11. CVIČENIE

ℓ^1 .... $(\ell^1)' \cong \ell^\infty$

pre $\forall f \in (\ell^1)'$ $\exists!$ $\{f_\ell\}_{\ell=1}^\infty \in \ell^\infty$ taký, že:

$$f(x) = \sum_{\ell=1}^\infty x_\ell \cdot f_\ell \quad ; \quad \forall x = \{x_\ell\}_{\ell=1}^\infty \in \ell^1$$

navyše, platí: $\|f\| = \|\{f_\ell\}\|_\infty = \sup_{\ell \in \mathbb{N}} |f_\ell|$

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c_0 .... $(c_0)' \cong \ell^1$

$\forall f \in (c_0)'$ $\exists!$ $\{f_\ell\}_{\ell=1}^\infty \in \ell^1$ taká, že:

$$f(x) = \sum_{\ell=1}^\infty x_\ell \cdot f_\ell \quad ; \quad \forall x = \{x_\ell\} \in c_0 \, ;$$

$$\|f\| = \|\{f_\ell\}\|_1 = \sum_{\ell=1}^\infty |f_\ell|$$

$\ell^1 := \left\{ \{a_\ell\}_{\ell=1}^\infty \subseteq \mathbb{R} \; ; \; \sum_{\ell=1}^\infty |a_\ell| < \infty \right\}$; $\|\{a_\ell\}\|_1 := \sum_{\ell=1}^\infty |a_\ell|$

$c_0 := \left\{ \{a_\ell\}_{\ell=1}^\infty \subseteq \mathbb{R} \; ; \; \lim_{\ell \to \infty} a_\ell = 0 \right\}$; $\|\{a_\ell\}\|_\infty := \sup_{\ell \in \mathbb{N}} |a_\ell|$

$c := \left\{ \{a_\ell\}_{\ell=1}^\infty \subseteq \mathbb{R} \; ; \; \lim_{\ell \to \infty} a_\ell \text{ existuje konečná} \right\}$; $\|\{a_\ell\}\|_\infty := \sup_{\ell \in \mathbb{N}} |a_\ell|$

$\ell^\infty := \left\{ \{a_\ell\}_{\ell=1}^\infty \subseteq \mathbb{R} \; ; \; \{a_\ell\} \text{ je ohraničená} \right\}$; $\|\{a_\ell\}\|_\infty := \sup_{\ell \in \mathbb{N}} |a_\ell|$

$c \ldots \quad (c)' \cong \ell^1$

$\forall f \in (c)' \ \exists ! \{f_\ell\}_{\ell=0}^{\infty} \in \ell^1$ taká, že:

$$f(x) = d_x \cdot f_0 + \sum_{\ell=1}^{\infty} x_\ell \cdot f_\ell \ ; \quad \forall x = \{x_\ell\}_{\ell=1}^{\infty} \in c \ ; \ d_x := \lim_{\ell \to \infty} x_\ell \ ;$$

navyše platí:

$$\|f\| = \|\{f_\ell\}\|_1 = \sum_{\ell=0}^{\infty} |f_\ell|$$

⇝) PRÍKLAD PRIESTORU X, KTORÝ NIE JE REFLEXÍVNY, HOCI JE $X \cong X''$ ⇝) JAMESOV PRIESTOR

$$X := \left\{ \{x_\ell\}_{\ell=1}^{\infty} \subseteq \mathbb{R} \ ; \ \lim_{\ell \to \infty} x_\ell = 0 \ ; \ \text{navyše platí:} \right.$$

$$S_{\{x_\ell\}} := \left\{ \sqrt{|x_{\ell_2} - x_{\ell_1}|^2 + |x_{\ell_3} - x_{\ell_2}|^2 + \ldots + |x_{\ell_n} - x_{\ell_{n-1}}|^2} \ , \text{ kde } \right.$$

$$\left. \ell_i \in \mathbb{N} \text{ pre } \forall i = 1, \ldots, n \ ; \ n \in \mathbb{N} \ , \ \ell_1 < \ell_2 < \ell_3 < \ldots < \ell_n \right\}$$

$$\left. \text{je ohraničená} \right\}$$

⇝) na X definujeme normu:

$$\|\{x_\ell\}\| := \frac{1}{\sqrt{2}} \sup S_{\{x_\ell\}}$$

⇝) platí:

- X je úplný priestor = BANACHOV PRIESTOR

- X je izometricky izomorfný s X''
- X nie je reflexívny priestor

$\Rightarrow$ platí, že $X \subseteq c_0$ v zmysle množinovej inklúzie, ale X sa nedá izometricky homomorfne vnoriť do c_0, ani do ℓ^1