

$$1) \iiint_V xz \, dx \, dy \, dz, \text{ kde } V: 0 \leq x \leq 1, 1 \leq y \leq 2, 0 \leq z \leq 2$$

$$a) \int_0^1 \left(\int_1^2 \left(\int_0^2 xz \, dz \right) dy \right) dx = \int_0^1 \left(\int_1^2 \left[x \frac{z^2}{2} \right]_0^2 dy \right) dx = \int_0^1 \left(\int_1^2 (2x - 0) dy \right) dx =$$

$$= \int_0^1 [2xy]_1^2 dx = \int_0^1 (4x - 2x) dx = \int_0^1 2x dx = [x^2]_0^1 = 1 //$$

$$b) \int_0^2 \left(\int_0^1 \left(\int_1^2 xz \, dy \right) dx \right) dz = \int_0^2 \left(\int_0^1 [xz y]_1^2 dx \right) dz = \int_0^2 \left(\int_0^1 (2xz - xz) dx \right) dz =$$

$$= \int_0^2 \left(\int_0^1 xz \, dx \right) dz = \int_0^2 \left[\frac{x^2}{2} z \right]_0^1 dz = \int_0^2 \frac{1}{2} z \, dz = \left[\frac{z^2}{4} \right]_0^2 = 1 //$$

$$c) \iiint_V xz \, dx \, dy \, dz = \int_0^1 x \, dx \cdot \int_1^2 dy \cdot \int_0^2 z \, dz = \left[\frac{x^2}{2} \right]_0^1 \cdot [y]_1^2 \cdot \left[\frac{z^2}{2} \right]_0^2 =$$

$$= \frac{1}{2} \cdot (2-1) \cdot 2 = 1 //$$

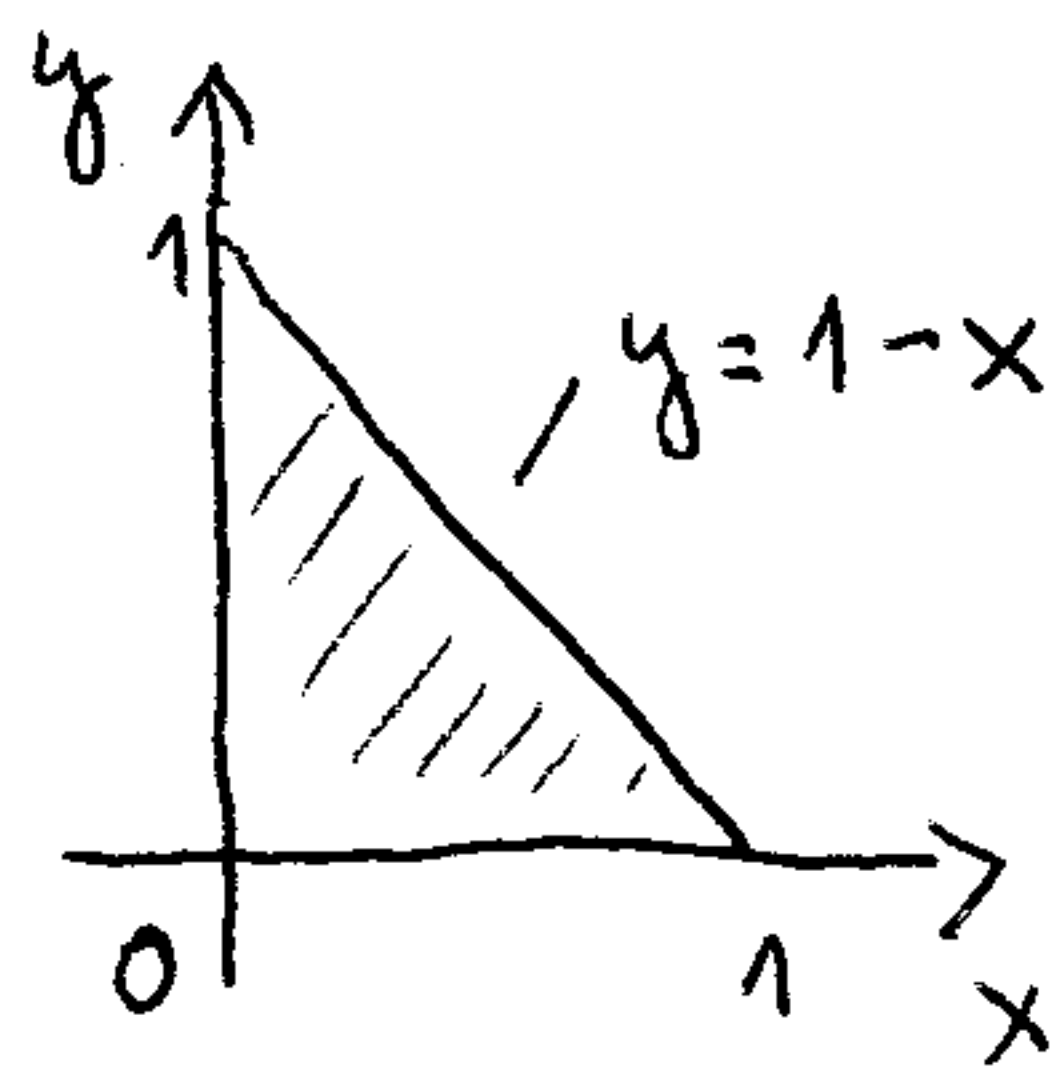
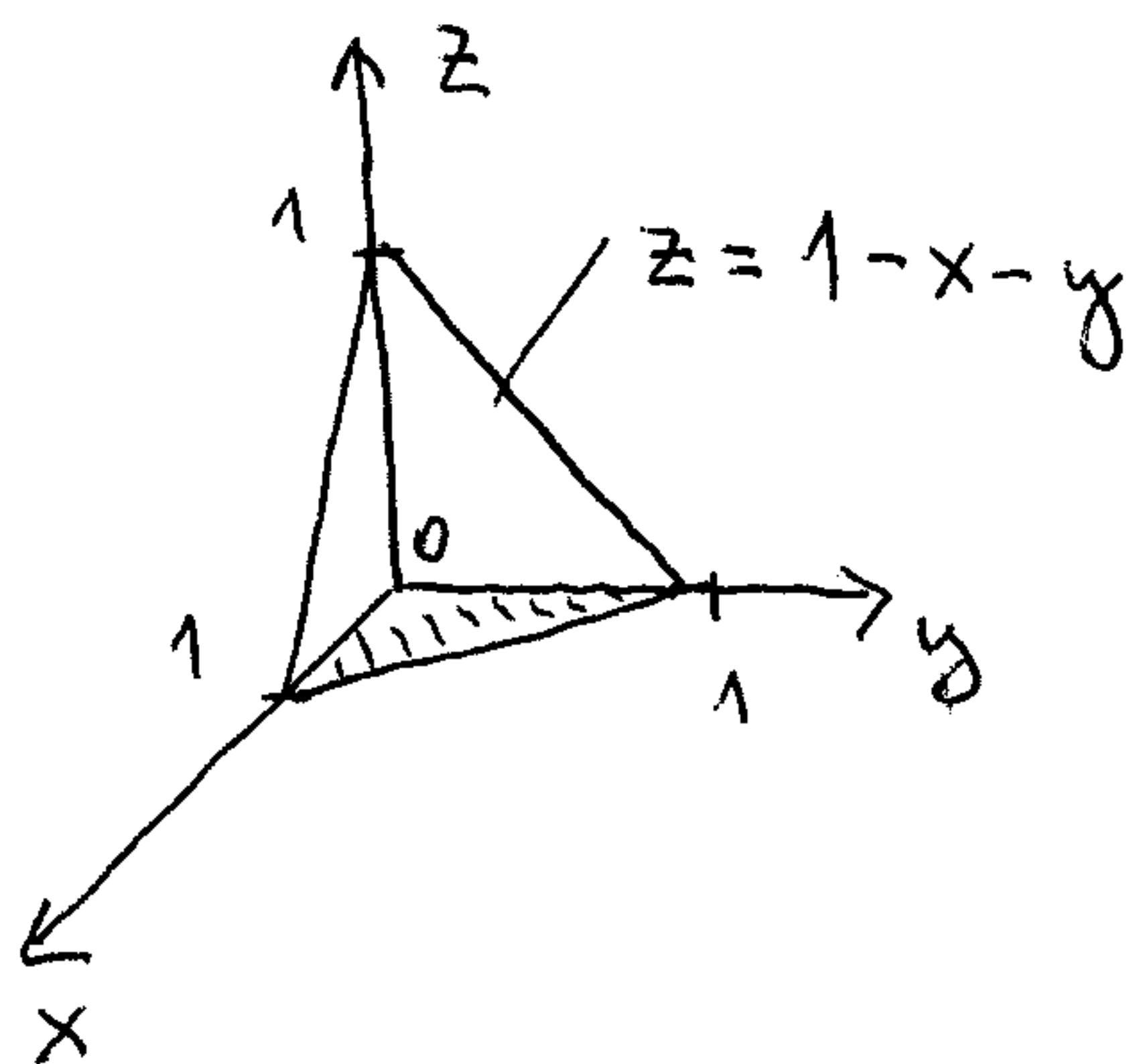
$$2) \iiint_V x^2 y z^3 \, dx \, dy \, dz, \text{ kde } V: 0 \leq x \leq 1, 0 \leq y \leq x, 0 \leq z \leq xy$$

$$\iiint_V x^2 y z^3 \, dx \, dy \, dz = \int_0^1 \left(\int_0^x \left(\int_0^{xy} x^2 y z^3 \, dz \right) dy \right) dx = \int_0^1 \left(\int_0^x \left[x^2 y \frac{z^4}{4} \right]_0^{xy} dy \right) dx =$$

$$= \int_0^1 \left(\int_0^x \frac{1}{4} x^6 y^5 \, dy \right) dx = \frac{1}{4} \int_0^1 \left[x^6 \frac{y^6}{6} \right]_0^x dx = \frac{1}{24} \int_0^1 x^{12} \, dx =$$

$$= \frac{1}{24} \left[\frac{x^{13}}{13} \right]_0^1 = \frac{1}{24} \cdot \frac{1}{13} = \frac{1}{312}$$

3) Vypočítejte objem tělesa V z příkladu 7.6 (sh. 86)



$$0 \leq x \leq 1$$

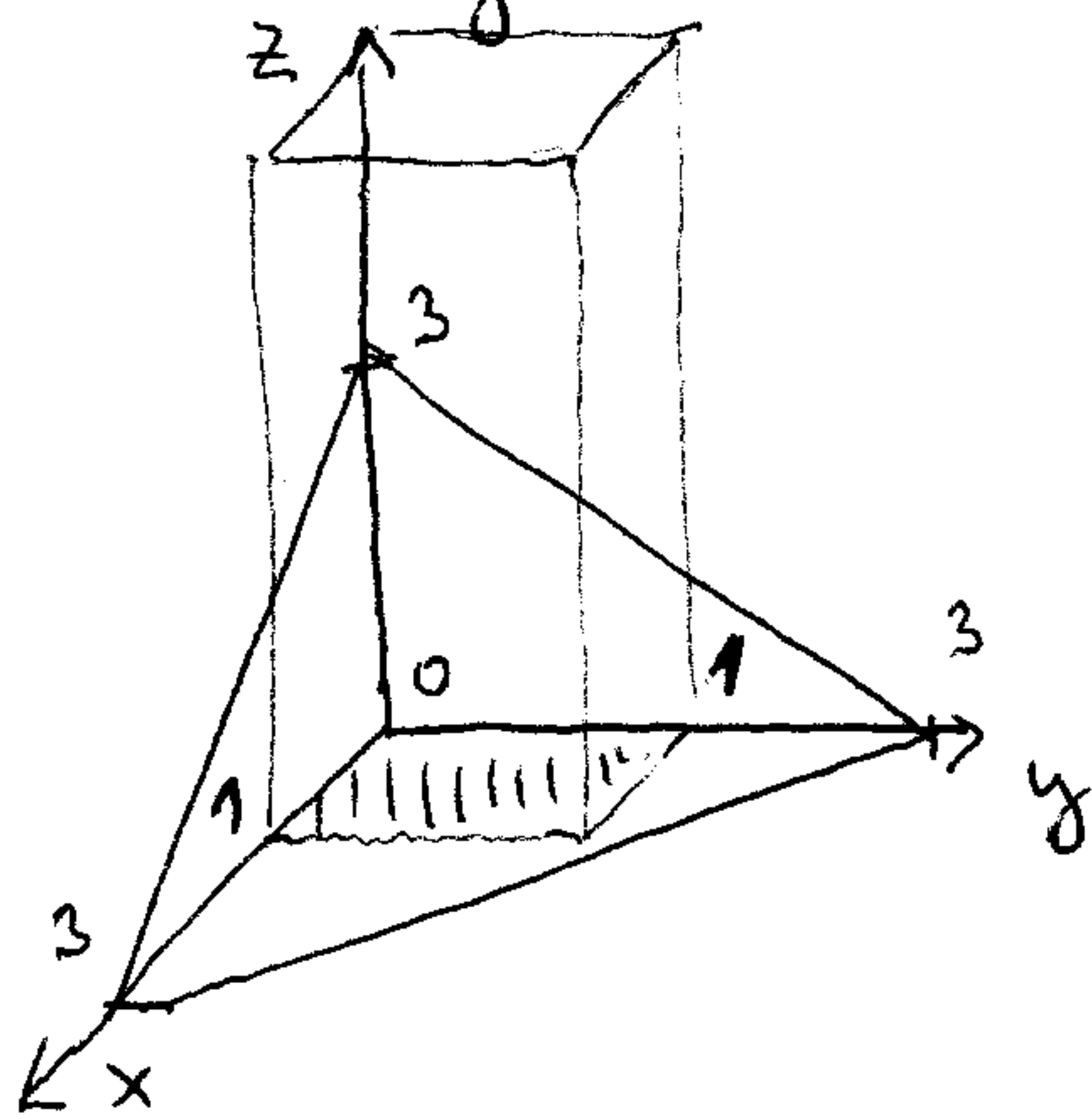
$$0 \leq y \leq 1 - x$$

$$0 \leq z \leq 1 - x - y$$

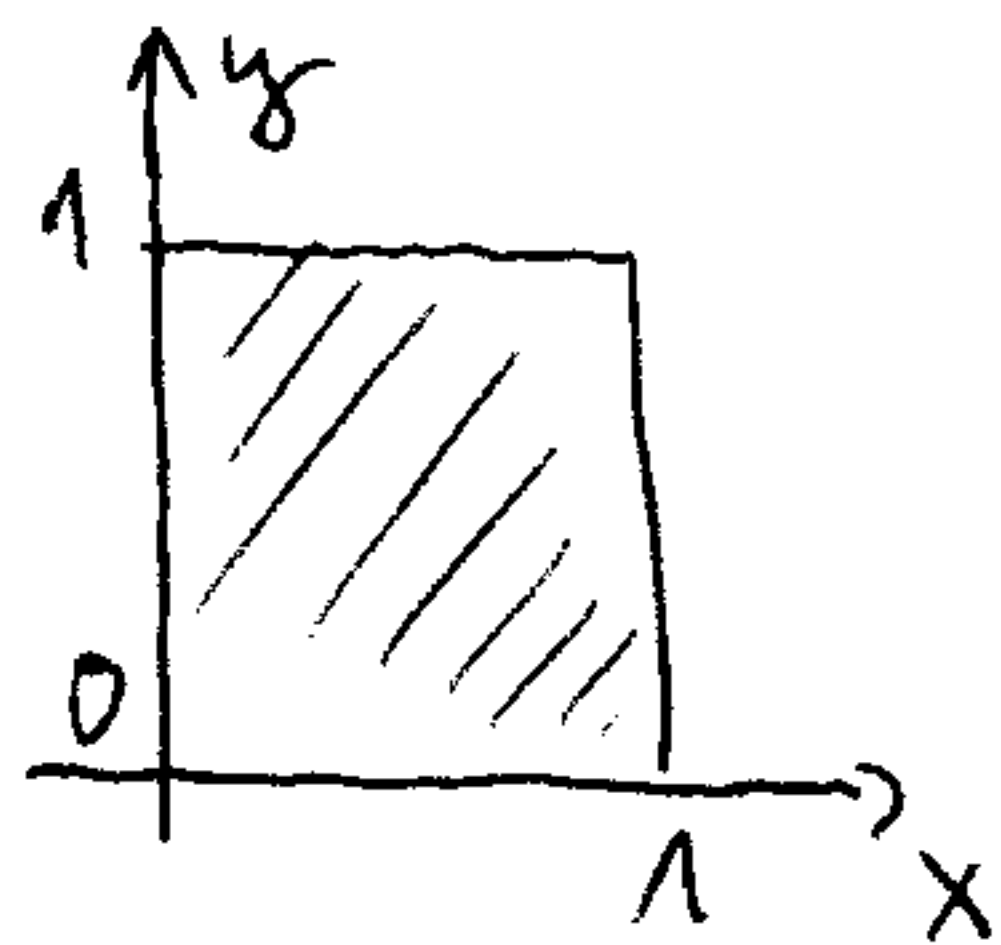
$$\begin{aligned} V &= \iiint_V 1 \, dx \, dy \, dz = \int_0^1 \left(\int_0^{1-x} \left(\int_0^{1-x-y} dz \right) dy \right) dx = \\ &= \int_0^1 \left(\int_0^{1-x} [z]_0^{1-x-y} dy \right) dx = \int_0^1 \left(\int_0^{1-x} (1-x-y) dy \right) dx = \int_0^1 \left[y - xy - \frac{y^2}{2} \right]_0^{1-x} dx \\ &= \int_0^1 \left(1-x - x(1-x) - \frac{1}{2}(1-x)^2 \right) dx = \int_0^1 \left(\frac{1}{2} - x + \frac{1}{2}x^2 \right) dx = \left[\frac{1}{2}x - \frac{x^2}{2} + \frac{x^3}{6} \right]_0^1 = \\ &= \frac{1}{2} - \frac{1}{2} + \frac{1}{6} = \frac{1}{6} \end{aligned}$$

4) Vypočítejte objem tělesa V , které je ohraničeno rovinami

$$x=0, y=0, z=0, x=1, y=1 \text{ a } x+y+z=3.$$



$$0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 3 - x - y$$



$$\begin{aligned} V &= \iiint_V dx \, dy \, dz = \int_0^1 \left(\int_0^1 \left(\int_0^{3-x-y} dz \right) dy \right) dx = \int_0^1 \left(\int_0^1 [z]_0^{3-x-y} dy \right) dx = \\ &= \int_0^1 \left(\int_0^1 (3-x-y) dy \right) dx = \int_0^1 \left[3y - xy - \frac{y^2}{2} \right]_0^1 dx = \int_0^1 \left(3 - x - \frac{1}{2} \right) dx = \\ &= \int_0^1 \left(\frac{5}{2} - x \right) dx = \left[\frac{5}{2}x - \frac{x^2}{2} \right]_0^1 = \frac{5}{2} - \frac{1}{2} = 2 \end{aligned}$$