha_j}), \qquad \tilde{\delta}\mu_{\delta_i} = \sum_{j=1}^{N} (\mu_{\delta_i} - \mu_{\delta_j}). \tag{10}$$

Now, with the new Gaia data we can even investigate very distant star clusters using proper motions

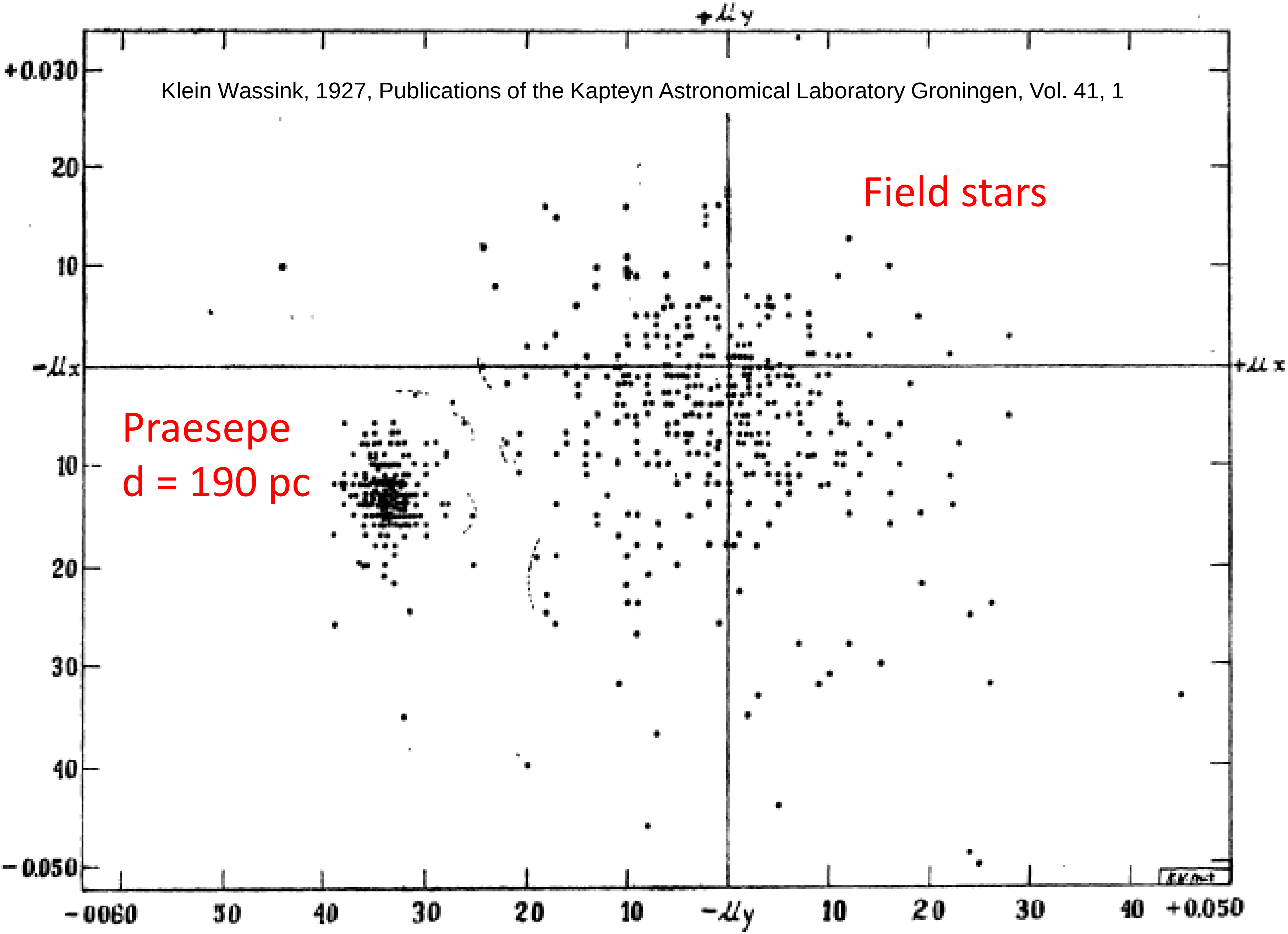
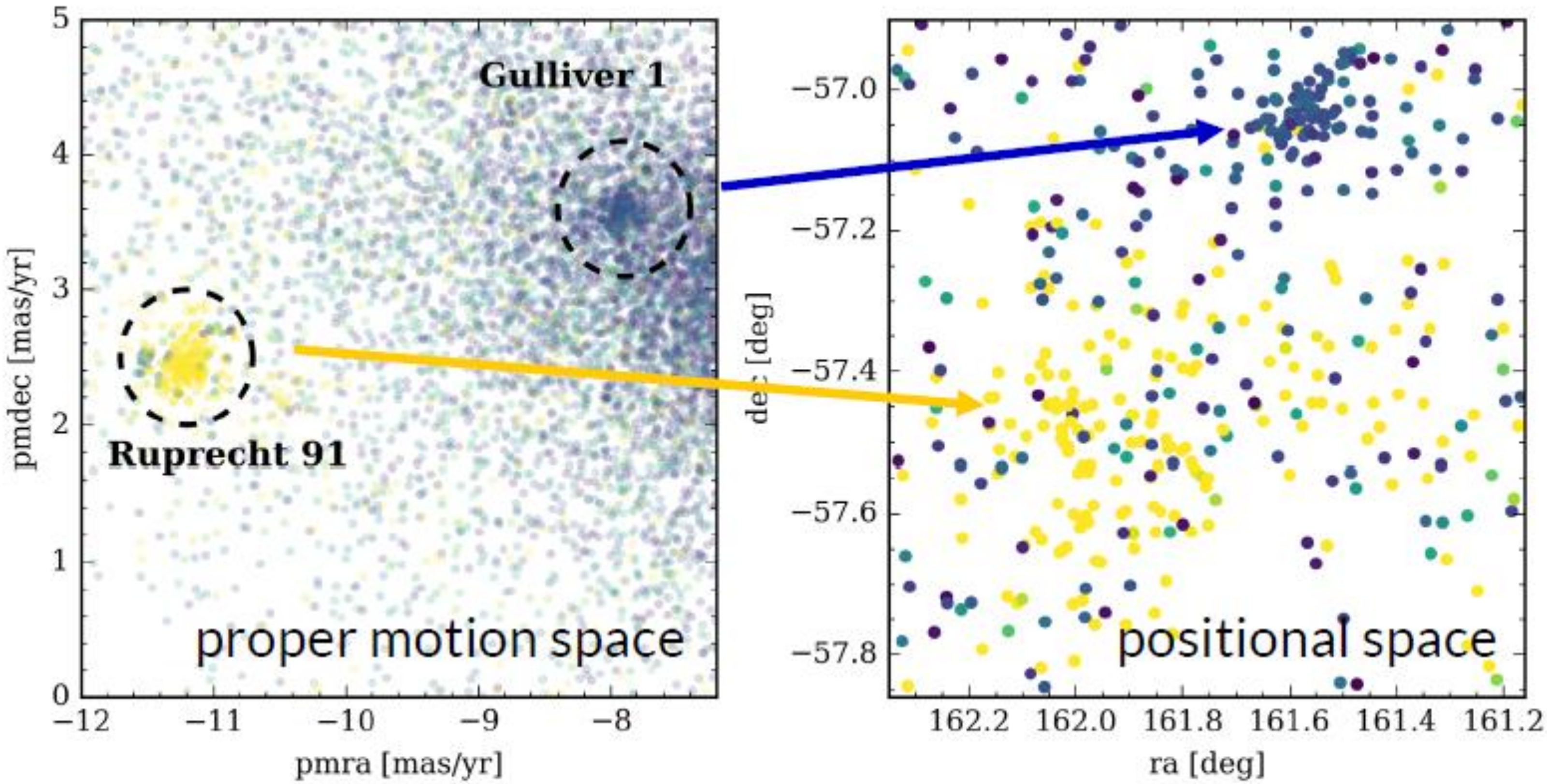


Figure 2. Diagram of the absolute proper motions of the Catalogue; photographic magnitude 6 to 14·0, numbers 1 to 531. The dotted lines separate the Praesepe stars from the backgroundstars.

Common proper motion



Credit: Tristan Cantat-Gaudin

Dramatic improvement by Gaia even for overlapping star clusters

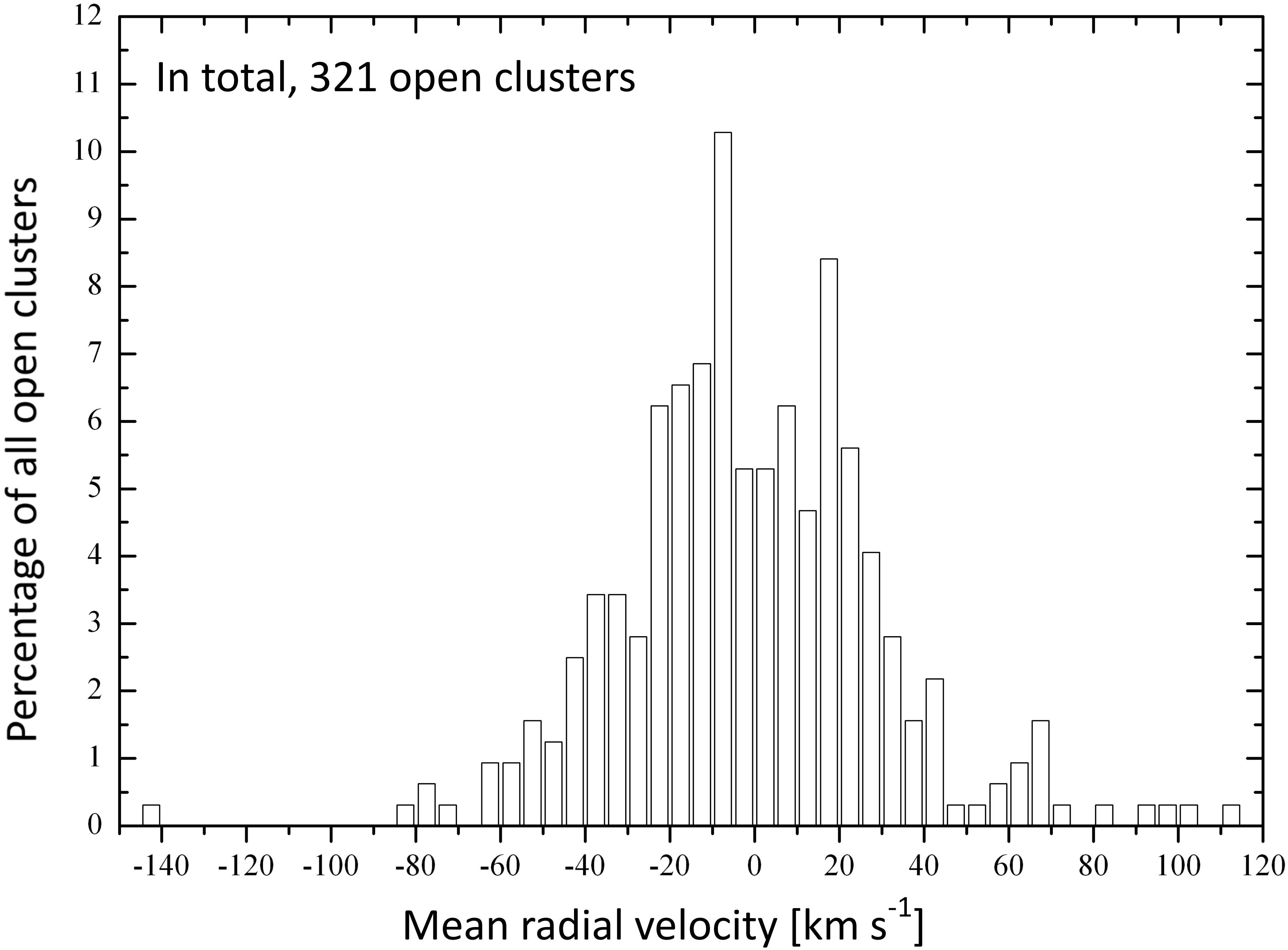
Radial velocities

- Advantages:

1. Correlated with the galactic rotation only
2. Possible to measure for most distant cluster members

- Disadvantages:

1. High-resolution high S/N spectrum needed
2. Faintness of members for distant clusters


In total, 321 open clusters
Percentage of all open clusters
Mean radial velocity [km s-1]
0
1
2
3
4
5
6
7
8
9
10
11
12
-140
-120
-100
-80
-60
-40
-20
0
20
40
60
80
100
120

Determination of the radial velocity

- Doppler shift of spectral lines

$$\Delta\lambda = \frac{v_R \lambda}{c}$$

- Determine the central wavelength of the shifted line
- Better accuracy if
  1. Instrumental resolution ($\lambda/\Delta\lambda$) is higher
  2. Signal-To-Noise ration (S/N) is higher
  3. vsini of star is lower
  4. The number of measured lines is higher

λ [Å] \ R_V [km s^{-1}]	5	10	30	100
3500	0,058	0,117	0,350	1,167
4000	0,067	0,133	0,400	1,333
4500	0,075	0,150	0,450	1,500
5000	0,083	0,167	0,500	1,667
5500	0,092	0,183	0,550	1,833
6000	0,100	0,200	0,600	2,000
6500	0,108	0,217	0,650	2,167
7000	0,117	0,233	0,700	2,333
7500	0,125	0,250	0,750	2,500
8000	0,133	0,267	0,800	2,667

Δλ [Å]

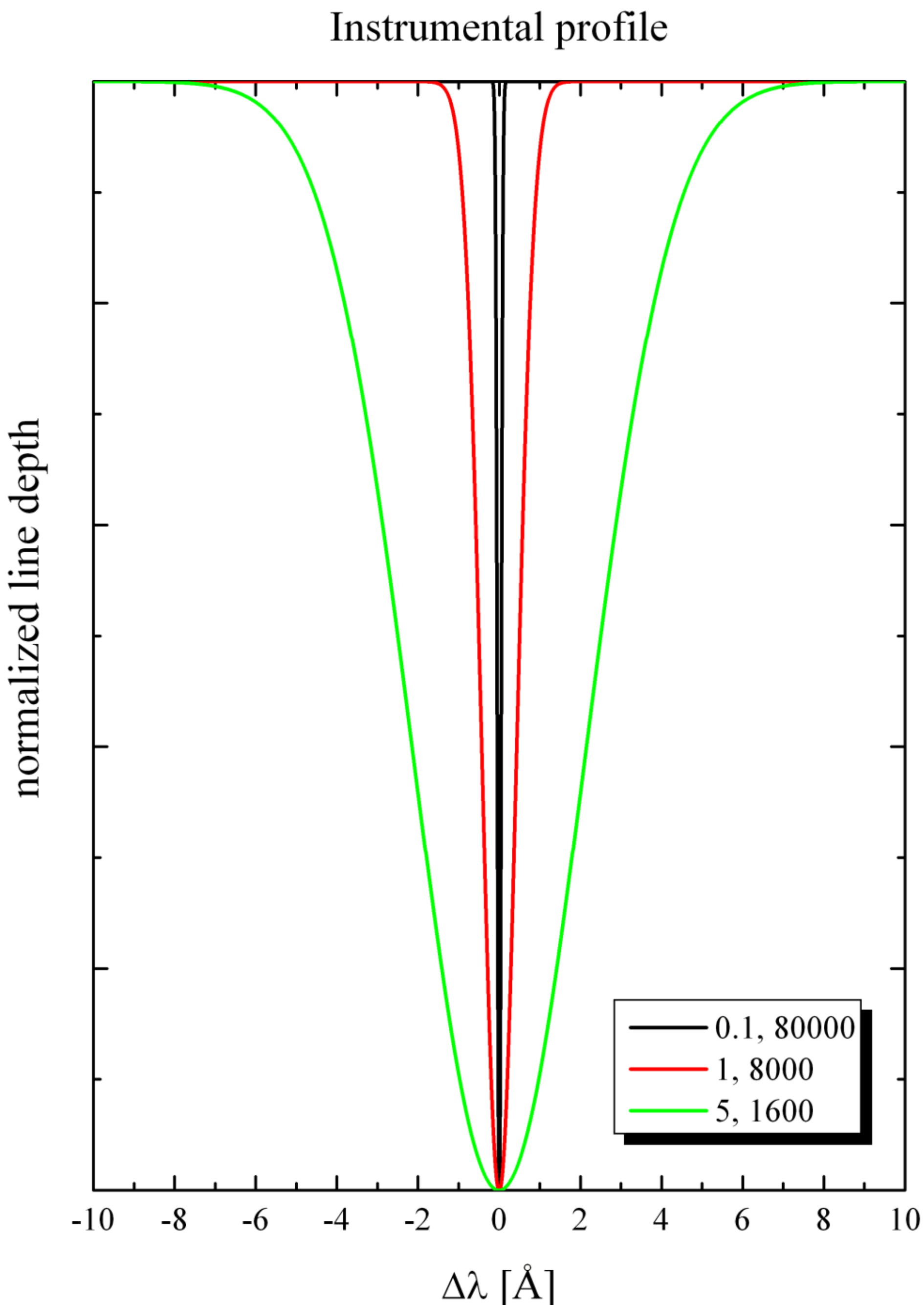
Instrumental profile defined by the resolution:

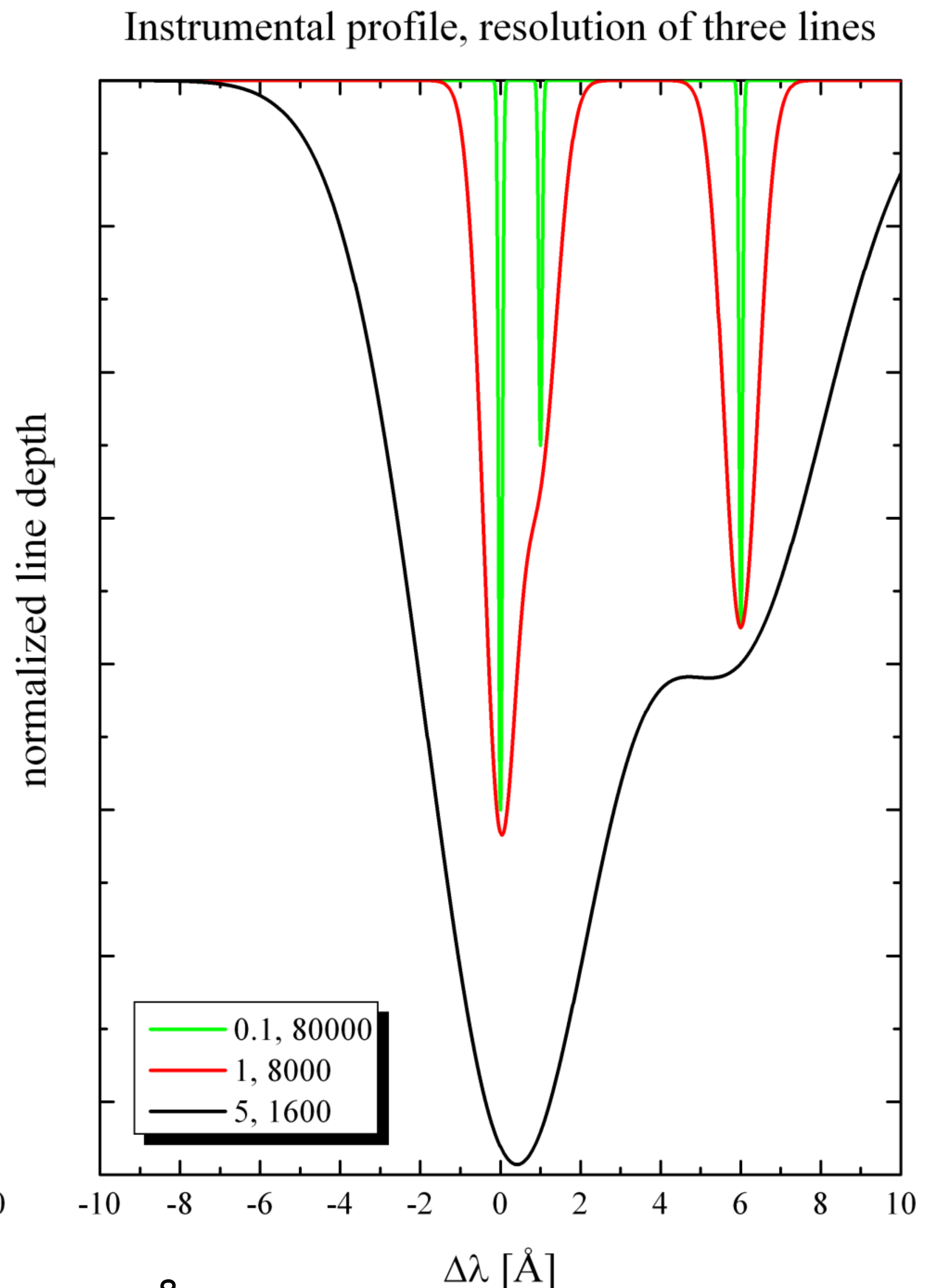
$$IP(\Delta\lambda) = \exp\left[-0.5\left(\frac{(\lambda - \Delta\lambda)}{\sigma}\right)^2\right] \text{ with } \sigma = \frac{FWHM}{2.355}$$

Rotational broadening:

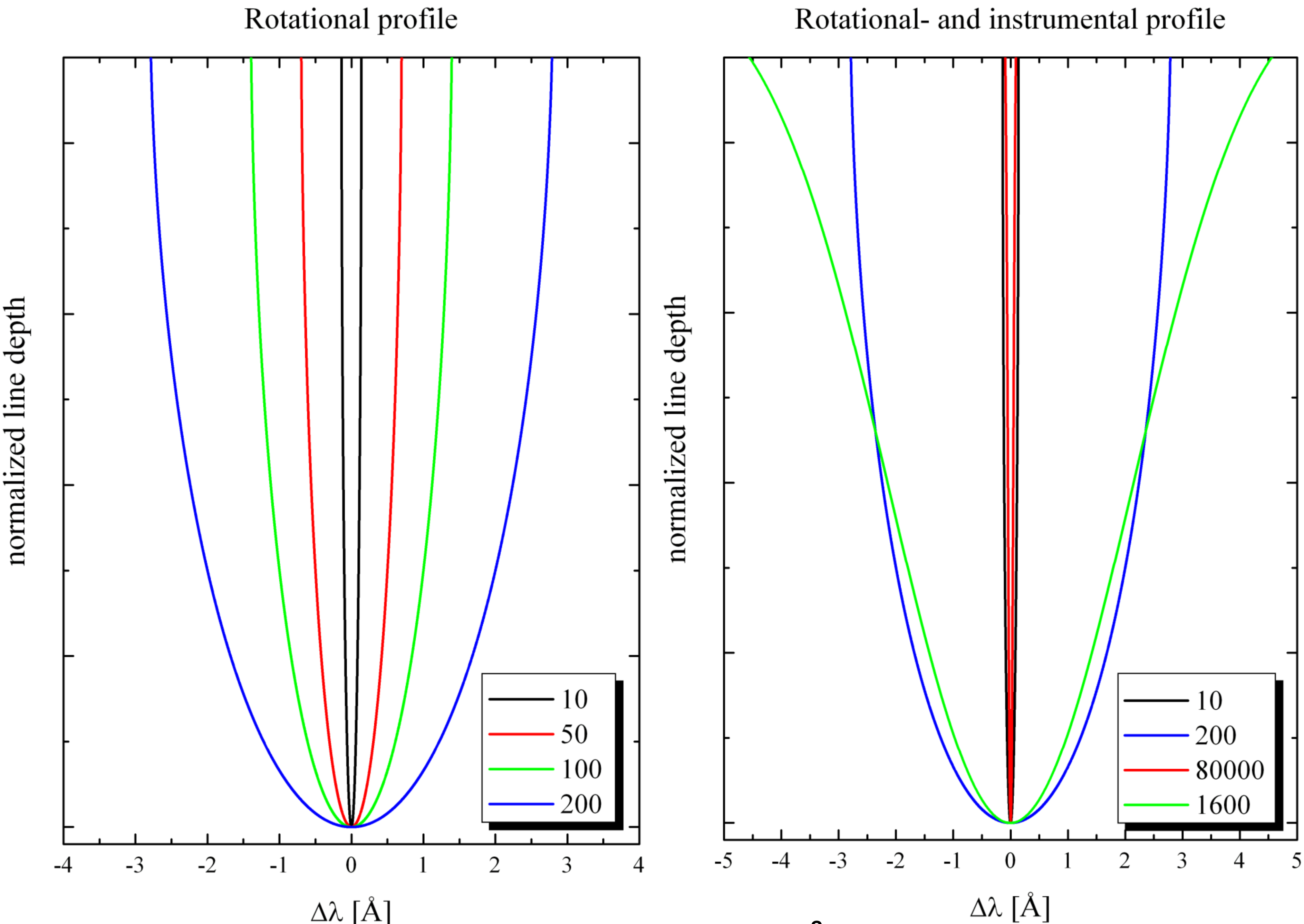
$$RP(\Delta\lambda) = c_1\sqrt{x} + c_2 x \quad \text{with} \quad x = 1 - \left(\frac{\Delta\lambda}{\Delta\lambda_L}\right)^2$$

$$\Delta\lambda_L = \lambda\frac{v \sin i}{c}$$





for λ = 4200Å



Rotational profile
normalized line depth
10
50
100
200
-4
-3
-2
-1
0
1
2
3
4
Δλ [Å]
Rotational- and instrumental profile
normalized line depth
10
200
80000
1600
-5
-4
-3
-2
-1
0
1
2
3
4
5
Δλ [Å]

for λ = 4200Å

Situation – Gaia DR2

A&A 619, A155 (2018)
https://doi.org/10.1051/0004-6361/201834020

Astronomy & Astrophysics

Open cluster kinematics with *Gaia* DR2★

C. Soubiran[1], T. Cantat-Gaudin[2], M. Romero-Gómez[2], L. Casamiquela[1], C. Jordi[2], A. Vallenari[3], T. Antoja[2], L. Balaguer-Núñez[2], D. Bossini[3], A. Bragaglia[4], R. Carrera[3], A. Castro-Ginard[2], F. Figueras[2], U. Heiter[6], D. Katz[7], A. Krone-Martins[5], J.-F. Le Campion[1], A. Moitinho[5], and R. Sordo[3]

Results. For the high-quality sample of 406 clusters, the median uncertainty of the weighted mean radial velocity is 0.5 km s^{-1}. The accuracy, assessed by comparison to ground-based high-resolution spectroscopy, is better than 1 km s^{-1}.

Can we use this for studying the internal velocity distribution (rms = 4.3 km s^{-1})?

How good can be mean RVs determined?

Situation – Gaia DR2

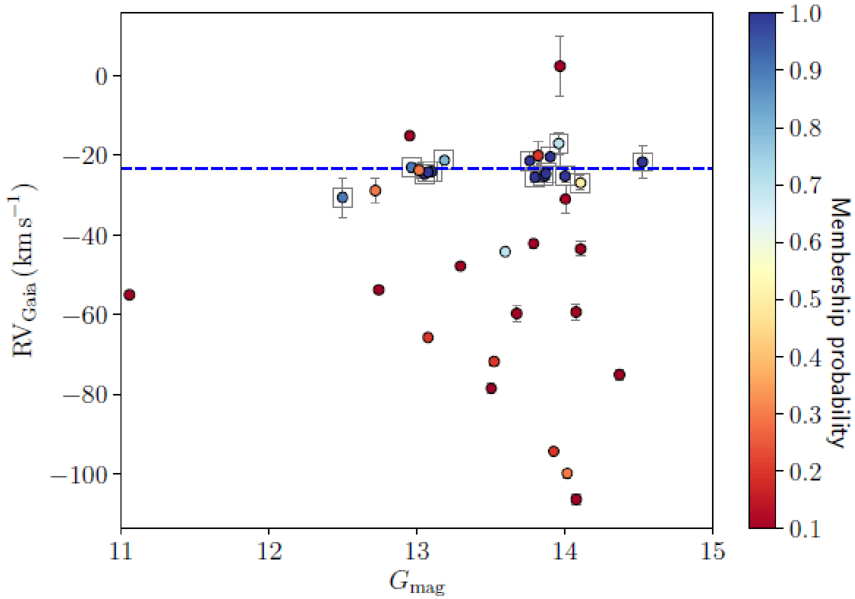


Fig. 2. Distribution of RV as a function of G magnitude for the 36 members of Skiff J0058+68.4

Huge spread and only a few members per cluster

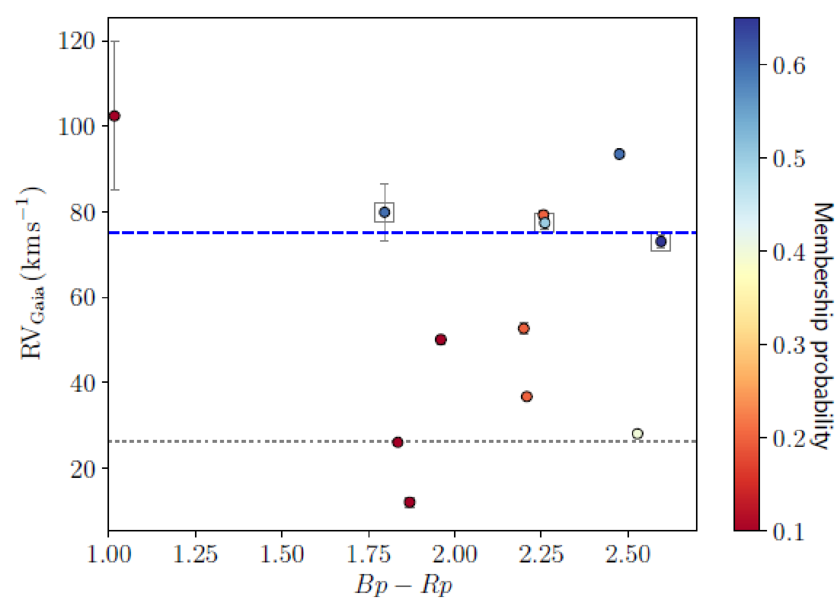


Fig. 4. Distribution of RV for members for NGC 2244.

Situation – Gaia DR2

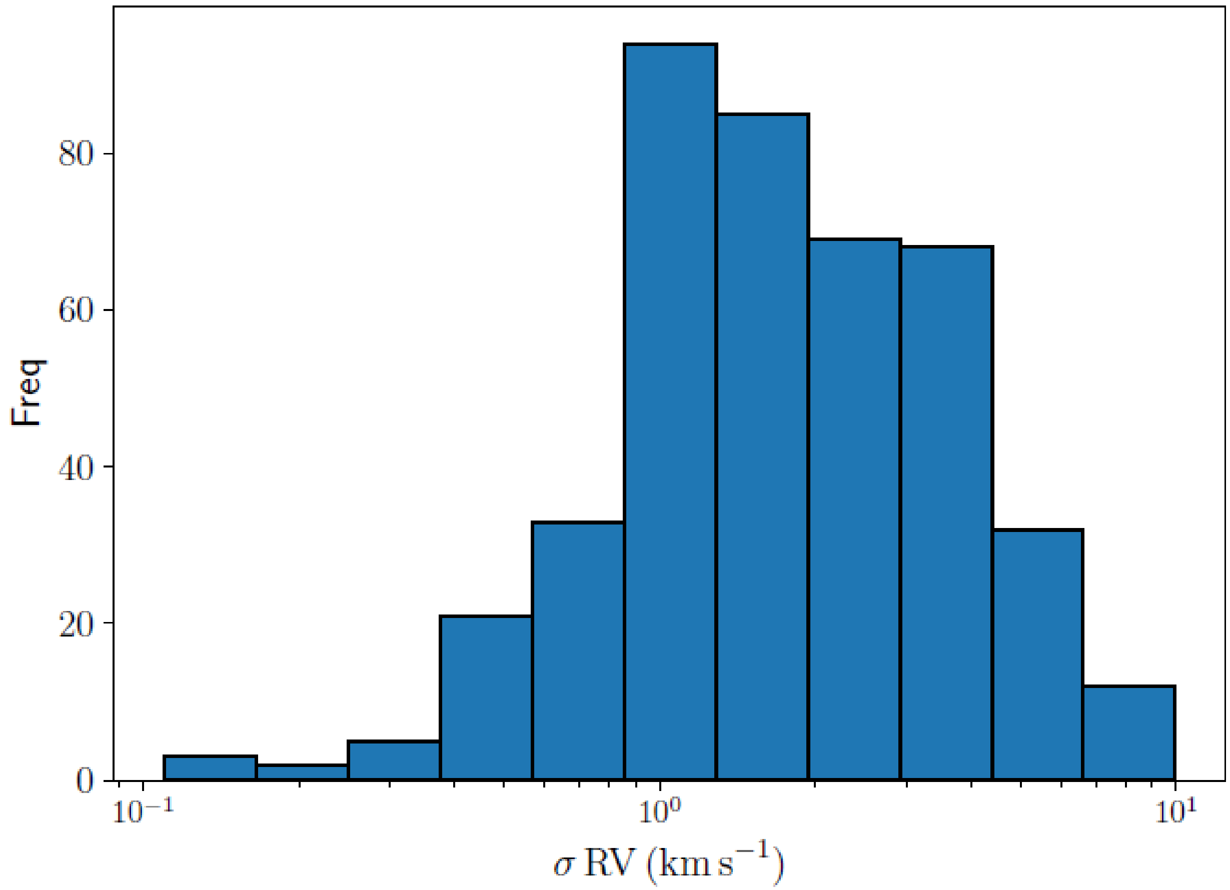


Fig. 3. Histogram of the RV standard deviation, in log scale, for the OCs with at least three members.

Not useable for internal distribution