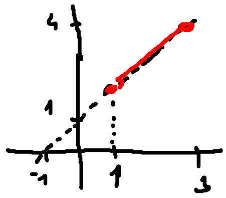
1) $\int_C (x+y)\,ds$, C je úsečka $x-y+1=0$, $\underline{x \in \langle 1,3 \rangle}$, $\underline{y = x+1}$



$$x = t,\quad y = t+1,\qquad t \in \langle 1,3 \rangle,\qquad x' = 1,\quad y' = 1$$

$$\int_1^3 (t+t+1)\sqrt{1^2+1^2} = \sqrt{2}\int_1^3 (2t+1)\,dt = \sqrt{2}\left[t^2+t\right]_1^3 = \sqrt{2}\left[(9+3)-(1+1)\right] = \underline{10\sqrt{2}}.$$

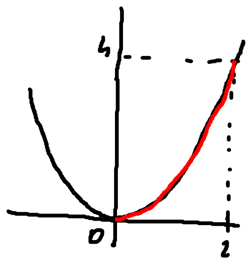
2) C, délka l $\qquad l = \int_C 1\,ds$

Vyp. délku úsečky KL, $K = [1,1,2]$, $L = [3,-1,2]$

$$\vec{s} = \overrightarrow{KL} = (2,-2,0)\qquad x = 1+2t,\quad y = 1-2t,\quad z = 2,\qquad t \in \langle 0,1 \rangle\qquad x' = 2,\quad y' = -2,\quad z' = 0$$

$$l = \int_0^1 \sqrt{2^2+(-2)^2+0^2}\,dt = \sqrt{8}\int_0^1 dt = \sqrt{8}\,\left[t\right]_0^1 = \sqrt{8} = 2\sqrt{2}$$

3) $\int_C 3x\,ds$, C je část paraboly $y = x^2$ mezi body $[0,0]$, $[2,4]$



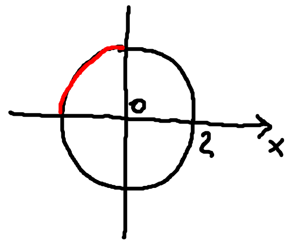
$$x = t,\quad y = t^2,\qquad t \in \langle 0,2 \rangle,\qquad x' = 1,\quad y' = 2t \qquad \frac{\frac{3}{8}}{\frac{3}{2}} =$$

$$\int_C 3x\,ds = \int_0^2 3t\sqrt{1^2+(2t)^2}\,dt = 3\int_0^2 t\sqrt{1+4t^2}\,dt = \left|\begin{array}{l} 1+4t^2 = z \\ 8t\,dt = dz \\ t\,dt = \frac{1}{8}\,dz \\ 0 \to 1 \\ 2 \to 17 \end{array}\right|$$

$$= 3\int_1^{17}\sqrt{z}\,\frac{1}{8}\,dz = \frac{3}{8}\int_1^{17}\sqrt{z}\,dz = \frac{3}{8}\left[\frac{z^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^{17} = \frac{1}{4}\left[\sqrt{z^3}\right]_1^{17}$$

$$= \frac{1}{4}\left(\sqrt{17^3}-\sqrt{1}\right) = \frac{1}{4}\left(17\sqrt{17}-1\right)$$

4) $\int_C (x+y)\,ds$, C je část kružnice $x^2+y^2=4$, $x \le 0,\ y \ge 0$



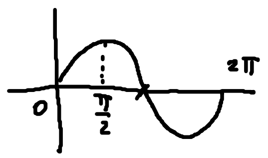
$x = 2\cos t$ $\quad t \in \langle \frac{\pi}{2}, \pi \rangle \quad$ $x' = -2\sin t$

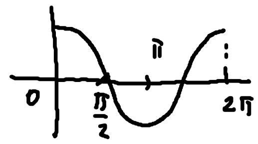
$y = 2\sin t$ $\quad\quad\quad\quad\quad\quad\quad$ $y' = 2\cos t$

$x^2+y^2 = 4\cos^2 t + 4\sin^2 t = 4(\cos^2 t + \sin^2 t) = 4$

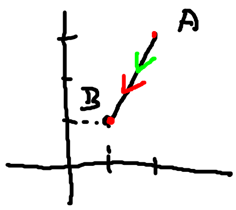
$$\int_{\frac{\pi}{2}}^{\pi} (2\cos t + 2\sin t)\sqrt{(-2\sin t)^2 + (2\cos t)^2}\,dt = 2\int_{\frac{\pi}{2}}^{\pi}(2\cos t + 2\sin t)\,dt =$$

$$4\left[\sin t - \cos t\right]_{\frac{\pi}{2}}^{\pi} = 4\left[(\overset{0}{\sin\pi} - \overset{-1}{\cos\pi}) - (\overset{1}{\sin\tfrac{\pi}{2}} - \overset{0}{\cos\tfrac{\pi}{2}})\right] = 4\cdot(1-1) = 0$$





5) $\int_C y\,dx + xy\,dy$, C je orientovaná úsečka AB , A = [2,3] poč. bod, B = [1,1] konc. bod



$\vec{s} = \vec{AB} = B - A = (-1,-2)$

$x = 2 - t$ $\quad t \in \langle 0,1 \rangle \quad$ $x' = -1$

$y = 3 - 2t$ $\quad\quad\quad\quad\quad$ $y' = -2$

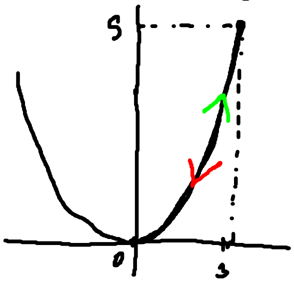
$$\int_0^1 (3-2t)(-1)\,dt + \int_0^1 (2-t)(3-2t)\cdot(-2)\,dt =$$

$$= \int_0^1 \left[-3+2t-2(6-3t-4t+2t^2)\right]dt = \int_0^1(-4t^2+16t-15)\,dt = \left[-4\frac{t^3}{3} + 8t^2 - 15t\right]_0^1 =$$

$\vec{F}(y, xy)$

$$= -\frac{4}{3} + 8 - 15 = -\frac{4}{3} - 7 = \frac{-4-21}{3} = -\frac{25}{3}$$

6) $\vec{F} = (x-y,\ y)$, $y = x^2$, z bodu $A=[3,9]$ do bodu $B=[0,0]$



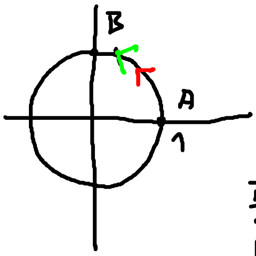
$x = t$, $y = t^2$, $t \in \langle 0,3 \rangle$, $x' = 1$, $y' = 2t$

$$\int_C (x-y)\,dx + y\,dy = \int_0^3 (t-t^2)\,1\,dt + \int_0^3 t^2\, 2t\,dt =$$

$$= \int_0^3 (2t^3 - t^2 + t)\,dt = \left[\frac{t^4}{2} - \frac{t^3}{3} + \frac{t^2}{2}\right]_0^3 = \frac{81}{2} - \frac{27}{3} + \frac{9}{2} = 45 - \frac{27}{3} = 36$$

Práce je -36.

7) $\int_C y\,dx - x\,dy$, C je část kružnice $x^2+y^2=1$ z bodu $A=[1,0]$ do $B=[0,1]$
(po horní půlkružnici)



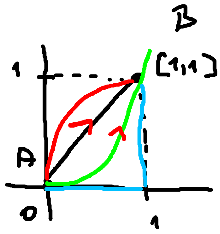
$x = 1\cos t$, $y = 1 \sin t$, $t \in \langle 0, \frac{\pi}{2} \rangle$, $x' = -\sin t$, $y' = \cos t$

$$\int_0^{\frac{\pi}{2}} \sin t\,(-\sin t)\,dt - \int_0^{\frac{\pi}{2}} \cos t \cos t\,dt = \int_0^{\frac{\pi}{2}} (-\sin^2 t - \cos^2 t)\,dt =$$

$$= \int_0^{\frac{\pi}{2}} -(\sin^2 t + \cos^2 t)\,dt = -\int_0^{\frac{\pi}{2}} dt = -\left[t\right]_0^{\frac{\pi}{2}} = -\frac{\pi}{2}$$

$\int_C P(x,y)\,dx + Q(x,y)\,dy$ Závisí? Nezávisí?

$P_y = Q_x$ nezávisí



$\int_C x\,dx - y\,dy$, po libovolné křivce z $[0,0]$ do $[1,1]$

$P = x$, $Q = -y$

$P_y = 0$, $Q_x = 0$

$P_y = Q_x$

1) úsečka AB $y = x$, $x \in \langle 0,1 \rangle$; $x = t$, $y = t$, $t \in \langle 0,1 \rangle$, $x' = 1$, $y' = 1$

$$\int_0^1 t\,1\,dt - \int_0^1 t\,1\,dt = 0$$

2) po parabole $y = x^2$; $x = t$, $y = t^2$, $t \in \langle 0,1 \rangle$, $x' = 1$, $y' = 2t$

$$\int_0^1 t\,dt - \int_0^1 t^2\,2t\,dt = \int_0^1 (2t^3 - t)\,dt = \left[\frac{t^4}{2} - \frac{t^2}{2}\right]_0^1 = 0$$

3) $y = \sqrt{x}$