1) $xy' - y = 0 \quad , \; y(1) = 2$

$xy' = y$

$x \frac{dy}{dx} = y \quad / : y$

$\frac{dy}{y} = \frac{dx}{x}$

$\ln|y| = \ln|x| + \ln C$

$\ln|y| = \ln C|x|$

$|y| = C|x|$

$y = Cx$

$2 = C \cdot 1$

$C = 2$

$\boxed{y = 2x}$

2) $y'' - 4y' + 4y = 0$

$\lambda^2 - 4\lambda + 4 = 0$

$(\lambda - 2)^2 = 0$

$\lambda_{1,2} = 2$

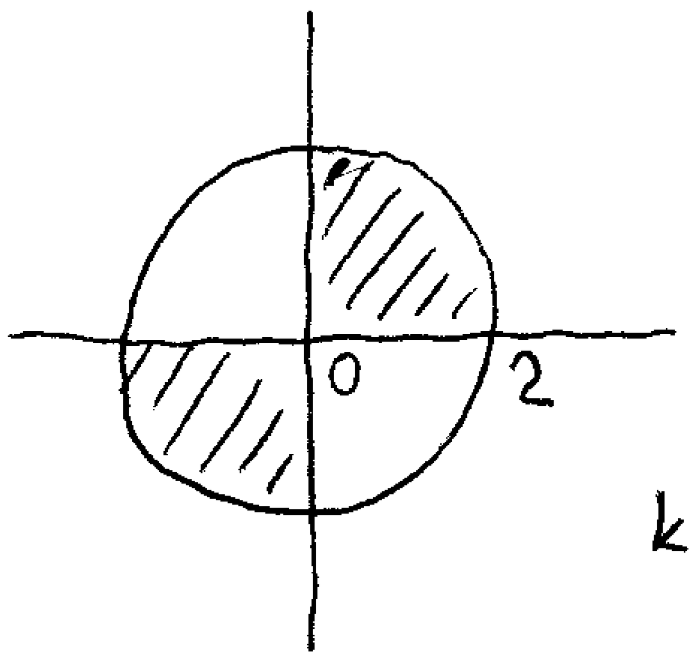
$y = C_1 e^{2x} + C_2 x e^{2x}$

3) $z = \sqrt{xy} + \ln(4 - x^2 - y^2)$

$xy \geq 0 \;\wedge\; 4 - x^2 - y^2 > 0$

$(x \geq 0 \wedge y \geq 0) \vee (x \leq 0 \wedge y \leq 0)$

$4 > x^2 + y^2$



kružnice do Df nepatří

4) $f(x,y) = \ln(x + y^2)$

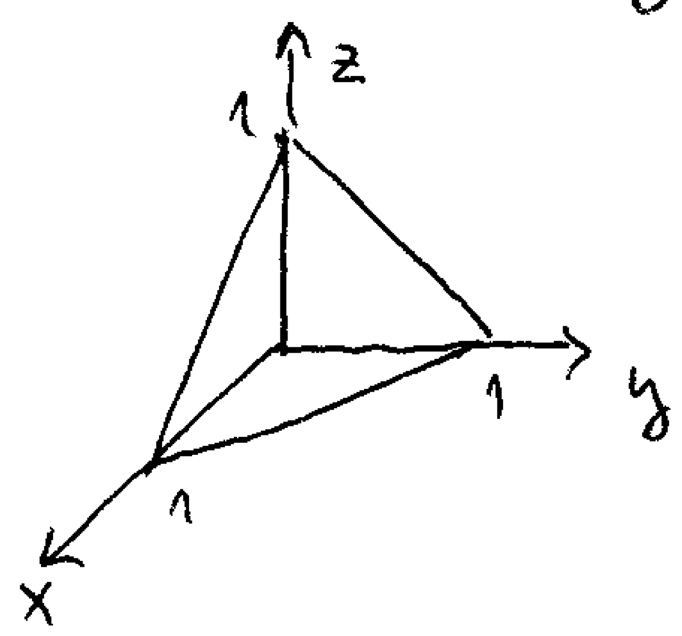
$f_x = \frac{1}{x + y^2} \cdot 1$

$f_y = \frac{1}{x + y^2} \cdot 2y$

$f_{xy} = \frac{0 \cdot (x+y^2) - 1 \cdot 2y}{(x+y^2)^2} = \frac{-2y}{(x+y^2)^2}$

$f_{yx} = \frac{0 \cdot (x+y^2) - 2y \cdot 1}{(x+y^2)^2} = \frac{-2y}{(x+y^2)^2}$

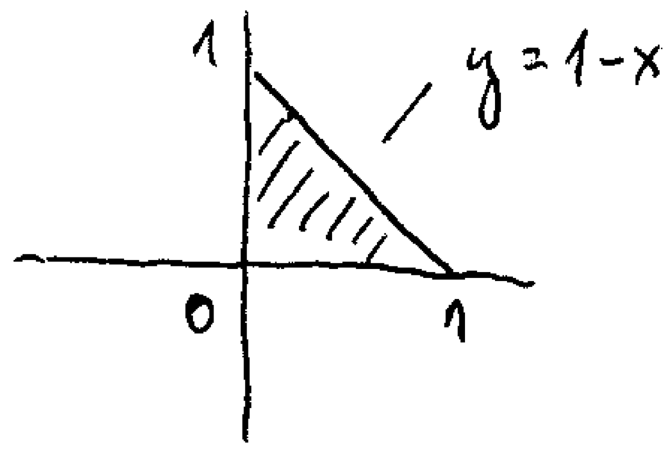
5) $f(x,y) = 1 - x - y$



$f_x = -1 \qquad f_x(0,0) = -1$

$f_y = -1 \qquad f_y(0,0) = -1$

M:



největší hodnota 1
v bodě [0,0]
nejmenší hodnota 0
v bodech úsečky $y = 1 - x$

6) $f(x,y) = \sqrt{x^2+y^2}$, [3,4]

$f_x = \frac{x}{\sqrt{x^2+y^2}} \qquad f_x(3,4) = \frac{3}{5}$, $f_y = \frac{y}{\sqrt{x^2+y^2}} \qquad f_y(3,4) = \frac{4}{5}$

$$df(3,4) = \frac{3}{5}dx + \frac{4}{5}dy$$

7) $f(x,y) = x^2+y^2$, $T = [1,1,2]$

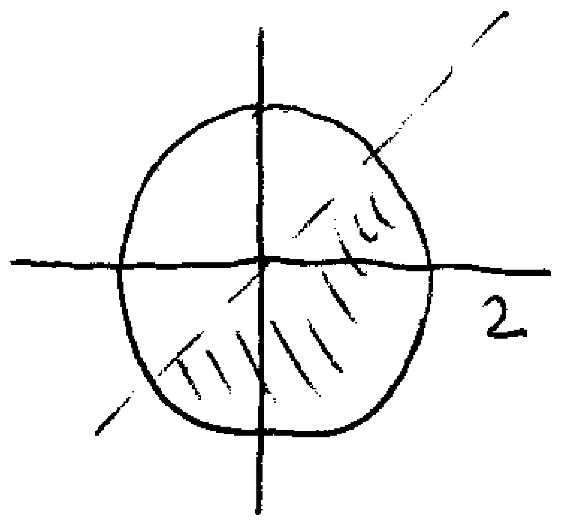
$f_x = 2x \qquad f_x(1,1) = 2$

$f_y = 2y \qquad f_y(1,1) = 2$

$$z - 2 = 2(x-1) + 2(y-1)$$

$$2x + 2y - z - 2 = 0$$

8) $x^2+y^2 \leq 4$, $\cancel{x \geq 0}$, $y \leq x$

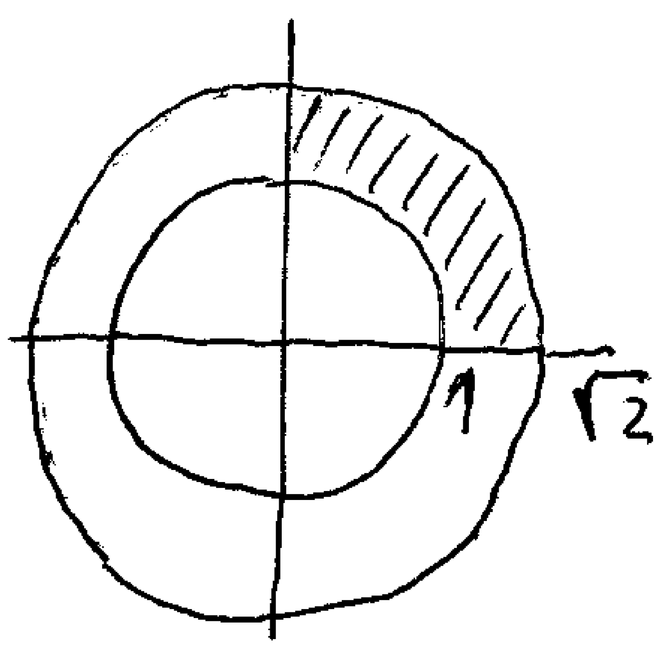


$0 \leq \varrho \leq 2$

$-\frac{3}{4}\pi \leq \varphi \leq \frac{\pi}{4}$

9) $\iint\limits_M \frac{dx\,dy}{x^2+y^2}$

$$\int_0^{\pi/2}\left(\int_1^{\sqrt{2}} \frac{\varrho\, d\varrho}{\varrho^2}\right)d\varphi = \int_0^{\pi/2}\left(\int_1^{\sqrt{2}} \frac{1}{\varrho}\, d\varrho\right)d\varphi =$$



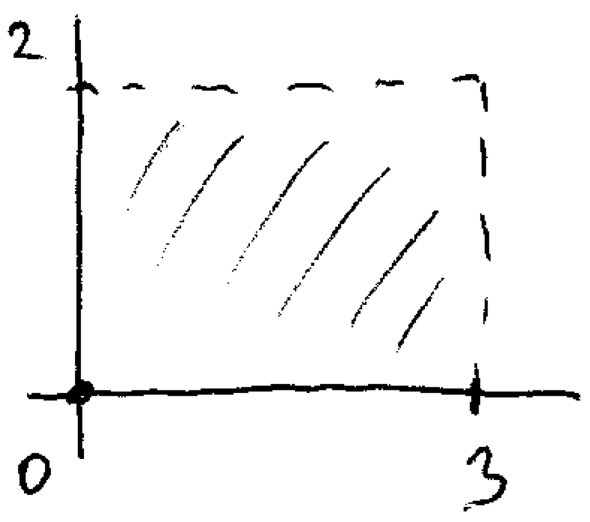
$1 \leq \varrho \leq \sqrt{2}$

$0 \leq \varphi \leq \frac{\pi}{2}$

$$= \int_0^{\pi/2} d\varphi \cdot \int_1^{\sqrt{2}} \frac{d\varrho}{\varrho} = [\varphi]_0^{\pi/2} \cdot [\ln \varrho]_1^{\sqrt{2}} =$$

$$= \frac{\pi}{2} \cdot (\ln\sqrt{2} - \ln 1) = \frac{\pi}{2} \ln\sqrt{2}$$

10) $\iint\limits_M xy\,dx\,dy$, M: obdélník ABCD



$0 \le x \le 3$
$0 \le y \le 2$

$$\int_0^3 \left(\int_0^2 xy\,dy \right) dx = \int_0^3 x\,dx \cdot \int_0^2 y\,dy =$$

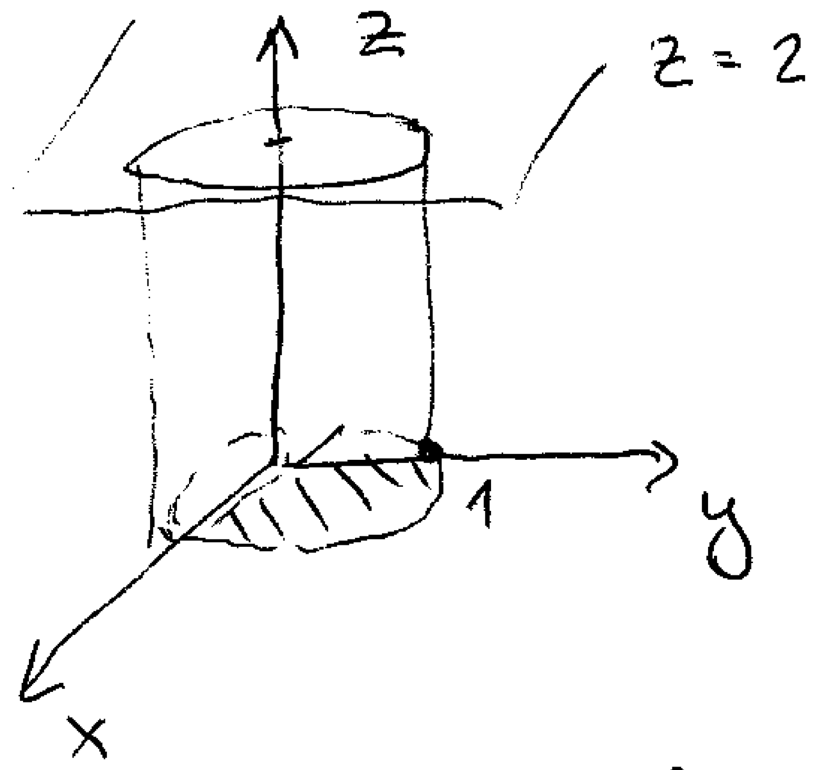
$$\left[\frac{x^2}{2}\right]_0^3 \cdot \left[\frac{y^2}{2}\right]_0^2 = \frac{9}{2} \cdot \frac{4}{2} = 9$$

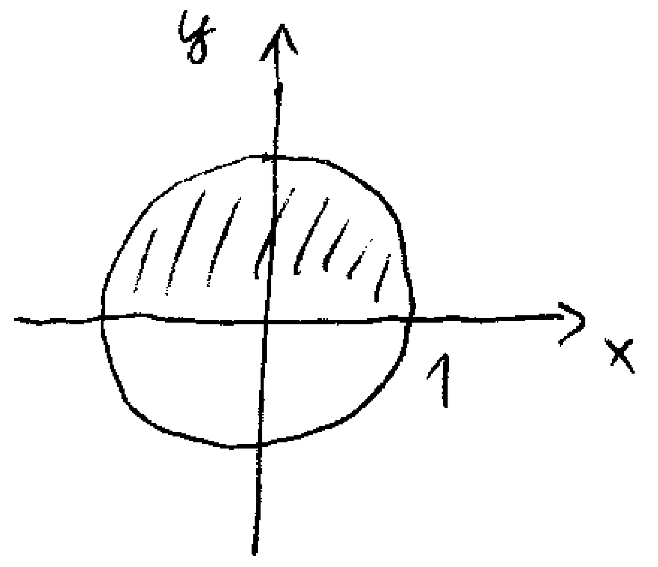
11) $$\iint\limits_M f(x,y)\,dx\,dy = \int_a^b \left(\int_{\varphi(x)}^{\psi(x)} f(x,y)\,dy \right) dx$$

$a \le x \le b$
$\varphi(x) \le y \le \psi(x)$

$\boxed{\text{Věta 6.4, str. 73}}$

12) $\iiint\limits_V (x^2+y^2)\,dx\,dy\,dz$, V: $x^2+y^2 \le 1$, $y \ge 0$, $0 \le z \le 2$

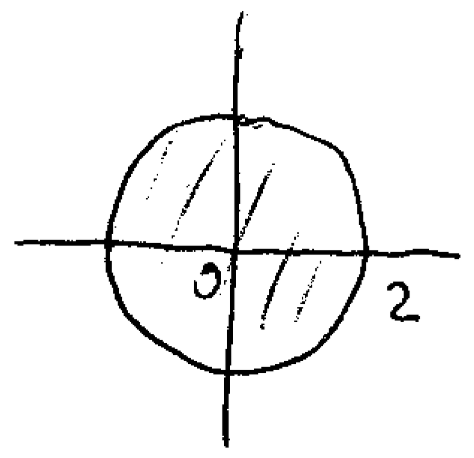




$0 \le \varrho \le 1$
$0 \le \varphi \le \pi$
$0 \le z \le 2$

$$\iiint\limits_V (x^2+y^2)\,dx\,dy\,dz = \int_0^\pi \left(\int_0^1 \left(\int_0^2 \varrho^2 \varrho\,dz \right) d\varrho \right) d\varphi$$

13)

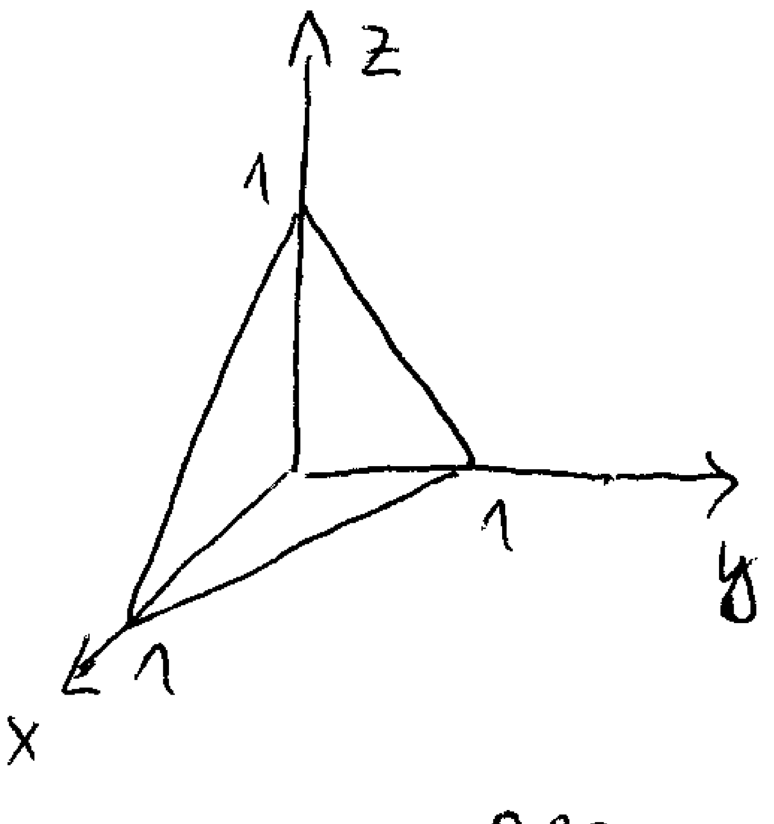


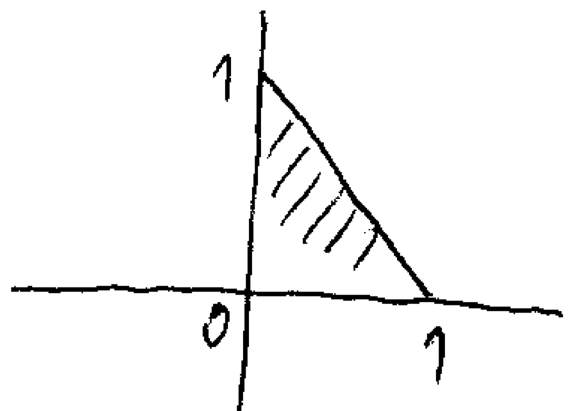
$0 \le \varrho \le 2$
$0 \le \varphi \le 2\pi$

$$S = \iint\limits_M 1\,dx\,dy = \int_0^{2\pi} \left(\int_0^2 \varrho\,d\varrho \right) d\varphi =$$

$$= \int_0^{2\pi} d\varphi \cdot \int_0^2 \varrho\,d\varrho = \left[\varphi\right]_0^{2\pi} \cdot \left[\frac{\varrho^2}{2}\right]_0^2 = 2\pi \cdot 2 = 4\pi$$

14)





$0 \le x \le 1$
$0 \le y \le 1-x$
$0 \le z \le 1-x-y$

$$V = \iiint\limits_V 1\,dx\,dy\,dz = \int_0^1 \left(\int_0^{1-x} \left(\int_0^{1-x-y} dz \right) dy \right) dx = \cdots = \frac{1}{6}$$

Viz priklady_chemici_5

15) $\int_C (2x+y)\,ds$, C: úsečka AB, $A=[0,0]$, $B=[1,2]$

$\vec{s} = \vec{AB} = B-A = (1,2)$ $\quad x = 0+1t \quad y = 0+2t \quad t \in \langle 0,1\rangle \quad x' = 1 \quad y' = 2$

$$\int_C (2x+y)\,ds = \int_0^1 (2t+2t)\sqrt{1^2+2^2}\,dt = \sqrt{5}\int_0^1 4t\,dt = \sqrt{5}\left[2t^2\right]_0^1 = 2\sqrt{5}$$

16) $\int_C 3x^2\cos y\,dx - (x^3\sin y + 3y)\,dy$

$P = 3x^2\cos y \qquad P_y = 3x^2(-\sin y) \qquad P_y = Q_x$

$Q = -x^3\sin y - 3y \qquad Q_x = -3x^2\sin y$

Integrál nezávisí na integrační cestě. Závisí pouze na počátečním a koncovém bodě. V případě uzavřené křivky je roven 0.

17) $\int_C P(x,y)\,dx + Q(x,y)\,dy$ Práce, kterou vykoná silové pole $\vec{F} = (P(x,y), Q(x,y))$ při přemístění hmotného bodu podél křivky C z jejího počátečního do koncového bodu.

1) $y' - xy = 2x e^{\frac{x^2}{2}}$ $\qquad y' + p(x) y = f(x)$

integrační faktor $\quad I(x) = e^{\int -x \, dx} = e^{-\frac{x^2}{2}}$

$$(y' - xy) e^{-\frac{x^2}{2}} = 2x e^{\frac{x^2}{2}} \cdot e^{-\frac{x^2}{2}} = 2x$$

$$\left(y e^{-\frac{x^2}{2}}\right)' = 2x$$

$$y \cdot e^{-\frac{x^2}{2}} = \int 2x \, dx$$

$$y \cdot e^{-\frac{x^2}{2}} = x^2 + C$$

$$\boxed{y = C e^{\frac{x^2}{2}} + x^2 e^{\frac{x^2}{2}}}$$

2) $y'' - 3y' + 2y = 2x + 5 \quad , y(0) = 4 \, , y'(0) = 0$

$\lambda^2 - 3\lambda + 2 = 0$

$(\lambda - 2)(\lambda - 1) = 0$

$$\boxed{y_0 = C_1 e^{x} + C_2 e^{2x}}$$

$y_p = Ax + B$

$y_p' = A$

$y_p'' = 0$

$0 - 3A + 2(Ax + B) = 2x + 5$

$2Ax + (-3A + 2B) = 2x + 5$

$A = 1 \, , \quad B = 4$

$$\boxed{y_p = x + 4}$$

$$\boxed{y = C_1 e^{x} + C_2 e^{2x} + x + 4}$$

$y' = C_1 e^{x} + 2C_2 e^{2x} + 1$

$4 = C_1 + C_2 + 4 \rightarrow C_1 + C_2 = 0$

$0 = C_1 + 2C_2 + 1 \rightarrow C_1 + 2C_2 = -1$

$C_1 = 1 \, , \quad C_2 = -1$

$$\boxed{y = e^{x} - e^{2x} + x + 4}$$

2) $z = 2x^3 - 6xy + 3y^2 - 6x - 6y + 6$

$z_x = 6x^2 - 6y - 6$ $\qquad 6x^2 - 6y - 6 = 0 \rightarrow x^2 - y - 1 = 0$

$z_y = -6x + 6y - 6$ $\qquad -6x + 6y - 6 = 0 \rightarrow x - y + 1 = 0$

$y = x^2 - 1$ $\qquad x - x^2 + 1 + 1 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

stacionární body

$[-1, 0]$, $[2, 3]$

$z_{xx} = 12x$, $z_{xy} = -6$, $z_{yx} = -6$, $z_{yy} = 6$

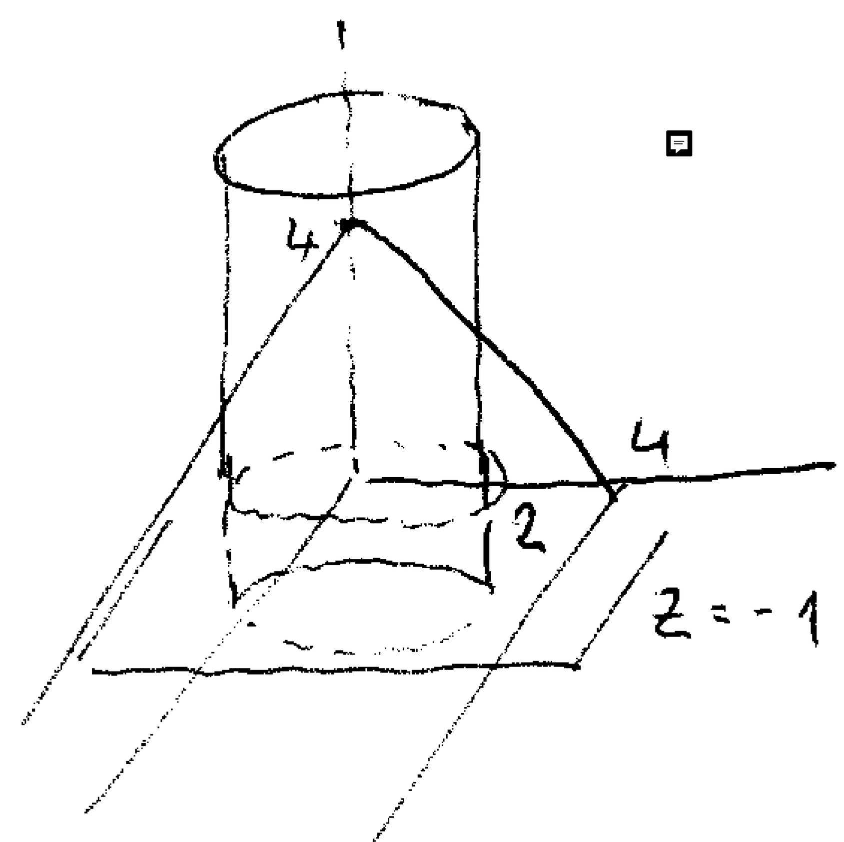
$$D(x,y) = 12x \cdot 6 - (-6)^2 = 72x - 36$$

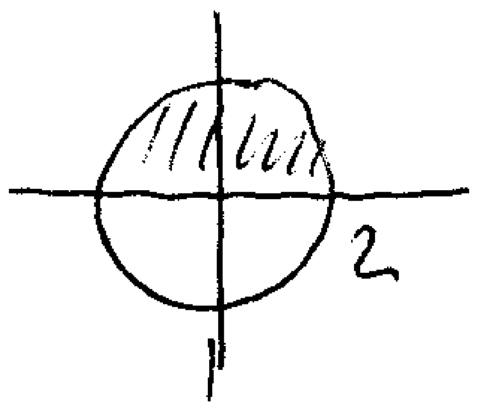
$D(-1,0) = 72 \cdot (-1) - 36 < 0$ není extrém

$D(2,3) = 72 \cdot 2 - 36 > 0$ je extrém $\qquad z_{xx}(2,3) = 12 \cdot 2 = 24 > 0$

v bodě $[2,3]$ lokální min.

3)





$0 \leq \varrho \leq 2$

$0 \leq \varphi \leq \pi$

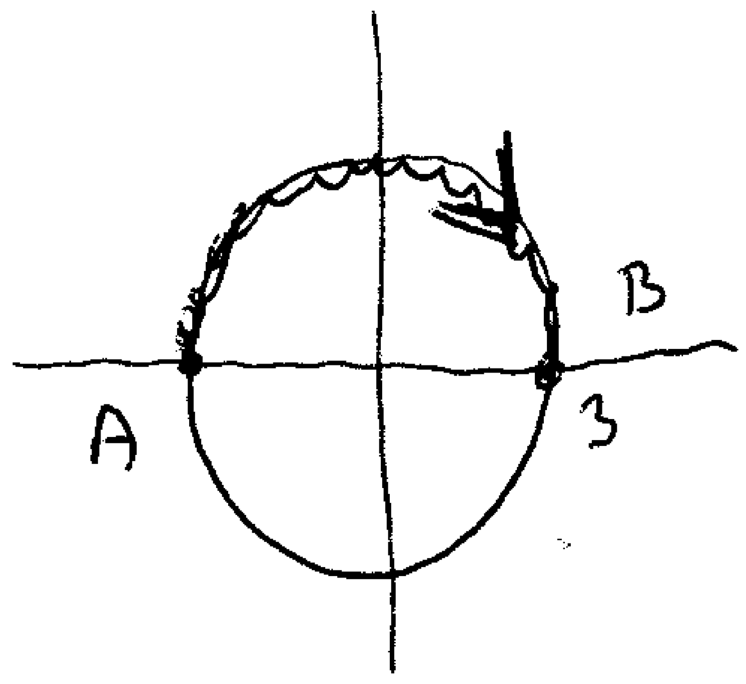
$-1 \leq z \leq \varrho \sin\varphi + 4$

$$V = \int_0^{\pi} \left(\int_0^2 \left(\int_{-1}^{\varrho\sin\varphi + 4} \varrho \, dz \right) d\varrho \right) d\varphi = \int_0^{\pi} \left(\int_0^2 \varrho \left[\varrho \sin\varphi + 4 + 1 \right] d\varrho \right) dz = \int_0^{\pi} \left(\int_0^2 (\varrho^2 \sin\varphi + 5\varrho) d\varrho \right) d\varphi$$

$$= \int_0^{\pi} \left[\frac{\varrho^3}{3} \sin\varphi + 5\frac{\varrho^2}{2} \right]_0^2 d\varphi = \int_0^{\pi} \left(\frac{8}{3} \sin\varphi + 10 \right) d\varphi =$$

$$= \left[-\frac{8}{3}\cos\varphi + 10\varphi \right]_0^{\pi} = -\frac{8}{3}(-1) + 10\pi + \frac{8}{3} + 0 = \frac{16}{3} + 10\pi$$

4) $\int_C x\,dx + (y-1)\,dy$, C: horní část kružnice $x^2+y^2=9$ pro $y \geq 0$ z bodu $A=[-3,0]$ do $B=[3,0]$



$x = 3\cos t$ $\quad t \in \langle 0, \pi \rangle \quad$ $x' = -3\sin t$

$y = 3\sin t$ $\qquad\qquad\qquad\qquad$ $y' = 3\cos t$

$$\int_C x\,dx + (y-1)\,dy = \int_0^{\pi} 3\cos t\,(-3\sin t)\,dt + \int_0^{\pi}(3\sin t - 1)\cdot 3\cos t\,dt =$$

$$= \int_0^{\pi}(-9\sin t\cos t + 9\sin t\cos t - 3\cos t)\,dt =$$

$$= \int_0^{\pi} -3\cos t\,dt = \left[-3\sin t\right]_0^{\pi} = -3\cdot 0 + 3\cdot 0 = 0$$