PROBLEMS FOR ASSIGNMENT 4

1. Problem 1

For each of the following functions, determine all its zeroes and all its isolated singular points *in the extended complex plane* $\overline{\mathbb{C}}$ (that is, the point ∞ must be investigated as well!). As long as a point is a zero or a pole, determine the order. Explain your answer!

a) $f(z) = \frac{1-\cos(nz^{n+5})}{(\sin z)^{n+4}} \cdot e^{\frac{1}{\pi - z}}$

b) $f(z) = \sqrt{z} \sin\left(\frac{1}{\sqrt{z^{2n+5}}}\right)$ (explain also why the function is holomorphic outside its isolated singular points; the choice of $\operatorname{Arg} z$ for the two roots is the same).

Remark. I remind that a removable singularity can be considered at the same time as an isolated zero! In this case, also determine the order.

2. Problem 2

Find Taylor/Laurent expensions of the function $f(z)$ in *all* (!) possible discs and annuli with the center $z = 0$:

$$f(z) = \frac{z^{n+3}}{z(z-n)(z-2n)}$$

After doing so, determine the types of isolated singularity at the points $z = 0$ and $z = \infty$ (including the order, if applicable).

Hint. Use partial fractions.

3. Problem 3

Using the theory of Laurent series, calculate

$$\lim_{\epsilon \to 0} \left(\int_{|z|=\epsilon} z^n e^{1/z} \right)$$