

$$A = [3, 1], B = [5, 4]$$

$$\vec{d} = \vec{AB} = B - A = (2, 3)$$

$$x = 3 + 2t$$

$$y = 1 + 3t$$

$$t \in \langle 0, 1 \rangle$$

$$\vec{n} = (3, -2)$$

$$ax + by + c = 0$$

$$3x - 2y + c = 0$$

$$3 \cdot 3 - 2 \cdot 1 + c = 0$$

$$7 + c = 0$$

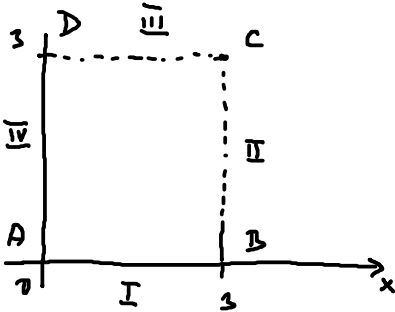
$$c = -7$$

$$\boxed{3x - 2y - 7 = 0}$$

$$2y = 3x - 7$$

$$y = \frac{3}{2}x - \frac{7}{2}, x \in \langle 3, 5 \rangle$$

$$A = [0, 0], B = [2, 0], C = [2, 3], D = [0, 3]$$



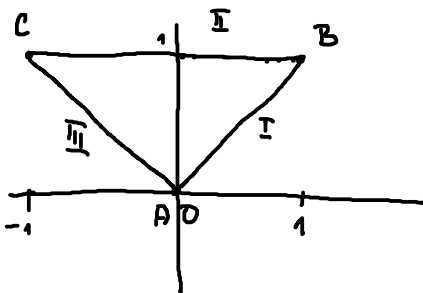
$$\text{I: } y = 0, x \in \langle 0, 2 \rangle$$

$$\text{II: } x = 2, y \in \langle 0, 3 \rangle$$

$$\text{III: } y = 3, x \in \langle 0, 2 \rangle$$

$$\text{IV: } x = 0, y \in \langle 0, 3 \rangle$$

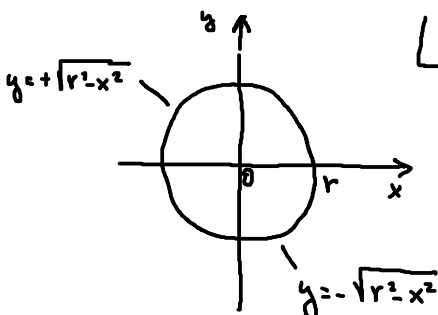
$$A = [0, 0], B = [1, 1], C = [-1, 1]$$



$$\text{II: } y = 1, x \in \langle -1, 1 \rangle$$

$$\text{I: } y = x, x \in \langle 0, 1 \rangle$$

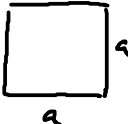
$$\text{III: } y = -x, x \in \langle -1, 0 \rangle$$




$$x^2 + y^2 = r^2$$

$$y^2 = r^2 - x^2$$

$$y = \pm \sqrt{r^2 - x^2}$$

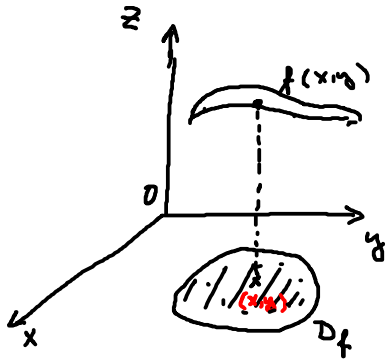

 $S = a^2$ $a > 0$
 $v = 4a$


 $S = a \cdot b$

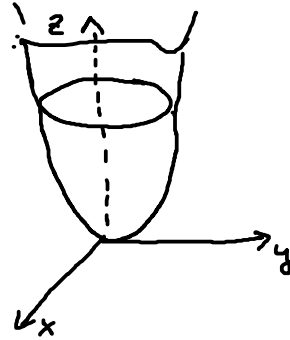
$y = x^2$

$z = x \cdot y$

$z = \ln(y^2 - x)$



$z = x^2 + y^2$ $D_f = \{(x, y) \in \mathbb{R}^2\}$



$z = f(x, y)$

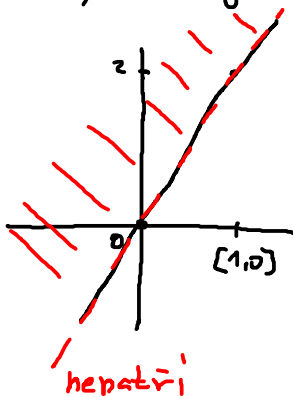
$z = x^2 + y^2$

$f: z = x^2 + y^2$

Ureďte def. obor a znázorněte je :

a) $z = \ln(y - 2x)$, $y - 2x > 0$

$D_f = \{(x, y) \in \mathbb{R}^2; y - 2x > 0\}$



$y - 2x > 0$

$y - 2x = 0 \rightarrow y = 2x$

$[1, 0]$, $x = 1$
 $y = 0$?

b) $z = \sqrt{4 - x^2 - y^2}$

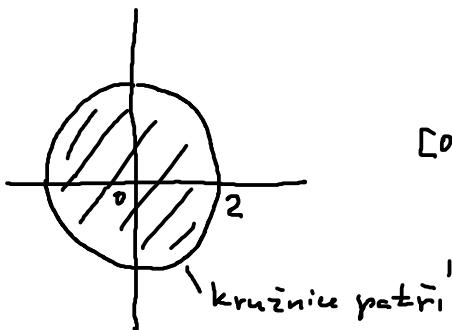
$4 - x^2 - y^2 \geq 0$

$D_f = \{(x, y) \in \mathbb{R}^2; 4 - x^2 - y^2 \geq 0\}$

$4 - x^2 - y^2 = 0$

$4 = x^2 + y^2$

$x^2 + y^2 = 4$ $r = 2$



$[0, 0]$ $4 - 0^2 - 0^2 \geq 0$

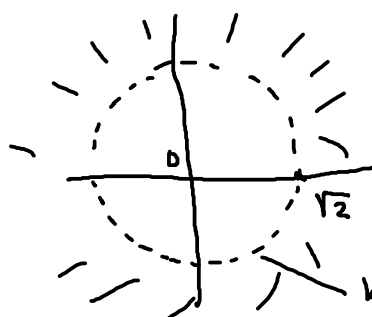
$$c) z = \frac{2x-y}{\sqrt{x^2+y^2-2}}$$

$$x^2+y^2-2 \geq 0 \wedge \sqrt{x^2+y^2-2} \neq 0$$

$$D_f = \{(x,y) \in \mathbb{R}^2; \underline{x^2+y^2-2 > 0}\}$$

$$x^2+y^2-2 = 0$$

$$x^2+y^2 = 2$$



[0,∞)

$$0^2+0^2-2 \neq 0$$

kružnice nepatří

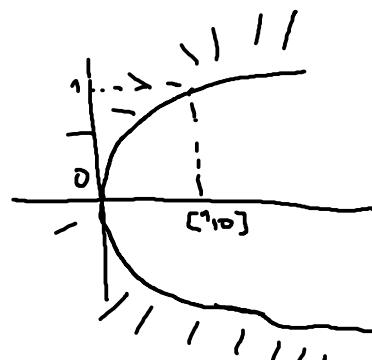
$$d) z = \sqrt{y^2-x}$$

$$y^2-x \geq 0, \quad D_f \dots$$

$$y^2-x=0$$

$$\boxed{x=y^2} \quad \nabla$$

$$y = \pm \sqrt{x}$$



parabola patří

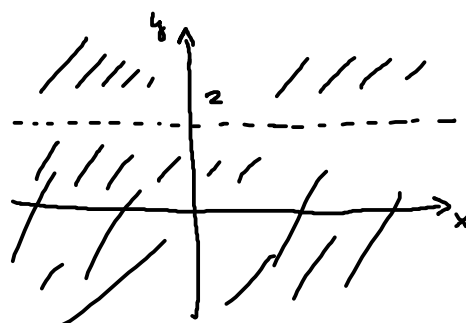
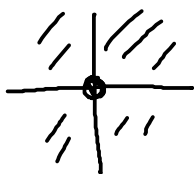
$$e) z = \frac{x+y}{x^2+y^2}$$

$$x^2+y^2 \neq 0$$

$$D_f = \{(x,y) \in \mathbb{R}^2, \underline{x^2+y^2 \neq 0}\}$$

$$x^2+y^2 = 0$$

$$\underline{(x,y) \neq (0,0)}$$



přímka nepatří

$$f) z = \frac{1}{y-2}$$

$$\boxed{\frac{y-2 \neq 0}{y \neq 2}}$$

$$y=2$$

$$g) z = \ln(y-x) + \sqrt{1-x^2-y^2}$$

$$D_f = \{(x,y) \in \mathbb{R}^2; \underline{y-x > 0} \wedge \underline{1-x^2-y^2 \geq 0}\}$$

$$\boxed{y-x > 0}$$

$$y-x=0$$

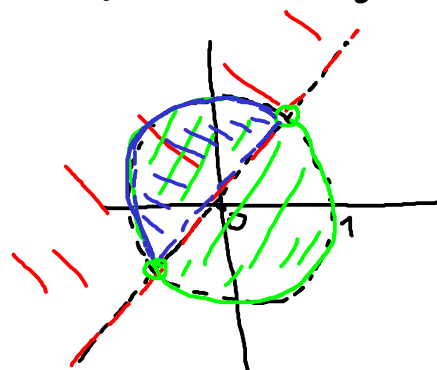
$$y=x$$

$$[1,0] \quad \times$$

$$\boxed{1-x^2-y^2 \geq 0}$$

$$x^2+y^2 = 1$$

$$[0,1] \quad \checkmark$$



$$z = f(x, y)$$

$$\underline{f_x, f'_x, z_x, z'_x} \quad \frac{\partial z}{\partial x}, \frac{\partial f}{\partial x}$$

$$a) z = x^2 y^4 \quad z_x = y^4 \cdot 2x = 2xy^4 \quad z_y = x^2 \cdot 4y^3 = 4x^2 y^3$$

$$b) z = x^2 + 2y^2 - 3xy^2 + 7x - y + 4$$

$$z_x = 2x + 0 - 3 \cdot 1 \cdot y^2 + 7 - 0 + 0 = 2x - 3y^2 + 7$$

$$z_y = 4y - 3x \cdot 2y - 1 = 4y - 6xy - 1$$

$$c) z = \frac{x+y}{x-y} \quad z_x = \frac{1 \cdot (x-y) - (x+y) \cdot 1}{(x-y)^2} = \frac{-2y}{(x-y)^2}$$

$$z_y = \frac{1(x-y) - (x+y)(-1)}{(x-y)^2} = \frac{2x}{(x-y)^2}$$

$$d) z = \frac{x}{y} \quad z_x = \frac{1 \cdot y - x \cdot 0}{y^2} = \frac{1}{y}$$

podzieli

$$z = \frac{x}{y+3} = x \cdot \frac{1}{y+3}$$

$$z = x \cdot \frac{1}{y} \quad z_y = x \cdot (-y^{-2}) = -\frac{x}{y^2}$$

$$e) z = (2x + 3y^2)^5$$

$$z_x = 5(2x + 3y^2)^4 \cdot 2 \quad z_y = 5(2x + 3y^2)^4 \cdot 6y$$

$$f) z = \sqrt{y-x} \quad z_x = \frac{1}{2\sqrt{y-x}} \cdot (-1) \quad z_y = \frac{1}{2\sqrt{y-x}} \cdot 1$$

$$g) z = \sqrt{x^2 y + yx} \quad z_x = \frac{1}{2\sqrt{x^2 y + yx}} \cdot (2xy + y) \quad z_y = \frac{1}{2\sqrt{x^2 y + yx}} \cdot (x^2 + x)$$

$$h) z = \ln(x^2 - y) \quad z_x = \frac{1}{(x^2 - y)} \cdot 2x \quad z_y = \frac{1}{(x^2 - y)} \cdot (-1)$$

$$i) z = e^{x \cdot y^2} \quad z_x = e^{x \cdot y^2} \cdot y^2 \quad z_y = e^{x \cdot y^2} \cdot 2xy$$

$$j) z = \cos(x-y) \quad z_x = -\sin(x-y) \cdot 1 \quad z_y = -\sin(x-y) \cdot (-1)$$

$$k) z = x \cdot e^{y^2} \quad z_x = 1 \cdot e^{y^2} \quad z_y = x \cdot e^{y^2} \cdot 2y$$

$$l) z = \ln \left(\frac{y}{x+y} \right)$$

$$z_x = \frac{1}{\frac{y}{x+y}} \cdot \frac{0 - y \cdot 1}{(x+y)^2} = \frac{x+y}{y} \cdot \frac{-y}{(x+y)^2} = \frac{-1}{x+y}$$
$$z_y = \frac{x+y}{y} \cdot \frac{1 \cdot (x+y) - y \cdot 1}{(x+y)^2} = \frac{x}{y(x+y)}$$