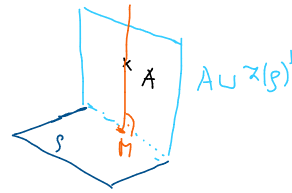
Příklad. 1. V $\mathbb{R}^4$ určete vzdálenost bodu $A = [4, 1, -4, -5]$ od roviny

$$\rho: \ \overset{B}{[3, -2, 1, 5]} + t\overset{u_1}{(2, 3, -2, -2)} + s\overset{u_2}{(4, 1, 3, 2)}.$$

Současně najděte bod $M \in \rho$ takový, že $\|M - A\| = \mathrm{dist}(A, \rho)$.



Skoro totožný príklad ako príklad 6 v cvičení 5, tu je ρ zadaná parametricky. Vyriešime spôsobom 2.

Z teórie vieme, že vektor $\overrightarrow{AM}$ leží v $\boxed{(Z(\rho)+Z(A))^\perp = (Z(\rho)+0)^\perp = Z(\rho)^\perp}$. (viď obrázok)

$\boxed{Z(\rho) = [u_1, u_2]}$

$x \in \boxed{Z(\rho)^\perp} \iff \langle x, u_1\rangle = 0, \ \langle x, u_2 \rangle = 0$ t.j. $\boxed{\begin{aligned} 2x_1 + 3x_2 - 2x_3 - 2x_4 &= 0 \\ 4x_1 + x_2 + 3x_3 + 2x_4 &= 0 \end{aligned}}$

$\boxed{\text{rovina } A \sqcup Z(\rho)^\perp}$ je daná implicitne rovnicami:
$$\begin{aligned} 2x_1 + 3x_2 - 2x_3 - 2x_4 &= a \\ 4x_1 + x_2 + 3x_3 + 2x_4 &= b \end{aligned}$$
$a, b \in \mathbb{R}$ zistíme dosadením A.

$$\begin{aligned} 2\cdot 4 + 3\cdot 1 - 2\cdot(-4) - 2(-5) &= 29 \\ 4\cdot 4 + 1\cdot 1 + 3\cdot(-4) + 2\cdot(-5) &= -5 \end{aligned} \implies \boxed{\begin{aligned} 2x_1 + 3x_2 - 2x_3 - 2x_4 &= 29 \\ 4x_1 + x_2 + 3x_3 + 2x_4 &= -5 \end{aligned}}$$

Hľadaný bod M leží v $(A \sqcup Z(\rho)^\perp) \cap \rho$ (viď obrázok):

$$\rho: \ \begin{aligned} x_1 &= 3 + 2t + 4s \\ x_2 &= -2 + 3t + s \\ x_3 &= 1 - 2t + 3s \\ x_4 &= 5 - 2t + 2s \end{aligned}$$

dosadíme

$$\begin{aligned} 2\cdot(3+2t+4s) + 3\cdot(-2+3t+s) - 2\cdot(1-2t+3s) - 2\cdot(5-2t+2s) &= 29 \\ 4\cdot(3+2t+4s) + (-2+3t+s) + 3\cdot(1-2t+3s) + 2\cdot(5-2t+2s) &= -5 \end{aligned}$$

$$\begin{aligned} 21\cdot t + s &= 41 \\ t + 30s &= -28 \end{aligned} \qquad (*)$$

$\boxed{\text{Cramerovo pravidlo}}$:

$$t = \frac{\det\begin{pmatrix} 41 & 1 \\ -28 & 30 \end{pmatrix}}{\det\begin{pmatrix} 21 & 1 \\ 1 & 30 \end{pmatrix}} = \frac{41\cdot 30 + 28}{21\cdot 30 - 1} = \frac{1258}{629} = 2$$

$s = 41 - 21\cdot 2 = -1$ (z 1. rovnice)

$$M = [3+4-4, -2+6-1, 1-4-3, 5-4-2] = [3, 3, -6, -1]$$

$$\mathrm{dist}(A, \rho) = \|A - M\| = \|(1, -2, 2, -4)\| = \sqrt{1+4+4+16} = 5.$$

$M = [3+4-4,\ -2+6-1,\ 1-4-3,\ 5-4-2] = [3, 3, -6, -1]$

$\mathrm{dist}(A, S) = \| A - M \| = \| (1, -2, 2, -4) \| = \sqrt{1+4+4+16} = 5.$

Poznámka. Všimnite si, že sústava (*) je vlastne

$$\boxed{\begin{aligned} \langle u_1, u_1 \rangle \cdot t + \langle u_2, u_1 \rangle \cdot s &= \langle A-B, u_1 \rangle \\ \langle u_1, u_2 \rangle \cdot s + \langle u_2, u_2 \rangle \cdot s &= \langle A-B, u_2 \rangle \end{aligned}}$$

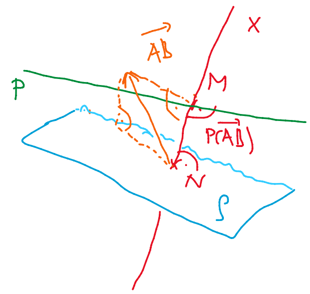
t.j. počítame nepriamo $\quad P(A-B) = t \cdot u_1 + s \cdot u_2$

$(A-B) - P(A-B) \perp u_1, u_2.$

Příklad. 2. V $\mathbb{R}^4$ určete vzdálenost přímky p od roviny ρ

$$p: \overset{A}{[5,4,4,5]} + r\overset{u}{(0,0,1,-4)}, \quad \rho: \overset{B}{[4,1,1,0]} + s\overset{v_1}{(1,-1,0,0)} + t\overset{v_2}{(2,0,-1,0)}$$

a body $M \in p$ a $N \in \rho$, v nichž se tato vzdálenost realizuje, tj. $\|M-N\| = \operatorname{dist}(p,\rho)$.



– vektor $\boxed{\overrightarrow{MN}}$ leží v $\boxed{(\mathcal{Z}(p)+\mathcal{Z}(\rho))^\perp}$ (je kolmý na $\mathcal{Z}(p)$ a $\mathcal{Z}(\rho)$)

$x \in (\mathcal{Z}(p)+\mathcal{Z}(\rho))^\perp \iff$ $\langle x,u\rangle = 0$, $\langle x,v_1\rangle = 0$, $\langle x,v_2\rangle = 0$ t.j.

$$\begin{aligned} x_3-4x_4 &= 0\\ x_1-x_2 &= 0\\ 2x_1 - x_3 &= 0 \end{aligned}$$

$$\begin{pmatrix} u\\ v_1\\ v_2\end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & -4\\ 1 & -1 & 0 & 0\\ 2 & 0 & -1 & 0\end{pmatrix} \sim \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 \end{matrix}\\ \begin{pmatrix} 1 & -1 & 0 & 0\\ 0 & 2 & -1 & 0\\ 0 & 0 & 1 & -4\end{pmatrix}\end{array}$$

$x_4 = a$
$x_3 = 4a$
$x_2 = 2a$
$x_1 = 2a$

$\boxed{X = a\cdot(2,2,4,1)}$

– platí $\boxed{P(\overrightarrow{AB}) = \overrightarrow{MN}}$, kde P je kolmá projekcia do $(\mathcal{Z}(p)+\mathcal{Z}(\rho))^\perp = [(2,2,4,1)] = [w]$

$P(\overrightarrow{AB}) = c\cdot w$

$\overrightarrow{AB} - P(\overrightarrow{AB}) = \overrightarrow{AB} - c\cdot w \perp w \iff \langle \overrightarrow{AB} - c\cdot w, w\rangle = 0$

$$c = \frac{\langle \overrightarrow{AB}, w\rangle}{\|w\|^2} = \frac{\langle(-1,-3,-3,-5),(2,2,4,1)\rangle}{2^2+2^2+4^2+1} = \frac{-2-6-12-5}{25} = -1$$

$\boxed{P(\overrightarrow{AB}) = -w}$

$\boxed{\operatorname{dist}(p,\rho)} = \|\overrightarrow{MN}\| = \|P(\overrightarrow{AB})\| = \|-w\| = \|w\| = \sqrt{25} = \boxed{5}$

Pretože $P(\overrightarrow{AB}) = \overrightarrow{MN}$ je $\boxed{\overrightarrow{MN} = -w}$, kde $M = A + r\cdot u$, $r \in \mathbb{R}$
$N = B + s\cdot v_1 + t\cdot v_2$, $s,t \in \mathbb{R}$

$N - M = -w$

$B + s\cdot v_1 + t\cdot v_2 - A - r\cdot u = -w$

$s\cdot v_1 + t\cdot v_2 - r\cdot u = A - B - w = (1,3,3,5) - (2,2,4,1) = (-1,1,-1,4)$

$$\begin{array}{c} \begin{matrix} s & t & r \end{matrix}\\ \left(\begin{array}{ccc|c} 1 & 2 & 0 & -1\\ -1 & 0 & 0 & 1\\ 0 & -1 & -1 & -1\\ 0 & 0 & 4 & 4\end{array}\right)\end{array} \Rightarrow \begin{aligned} r &= 1\\ t &= 0\\ s &= -1\end{aligned}$$

$\boxed{M} = A + u = [5,4,4,5] + (0,0,1,-4) = \boxed{[5,4,5,1]}$

$\boxed{N} = B - v_1 = [4,1,1,0] - (1,-1,0,0) = \boxed{[3,2,1,0]}$

Pozn. Úlohu vieme vyriešiť aj bez znalosti, že platí $\boxed{P(\overrightarrow{AB}) = \overrightarrow{MN}}$ (podobne ako príklad 1, používame iba $\overrightarrow{MN} \in (\mathcal{Z}(p)+\mathcal{Z}(\rho))^\perp$)

$M - N = c\cdot w$

$B + s\cdot v_1 + t\cdot v_2 - A - r\cdot u = c\cdot w$

$\overrightarrow{MN} \in (Z(p)+Z(\varrho))$)

$$M - N = c \cdot w$$

$$B + s \cdot v_1 + t \cdot v_2 - A - r \cdot u = c \cdot w$$

$$s \cdot v_1 + t \cdot v_2 - r \cdot u - c \cdot w = A - B$$

$$\begin{array}{c} \begin{array}{cccc} s & t & r & c \end{array} \\ \left(\begin{array}{cccc|c} 1 & 2 & 0 & -2 & 1 \\ -1 & 0 & 0 & -2 & 3 \\ 0 & -1 & -1 & -4 & 3 \\ 0 & 0 & 4 & -1 & 5 \end{array}\right) \end{array} \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -3 \\ 0 & 2 & 0 & -4 & 4 \\ 0 & 1 & 1 & 4 & -3 \\ 0 & 0 & 4 & -1 & 5 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -3 \\ 0 & 1 & 1 & 4 & -3 \\ 0 & 0 & -2 & -12 & 10 \\ 0 & 0 & 4 & -1 & 5 \end{array}\right) \sim \begin{array}{c} \begin{array}{cccc} s & t & r & c \end{array} \\ \left(\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -3 \\ 0 & 1 & 1 & 4 & -3 \\ 0 & 0 & 1 & 6 & -5 \\ 0 & 0 & 0 & -25 & 25 \end{array}\right) \end{array}$$

$$\boxed{\begin{array}{c} c = -1 \\ r = 1 \\ t = 0 \\ s = -1 \end{array}}$$

rovnaké ako predtým

Prípadne môžete postupovať bez výpočtu w:

$$\overrightarrow{MN} \in (Z(p)+Z(\varrho))^{\perp} \quad \text{t.j.} \quad \left.\begin{array}{l} \langle \overrightarrow{MN}, u \rangle = 0 \\ \langle \overrightarrow{MN}, v_1 \rangle = 0 \\ \langle \overrightarrow{MN}, v_2 \rangle = 0 \end{array}\right\} \text{ 3 rovnice o 3 neznámych}$$

$$M = A + r \cdot u$$

$$N = B + s \cdot v_1 + t \cdot v_2$$

Příklad. 3. V $\mathbb{R}^4$ určete vzdálenost rovin σ a τ a body, v nichž se realizuje.

$$\sigma : \overset{A}{[4,5,3,2]} + s\overset{u_1}{(1,2,2,2)} + t\overset{u_2}{(2,0,2,1)},$$
$$\tau : \underset{B}{[1,-2,1,-3]} + p\underset{v_1}{(2,-2,1,2)} + q\underset{v_2}{(1,-2,0,-1)}.$$

Postupujeme obdobne ako v príklade 2.

Nech $M \in \sigma$, $N \in \tau$ také, že $dist(\sigma,\tau) = \|M-N\|$, potom $\boxed{\overrightarrow{MN} \in (Z(\sigma)+Z(\tau))^{\perp}}$

$$x \in (Z(\sigma)+Z(\tau))^{\perp} \iff \begin{matrix} \langle x,u_1\rangle = 0 \\ \langle x,u_2\rangle = 0 \\ \langle x,v_1\rangle = 0 \\ \langle x,v_2\rangle = 0 \end{matrix} \iff \underset{4\times 4}{\begin{pmatrix} u_1 \\ u_2 \\ v_1 \\ v_2 \end{pmatrix}} \cdot \underset{4\times 1}{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}} = \underset{4\times 1}{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}}$$

$$\begin{pmatrix} u_1 \\ u_2 \\ v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 2 & 2 \\ 2 & 0 & 2 & 1 \\ 2 & -2 & 1 & 2 \\ 1 & -2 & 0 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 0 & -1 \\ 0 & 4 & 2 & 3 \\ \cancel{0} & \cancel{4} & \cancel{2} & \cancel{3} \\ 0 & 2 & 1 & 4 \end{pmatrix} \begin{matrix} R4 \\ R1-R4 \\ R2-2\cdot R4 \\ R3-2\cdot R4 \end{matrix} \sim \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 \end{matrix} \\ \begin{pmatrix} 1 & -2 & 0 & -1 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & 0 & -5 \end{pmatrix} \end{array}$$

$x_4 = 0$
$x_3 = 2a$
$x_2 = -a$
$x_1 = -2a$

$$(Z(\sigma)+Z(\tau))^{\perp} = [(-2,-1,2,0)] = [w]$$

Využijeme, že platí $\boxed{P(\overrightarrow{AB}) = \overrightarrow{MN} \in [w]}$ $\quad$ P je kolmá projekcia do $(Z(\sigma)+Z(\tau))^{\perp}$

$$P(\overrightarrow{AB}) = c \cdot w$$
$$\overrightarrow{AB} - P(\overrightarrow{AB}) = \overrightarrow{AB} - c\cdot w \perp w$$
$$c = \frac{\langle \overrightarrow{AB}, w\rangle}{\|w\|^2} = \frac{\langle (-3,-7,-2,-5), (-2,-1,2,0)\rangle}{2^2+1+2^2} = \frac{6+7-4}{9} = 1$$

$\boxed{P(\overrightarrow{AB}) = w}$

$\boxed{dist(\sigma,\tau)} = \|P(\overrightarrow{AB})\| = \|w\| = \sqrt{9} = \boxed{3}$

Výpočet bodov M a N

$$\overrightarrow{MN} = P(\overrightarrow{AB}) = w$$
$$N - M = w$$
$$B + p\cdot v_1 + q\cdot v_2 - A - s\cdot u_1 - t u_2 = w$$
$$p\cdot v_1 + q\cdot v_2 - s\cdot u_1 - t u_2 = w - \overrightarrow{AB} = (-2,-1,2,0) + (3,7,2,5) = (1,6,4,5)$$

$$\left(\begin{array}{cccc|c} 2 & 1 & -1 & -2 & 1 \\ -2 & -2 & -2 & 0 & 6 \\ 1 & 0 & -2 & -2 & 4 \\ 2 & -1 & -2 & -1 & 5 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & -2 & -2 & 4 \\ 0 & -1 & -3 & -2 & 7 \\ 0 & -3 & -4 & -1 & 11 \\ 0 & -2 & -6 & -4 & 14 \end{array}\right) \begin{matrix} R3 \\ R1+R2 \\ R2+R4 \\ 2\cdot R3+R2 \end{matrix} \sim \left(\begin{array}{cccc|c} 1 & 0 & -2 & -2 & 4 \\ 0 & 1 & 3 & 2 & -7 \\ 0 & 0 & 5 & 5 & -10 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \sim \begin{array}{c} \begin{matrix} p & q & s & t \end{matrix} \\ \left(\begin{array}{cccc|c} 1 & 0 & -2 & -2 & 4 \\ 0 & 1 & 3 & 2 & -7 \\ 0 & 0 & 1 & 1 & -2 \end{array}\right) \end{array}$$

$s = -2 - t$
$q = -7 - 2t - 3(-2-t) = -1 + t$
$p = 4 + 2t + 2(-2-t) = 0$

$\boxed{M_t} = A + s\cdot u_1 + t\cdot u_2 = [4,5,3,2] - (2+t)\cdot(1,2,2,2) + t\cdot(2,0,2,1)$
$= \boxed{[2,1,-1,-2] + t\cdot(1,-2,0,-1)}$

$\boxed{N_t} = B + (-1+t)\cdot v_2 = [1,-2,1,-3] + (-1+t)\cdot(1,-2,0,-1)$

$p = 4 + 2t + 2(-2-t) = 0$

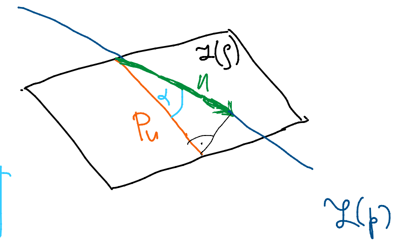
$\boxed{N_t} = B + (-1+t)\cdot v_2 = [1,-2,1,-3] + (-1+t)\cdot(1,-2,0,-1)$

$= \boxed{[0,0,1,-2] + t\cdot(1,-2,0,-1)}$

Vzdialenosť sa realizuje medzi dvojicami bodov M_t, N_t, ktoré ležia na 2 rovnobežných priamkach

Příklad. 4. Určete odchylku přímky $p:\ [1,2,3,4]+t\underset{u}{(-3,15,1,-5)}$ od roviny

$$\rho:\ [0,0,0,0]+r\underbrace{(1,-5,-2,10)}_{v_1}+s\underbrace{(1,8,-2,-16)}_{v_2}.$$



$$\boxed{\measuredangle\, p,\rho := \measuredangle\, Z(p), Z(\rho)}$$

$$P_u = a\cdot v_1 + b\cdot v_2$$

$$u - P_u = u - a\cdot v_1 - b\cdot v_2 \perp v_1, v_2$$

$$\langle u - a v_1 - b v_2, v_1\rangle = 0 \iff a\langle v_1, v_1\rangle + b\cdot\langle v_2, v_1\rangle = \langle u, v_1\rangle$$

$$\langle u - a v_1 - b_2, v_2\rangle = 0 \iff a\cdot\langle v_2, v_1\rangle + b\cdot\langle v_2, v_2\rangle = \langle u, v_2\rangle$$

$$\langle v_1, v_1\rangle = 1^2 + 5^2 + 2^2 + 10^2 = 130$$

$$\langle v_2, v_1\rangle = \langle v_1, v_2\rangle = 1 - 40 + 4 - 160 = -195$$

$$\langle v_2, v_2\rangle = 1 + 8^2 + (-2)^2 + (-16)^2 = 325$$

$$\langle u, v_1\rangle = -3 - 75 - 2 - 50 = -130$$

$$\langle u, v_2\rangle = -3 + 120 - 2 + 80 = 195$$

$$\begin{array}{cc|c} a & b & \\ \hline 130 & -195 & -130 \\ -195 & 325 & 195 \end{array}$$

Cramerovo pravidlo:

$$a = \frac{\det\begin{pmatrix}-130 & -195\\ 195 & 325\end{pmatrix}}{\det\begin{pmatrix}130 & -195\\ -195 & 325\end{pmatrix}} = \frac{-\cancel{\det\begin{pmatrix}130 & -195\\ -195 & 325\end{pmatrix}}}{\cancel{\det\begin{pmatrix}130 & -195\\ -195 & 325\end{pmatrix}}} = -1$$

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$$b = \frac{\det\begin{pmatrix}130 & -130\\ -195 & 195\end{pmatrix}}{\det\begin{pmatrix}130 & -195\\ -195 & 325\end{pmatrix}} = 0$$

$$\boxed{P_u = -v_1}$$

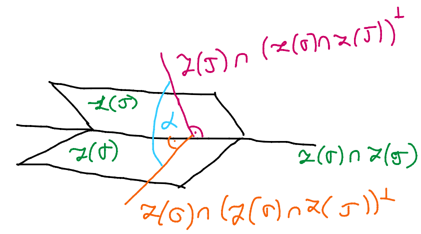
$$\cos(\measuredangle\, p,\rho) = \frac{\|P_u\|}{\|u\|} = \frac{\|-v_1\|}{\|u\|} = \frac{\|v_1\|}{\|u\|} = \frac{\sqrt{1+5^2+2^2+10^2}}{\sqrt{3^2+15^2+1^2+5^2}} = \frac{\sqrt{130}}{\sqrt{260}} = \frac{1}{\sqrt{2}}.$$

nezávisí na násobku u

$$\boxed{\measuredangle\, p,\rho = \frac{\pi}{4}}$$

Příklad. 5. V $\mathbb{R}^4$ určete odchylku rovin τ a σ.

$$\sigma : [2,1,0,1] + \underbrace{\overset{u_1}{s(1,1,1,1)} + \overset{u_2}{t(1,-1,1-1)}}_{\mathcal{Z}(\rho)},$$
$$\tau : [1,0,1,1] + \underbrace{\underset{v_1}{p(2,2,1,0)} + \underset{v_2}{q(1,-2,2,0)}}_{\mathcal{Z}(\sigma)}.$$



$\measuredangle \rho,\sigma$ je najmenší uhol $\alpha\in[0,\pi/2]$ určený $\cos\alpha = \dfrac{|\langle u,v\rangle|}{\|u\|\cdot\|v\|}$, kde $u\in \mathcal{Z}(\sigma)\cap(\mathcal{Z}(\rho)\cap\mathcal{Z}(\sigma))^\perp$,
$v\in \mathcal{Z}(\rho)\cap(\mathcal{Z}(\rho)\cap\mathcal{Z}(\sigma))^\perp$
$u,v\neq 0$

- K tomu potrebujeme zistiť $\boxed{\mathcal{Z}(\rho)\cap\mathcal{Z}(\sigma)}$.

Riešime systém: $w = p\cdot v_1 + q\cdot v_2 = s\cdot u_1 + t\cdot u_2$ $\quad p,q,s,t\in\mathbb{R}$

$$p\cdot v_1 + q\cdot v_2 - s\cdot u_1 - t\cdot u_2 = 0$$

$$\begin{pmatrix} v_1 & v_2 & -u_2 & -u_2\end{pmatrix} = \begin{pmatrix} 2 & 1 & -1 & -1\\ 2 & -2 & -1 & 1\\ 1 & 2 & -1 & -1\\ 0 & 0 & -1 & 1\end{pmatrix} \sim \begin{pmatrix} 1 & 2 & -1 & -1\\ 0 & -6 & 1 & 3\\ 0 & -3 & 1 & 1\\ 0 & 0 & -1 & 1\end{pmatrix} \sim \begin{array}{c} \begin{matrix} p & q & s & t\end{matrix}\\ \begin{pmatrix} 1 & 2 & -1 & -1\\ 0 & -3 & 1 & 1\\ 0 & 0 & -1 & 1\\ \cancel{0} & \cancel{0} & \cancel{-1} & \cancel{1}\end{pmatrix}\end{array}$$

$-s+t=0$
$\boxed{t=s}$

$w = s\cdot u_1 + s\cdot u_2 = s\cdot(u_1+u_2) = s\cdot(2,0,2,0) \Rightarrow \mathcal{Z}(\rho)\cap\mathcal{Z}(\sigma) = [(1,0,1,0)]$

- Z definície $\boxed{(\mathcal{Z}(\rho)\cap\mathcal{Z}(\sigma))^\perp}$ je $\boxed{[w]^\perp = \{(x_1,x_2,x_3,x_4)\in\mathbb{R}^4 \mid x_1+x_3=0\}}$.

- Spočítame $U = \mathcal{Z}(\rho)\cap(\mathcal{Z}(\rho)\cap\mathcal{Z}(\sigma))^\perp$.

$\left.\begin{aligned} x_1 &= s+t\\ x_3 &= s+t\end{aligned}\right\}$ dosadíme

$(\mathcal{Z}(\rho)\cap\mathcal{Z}(\sigma))^\perp$ je dané $x_1+x_3=0$

$2\cdot(s+t) = 0 \Rightarrow \boxed{s=-t}$

prienik je $s\cdot u_1 - s\cdot u_2 = s\cdot(u_1-u_2) = s\cdot(0,2,0,2)$

$\boxed{U = [(0,1,0,1)]}$

- Spočítame $V = \mathcal{Z}(\sigma)\cap(\mathcal{Z}(\rho)\cap\mathcal{Z}(\sigma))^\perp$

$(\mathcal{Z}(\rho)\cap\mathcal{Z}(\sigma))^\perp$ je dané $x_1+x_3=0$

$\left.\begin{aligned} x_1 &= 2p+q\\ x_3 &= p+2q\end{aligned}\right\}$ dosadíme

$(2p+q)+(p+2q)=0$
$3(p+q)=0 \Rightarrow \boxed{p=-q}$

prienik je $p\cdot v_1 - p\cdot v_2 = p\cdot(v_1-v_2) = p\cdot(1,4,-1,0)$

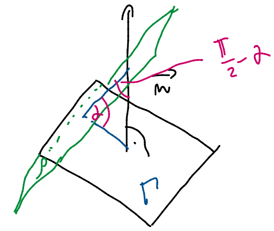
$\boxed{V = [(1,4,-1,0)]}$

$$\cos\alpha = \frac{|\langle \overbrace{a\cdot(0,1,0,1)}^{u}, \overbrace{b\cdot(1,4,-1,0)}^{v}\rangle|}{\|a\cdot(0,1,0,1)\|\cdot\|b\cdot(1,4,-1,0)\|} = \frac{\cancel{|a|\cdot|b|}\cdot|\langle(0,1,0,1),(1,4,-1,0)\rangle|}{\cancel{|a|\cdot|b|}\cdot\sqrt{1+1}\cdot\sqrt{1+4^2+1}} = \frac{4}{\sqrt{2}\cdot\sqrt{18}} = \frac{2}{3}$$

nezáleží na násobku, ide o odchýlku priamok

$\boxed{\alpha = \arccos^{-1}\left(\tfrac{2}{3}\right).}$ a $\alpha\in[0,\pi/2]$.

Příklad. 6. V $\mathbb{R}^5$ spočítejte odchylku roviny ρ a nadroviny Γ.



$$\rho : s\overset{v_1}{(1,-1,1,1,3)} + t\overset{v_2}{(1,-3,-3,-3,-9)},$$
$$\Gamma : \underline{1}x_1 + \underline{2}x_2 - \underline{1}x_3 + \underline{3}x_4 + \underline{1}x_5 = 0.$$

Γ je nadrovina v $\mathbb{R}^5$ ($\dim \Gamma = 5-1=4$), preto je výhodné využiť formulky:

$\boxed{\measuredangle \rho,\Gamma = \frac{\pi}{2} - \measuredangle \rho,[\vec{n}]}$ (viď obrázok), kde $\vec{n}$ je normálový vektor nadroviny Γ.

Výpočet $\measuredangle \rho[\vec{n}]$ prevedieme postupom z príkladu 4:

$\vec{n} = (1,2,-1,3,1)$

Počítajme kolmú projekciu $\vec{n}$ do $Z(\rho)$:

$$P_{\vec{n}} = s\cdot v_1 + t v_2$$

$$\vec{n} - P_{\vec{n}} = \vec{n} - s\cdot v_1 - t\cdot v_2 \perp v_1, v_2 \iff \begin{aligned} s\langle v_1,v_1\rangle + t\cdot\langle v_2,v_1\rangle &= \langle \vec{n}, v_1\rangle \\ s\langle v_1,v_2\rangle + t\cdot\langle v_2,v_2\rangle &= \langle \vec{n}, v_2\rangle \end{aligned}$$

$$\begin{array}{cc} s & t \end{array}$$
$$\left(\begin{array}{cc|c} 13 & -29 & 4 \\ -29 & 109 & -20 \end{array}\right)$$

$\langle v_1,v_1\rangle = 1^2 + (-1)^2 + 1^2 + 1^2 + 3^2 = 13$

$\langle v_2,v_1\rangle = \langle v_1,v_2\rangle = 1+3-3-3-27 = -29$

$\langle v_2,v_2\rangle = 1 + (-3)^2 + (-3)^2 + (-3)^2 + (-9)^2 = 109$

$\langle \vec{n},v_1\rangle = 1-2-1+3+3 = 4$

$\langle \vec{n},v_2\rangle = 1-6+3-9-9 = -20$

Cramerovo pravidlo:

$$s = \frac{\det\begin{pmatrix} 4 & -29 \\ -20 & 109 \end{pmatrix}}{\det\begin{pmatrix} 13 & -29 \\ -29 & 109 \end{pmatrix}} = \frac{4\cdot 109 - 20\cdot 29}{13\cdot 109 - 29^2} = \frac{-144}{576} = \boxed{-\frac{1}{4}}$$

$$t = \frac{\det\begin{pmatrix} 13 & 4 \\ -29 & -20 \end{pmatrix}}{\det\begin{pmatrix} 13 & -29 \\ -29 & 109 \end{pmatrix}} = \frac{13\cdot(-20)+4\cdot 29}{576} = \frac{-144}{576} = \boxed{-\frac{1}{4}}$$

$$P_{\vec{n}} = -\frac{1}{4}(v_1+v_2) = -\frac{1}{4}(2,-4,-2,-2,-6)$$
$$= \boxed{\frac{1}{2}(-1,2,1,1,3)}$$

$$\cos(\measuredangle \rho,[\vec{n}]) = \frac{\|P_{\vec{n}}\|}{\|\vec{n}\|} = \frac{\frac{1}{2}\|(-1,2,1,1,3)\|}{\|(1,2,-1,3,1)\|} = \frac{\frac{1}{2}\sqrt{(-1)^2+\cancel{2^2+1^2}+1^2+3^2}}{\sqrt{1^2+\cancel{2^2+(-1)^2}+3^2+1^2}} = \frac{1}{2} \Rightarrow \boxed{\measuredangle \rho,[\vec{n}] = \frac{\pi}{3}}$$

$$\boxed{\measuredangle \rho,\Gamma = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}}$$