$f(x,y) = 2x^3 - 3x^2 + y^2 - 2$

$f_x = 6x^2 - 6x$

$f_y = 2y$

$$6x^2 - 6x = 0$$
$$2y = 0$$
$$x^2 - x = 0 \qquad x(x-1) = 0$$
$$y = 0$$

$[0,0]$, $[1,0]$

$f_{xx} = 12x - 6$

$f_{xy} = 0 = f_{yx}$

$f_{yy} = 2$

$D(0,0) = \begin{vmatrix} -6 & 0 \\ 0 & 2 \end{vmatrix} < 0$ není extrém

$D(1,0) = \begin{vmatrix} 6 & 0 \\ 0 & 2 \end{vmatrix} = 12 > 0$ je extrém $\Big\}$ Lok. Min $[1,0]$

$6 > 0$

$z = 2x^3 + 3xy + 3y^2 - 3x - 6y + 9$

$z_x = 6x^2 + 3y - 3$

$z_y = 3x + 6y - 6$

$$6x^2 + 3y - 3 = 0$$
$$3x + 6y - 6 = 0$$
$$2x^2 + y - 1 = 0 \rightarrow y = 1 - 2x^2$$
$$x + 2y - 2 = 0$$
$$x + 2(1 - 2x^2) - 2 = 0$$
$$x + 2 - 4x^2 - 2 = 0$$
$$-4x^2 + x = 0$$
$$x(-4x + 1) = 0$$

$x_1 = 0$

$-4x + 1 = 0$

$x_2 = \frac{1}{4}$

$1 - 2 \cdot \frac{1}{16} =$

$1 - \frac{1}{8} = \frac{7}{8}$

$[0,1]$, $\left[\frac{1}{4}, \frac{7}{8}\right]$

$z_{xx} = 12x$

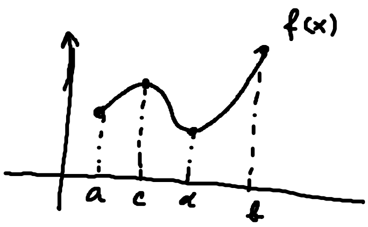
$z_{xy} = 3 = z_{yx}$

$z_{yy} = 6$

$D(0,1) = \begin{vmatrix} 0 & 3 \\ 3 & 6 \end{vmatrix} = -9 < 0$ není extrém

$D\left(\frac{1}{4}, \frac{7}{8}\right) = \begin{vmatrix} 3 & 3 \\ 3 & 6 \end{vmatrix} = 18 - 9 > 0$ je extrém $\left[\frac{1}{4}, \frac{7}{8}\right]$

$3 > 0$ Lok. min.

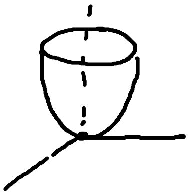


$f(a)$, $f(b)$, $f(c)$, $f(d)$

$f(b)$ – AMax

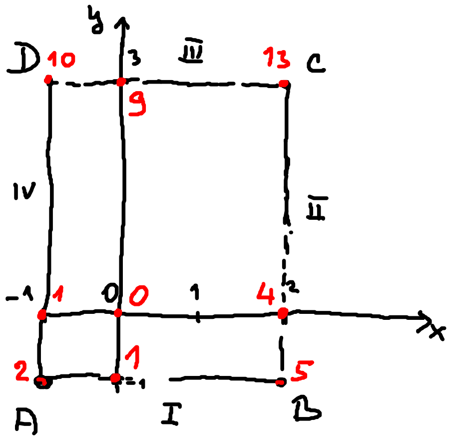
$f(d)$ – AMin

$f: z = x^2 + y^2$



ABCD

$A = [-1,-1]$, $B = [2,-1]$, $C = [2,3]$, $D = [-1,3]$



$z = x^2 + y^2$

$z_x = 2x$

$z_y = 2y$

$[0,0]$

$f(0,0) = 0$

$f(-1,-1) = 2$

$f(2,-1) = 5$

$f(2,3) = 13$

$f(-1,3) = 10$

I: $y=-1$, $x \in \langle -1, 2 \rangle$

II: $x = 2$, $y \in \langle -1, 3 \rangle$

III: $y = 3$, $x \in \langle -1, 2 \rangle$

IV: $x = -1$, $y \in \langle -1, 3 \rangle$

ad I: $y = -1$, $x \in \langle -1, 2 \rangle$

$z = x^2 + (-1)^2 = x^2 + 1$, $x \in \langle -1, 2 \rangle$

$z_x = 2x = 0 \qquad x = 0$

$f(0,-1) = 1$

ad II: $x = 2$, $y \in \langle -1, 3 \rangle$

$z = 2^2 + y^2 = y^2 + 4$, $y \in \langle -1, 3 \rangle$

$z_y = 2y = 0 \qquad y = 0$

$f(2,0) = 4$

ad III: $y = 3$, $x \in \langle -1, 2 \rangle$

$z = x^2 + 9$

$z_x = 2x = 0 \qquad x = 0$

$f(0,3) = 9$

ad IV: $x = -1$, $y \in \langle -1, 3 \rangle$

$z = (-1)^2 + y^2 = y^2 + 1$

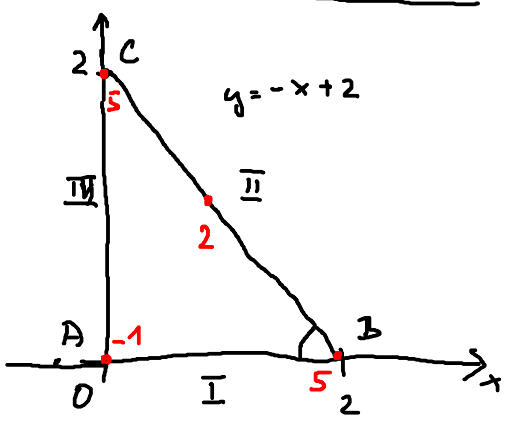
$z_y = 2y = 0 \qquad y = 0$

$f(-1,0) = 1$

V bodě $[0,0]$ je nejmenší hodnota (AMin) rovna 0

V bodě $[2,3]$ je největší hodnota (AMax) rovna 13

$z = x^2 + y^2 - xy + x + y - 1$, $\triangle ABC$ $A = [0,0]$, $B = [2,0]$, $C = [0,2]$



$z_x = 2x - y + 1$

$z_y = 2y - x + 1$

$2x - y + 1 = 0$

$-x + 2y + 1 = 0 \quad / \cdot 2$

$2x - y + 1 = 0$

$-2x + 4y + 2 = 0$

$3y + 3 = 0$

$y = -1$

$-x + 2(-1) + 1 = 0$

$-x - 2 + 1 = 0$

$x = -1$

$[-1,-1]$

$f(0,0) = -1 \qquad f(2,0) = 5 \qquad f(0,2) = 5$

ad I: $y = 0$, $x \in \langle 0,2 \rangle$

$z = x^2 + x - 1$

$z_x = 2x + 1 = 0 \qquad x = -\frac{1}{2}$

ad IV: $x = 0$, $y \in \langle 0,2 \rangle$

$z = y^2 + y - 1$

$z_y = 2y + 1 = 0 \qquad y = -\frac{1}{2}$

ad II: $y = -x + 2$, $x \in \langle 0,2 \rangle$

$z = x^2 + (2-x)^2 - x(2-x) + x + (2-x) - 1$

$z = x^2 + 4 - 4x + x^2 - 2x + x^2 + x + 2 - x - 1$

$z = 3x^2 - 6x + 5$, $x \in \langle 0,2 \rangle \qquad f(1,1) = 2$

$z_x = 6x - 6 = 0 \rightarrow x = 1, \ y = 1$

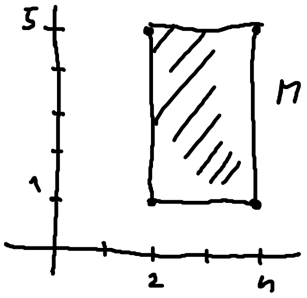
Abs. minimum -1 je v bodě $[0,0]$

Abs. maximum 5 je v bodech $[2,0]$, $[0,2]$

$$\iint_M f(x,y)\,dx\,dy \quad , \quad M \qquad a \le x \le b$$
$$c \le y \le d$$

Popište oblasti M :

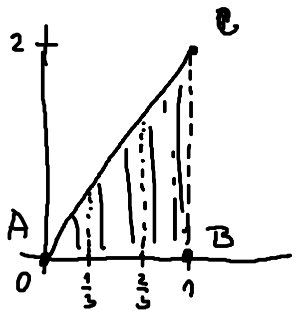
1) obdélník $ABCD$, $A=[2,1]$, $B=[4,1]$, $C=[4,5]$, $D=[2,5]$



$$2 \le x \le 4$$
$$1 \le y \le 5$$

$$\iint_M f(x,y)\,dx\,dy = \int_2^4 \left(\int_1^5 f(x,y)\,dy \right) dx$$

2) $\triangle ABC$, $A=[0,0]$, $B=[1,0]$, $C=[1,2]$



$$0 \le x \le 1$$
$$0 \le y \le 2x$$

AC ?

$\vec{s} = \vec{AC} = C - A = (1,2)$

$\vec{n} = (2,-1)$

$2x - y + c = 0 \qquad 2\cdot 0 - 0 + c = 0$

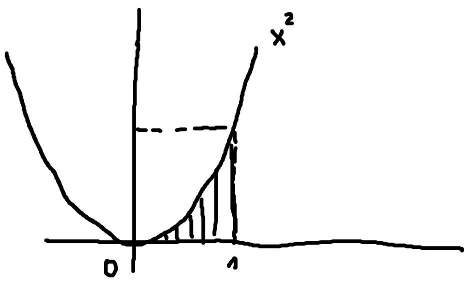
$c = 0$

$2x - y = 0$

$\boxed{y = 2x}$

$$\iint_M f(x,y)\,dx\,dy = \int_0^1 \left(\int_0^{2x} f(x,y)\,dy \right) dx$$

3) M je ohraničená grafem fce $y = x^2$, na intervalu $\langle 0,1 \rangle$, a osou x

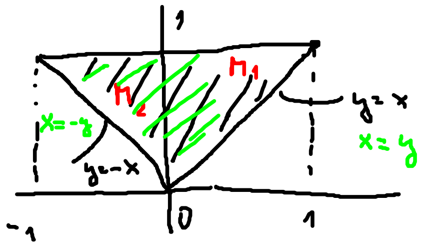


$$0 \le x \le 1$$
$$0 \le y \le x^2$$

$$\iint_M f(x,y)\,dx\,dy = \int_0^1 \left(\int_0^{x^2} f(x,y)\,dy \right) dx$$

4) obrázkem

$$M = M_1 \cup M_2$$



$M_1: \quad 0 \le x \le 1$
$\qquad x \le y \le 1$

$M_2: \quad -1 \le x \le 0$
$\qquad -x \le y \le 1$

$$\iint_M f\,dx\,dy = \iint_{M_1} + \iint_{M_2}$$

$$\boxed{M: \quad 0 \le y \le 1, \quad -y \le x \le y}$$

$$\iint_M f(x,y)\,dx\,dy = \int_0^1 \left(\int_{-y}^{y} f(x,y)\,dx \right) dy$$