$$(z+y-x)\,u_x + (z+x-y)\,u_y + z\,u_z = 0$$

$$\begin{aligned} \dot x &= -x + y + z \\ \dot y &= x - y + z \\ \dot z &= z \end{aligned}$$

$$\dot x + \dot y - 2\dot z = 0 \;\rightarrow\; x + y - 2z = \text{const}$$

$$\dot x - \dot y = -2x + 2y \;\rightarrow\; \frac{\dot x - \dot y}{x-y} = -2 = -2\frac{\dot z}{z}$$

$$\ln|x-y| = -2\ln|z| + \text{const}$$

$$z^2(x-y) = \text{const}$$

$$u(x,y,z) = \Phi\left(x+y-2z,\; z^2(x-y)\right)$$

Řešení systému:

$$\begin{vmatrix} -1-\lambda & 1 & 1 \\ 1 & -1-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)\left((1+\lambda)^2 - 1\right) = (1-\lambda)(\lambda^2 + 2\lambda) = \lambda(1-\lambda)(\lambda+2)$$

$$\lambda_{1,2,3} = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$$

$$\lambda = 0: \quad \begin{pmatrix} -1 & 1 & 1 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\lambda = 1: \quad \begin{pmatrix} -2 & 1 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & -3 & 3 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\lambda = -2: \quad \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad v_3 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + Be^{s}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + Ce^{-2s}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

Okrajová podmínka $u(x,y,1) = xy$, tj.

$$\begin{aligned} x &= \sigma_1 \\ y &= \sigma_2 \\ z &= 1 \\ u &= \sigma_1\sigma_2 \end{aligned}$$

$$\begin{pmatrix} 1 & 1 & 1 & \sigma_1 \\ 1 & 1 & -1 & \sigma_2 \\ 0 & 1 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & \sigma_1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & \sigma_1 - \sigma_2 \end{pmatrix} \rightarrow \quad \begin{aligned} C &= \tfrac{1}{2}(\sigma_1 - \sigma_2) \\ B &= 1 \\ A &= \sigma_1 - \tfrac{1}{2}(\sigma_1 - \sigma_2) - 1 = \tfrac{1}{2}(\sigma_1 + \sigma_2) - 1 \end{aligned}$$

$$\begin{aligned} x &= e^{s} + \tfrac{1}{2}(\sigma_1 - \sigma_2)e^{-2s} + \tfrac{1}{2}(\sigma_1 + \sigma_2) - 1 \\ y &= e^{s} - \tfrac{1}{2}(\sigma_1 - \sigma_2)e^{-2s} + \tfrac{1}{2}(\sigma_1 + \sigma_2) - 1 \\ z &= e^{s} \end{aligned}$$

$$x - z = \frac{1}{2}(\sigma_1 - \sigma_2)\frac{1}{z^2} + \frac{1}{2}(\sigma_1 + \sigma_2) - 1$$

$$y - z = -\frac{1}{2}(\sigma_1 - \sigma_2)\frac{1}{z^2} + \frac{1}{2}(\sigma_1 + \sigma_2) - 1$$

$$x + y - 2z = \sigma_1 + \sigma_2 - 2 \qquad x - y = (\sigma_1 - \sigma_2)\frac{1}{z^2}$$

$$\sigma_1 + \sigma_2 = x + y - 2z + 2 \qquad \sigma_1 - \sigma_2 = z^2(x - y)$$

$$2\sigma_1 = x + y - 2z + 2 + z^2(x - y)$$

$$2\sigma_2 = x + y - 2z + 2 - z^2(x - y)$$

$$u = \sigma_1 \sigma_2 = \underline{\underline{\frac{1}{4}\left((x + y - 2z + 2)^2 - z^4(x - y)^2\right)}}$$

z obecného řešení: $xy = \Phi(x + y - 2,\ x - y) \longrightarrow \Phi(\xi, \eta) = \left((\xi + 2)^2 - \eta^2\right)\frac{1}{4}$

$$u = \frac{1}{4}\left((x + y - 2z + 2)^2 - z^4(x - y)\right)$$

jiná okrajová podmínka $u(x, y, 0) = xy$

$$\begin{pmatrix} 1 & 1 & 1 & \sigma_1 \\ 1 & 1 & -1 & \sigma_2 \\ 0 & 1 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & \sigma_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & \sigma_1 - \sigma_2 \end{pmatrix}$$

$$C = \frac{1}{2}(\sigma_1 - \sigma_2)$$

$$B = 0$$

$$A = \sigma_1 - \frac{1}{2}(\sigma_1 - \sigma_2) = \frac{1}{2}(\sigma_1 + \sigma_2)$$

$$x = \frac{1}{2}(\sigma_1 - \sigma_2)e^{-2s} + \frac{1}{2}(\sigma_1 + \sigma_2)$$

$$y = -\frac{1}{2}(\sigma_1 - \sigma_2)e^{-2s} + \frac{1}{2}(\sigma_1 + \sigma_2)$$

$$z = 0$$

$$2x = \sigma_1 e^{-2s} + \sigma_1 - \sigma_2 e^{-2s} + \sigma_2 = \sigma_1(e^{-2s} + 1) - \sigma_2(e^{-2s} - 1)$$

$$2y = -\sigma_1 e^{-2s} + \sigma_2 e^{-2s} + \sigma_1 + \sigma_2 = -\sigma_1(e^{-2s} - 1) + \sigma_2(e^{-2s} + 1)$$

$$\begin{vmatrix} (e^{-2s} + 1) & -(e^{-2s} - 1) \\ -(e^{-2s} - 1) & (e^{-2s} + 1) \end{vmatrix} = (e^{-2s} + 1)^2 - (e^{-2s} - 1)^2 = 4e^{-2s}$$

$$\begin{vmatrix} 2x & -e^{-2s} + 1 \\ 2y & e^{-2s} + 1 \end{vmatrix} = 2x(e^{-2s} + 1) + 2y(e^{-2s} - 1) \rightarrow \sigma_1 = \frac{1}{2}\left(x(1 + e^{2s}) + y(1 - e^{2s})\right)$$

$$\begin{vmatrix} e^{-2s} + 1 & 2x \\ -(e^{-2s} - 1) & 2y \end{vmatrix} = 2y(e^{-2s} + 1) + 2x(e^{-2s} - 1) \rightarrow \sigma_2 = \frac{1}{2}\left(x(1 - e^{2s}) + y(1 + e^{2s})\right)$$

nelze eliminovat s