

PROBLEMS FOR HOMEWORK 3

1. PROBLEM 1

Calculate the integral:

$$\int_{|z|=\frac{2}{n}} \frac{z+1}{\sin(n^2 z^2)} dz.$$

2. PROBLEM 2

Using the residue calculus, evaluate the integral

$$\int_{-\infty}^{\infty} \frac{x \sin(nx)}{x^2 + 2x + 2} dx$$

(for n even), or the integral

$$\int_{-\infty}^{\infty} \frac{x \cos(nx)}{x^2 - 2x + 2} dx$$

(for n odd).

3. PROBLEM 3

Using the residue calculus, evaluate the integral

$$\int_0^{\infty} \frac{x+n}{\sqrt[3]{x}(x^2+1)} dx.$$

4. PROBLEM 4

Using the residue calculus, evaluate the integral

$$\int_0^{\infty} \frac{\ln x}{\sqrt[k]{x}(x^2-1)} dx, \quad k = n+2.$$

5. PROBLEM 5

Using the residue calculus, evaluate the integral

$$\int_{|z|=n+100} \frac{1}{nz+1} \cos\left(\frac{1}{(z-1)(z-2)\cdots(z-99-n)}\right) dz.$$

6. PROBLEM 6

Using the residue calculus, find the sum of the series

$$\sum_{k=n+1}^{\infty} \frac{k^2+n}{k^4-n^2k^2}$$

Problems 1,2 count for 1 point each, problems 3-6 count for 2 points each.