M7116 Strukturované populační modely

Populace s interní variabilitou

10. 5. 2021

Populace strukturovaná podle plodnosti

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \qquad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} \lambda = \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \\ = \frac{1}{2}\left(\sigma_1(1-\gamma) + \sigma_2 + \sqrt{\big(\sigma_1(1-\gamma) - \sigma_2\big)^2 + 4\sigma_1\gamma\varphi}\right) \end{aligned}$$

$$\lambda > 1 \;\Leftrightarrow\; \sigma_1\gamma\varphi \geq (1-\sigma_2)\big(1 - (\sigma_1(1-\gamma))\big)$$

Populace strukturovaná podle plodnosti

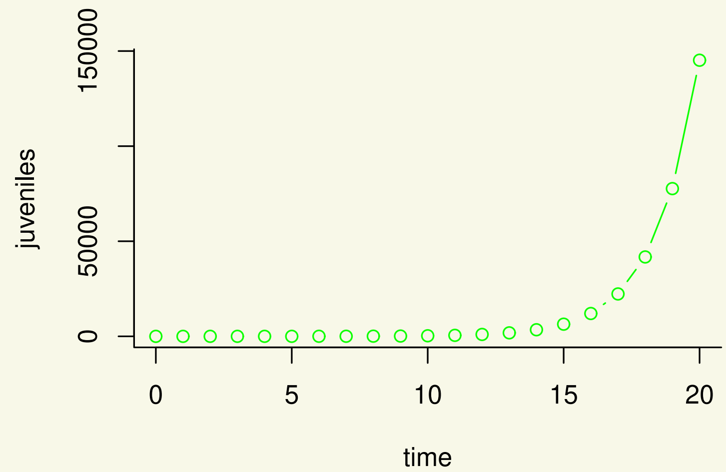
$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \quad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

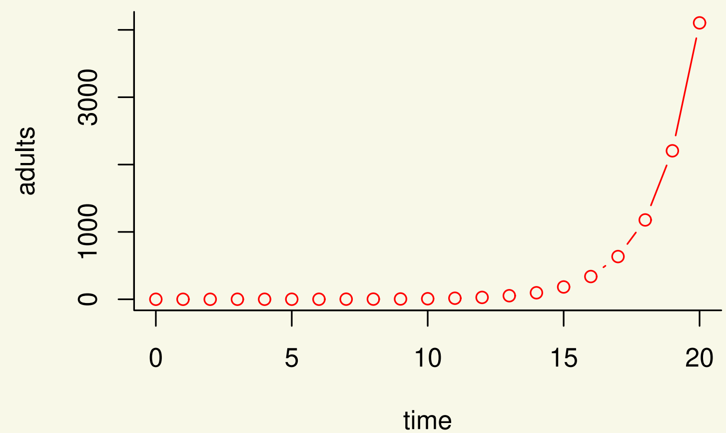
$$\lambda = \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \\ = \frac{1}{2}\left(\sigma_1(1-\gamma) + \sigma_2 + \sqrt{\left(\sigma_1(1-\gamma) - \sigma_2\right)^2 + 4\sigma_1\gamma\varphi}\right)$$

$$\lambda > 1 \;\Leftrightarrow\; \sigma_1\gamma\varphi \geq (1-\sigma_2)\big(1-(\sigma_1(1-\gamma))\big)$$

$\sigma_1 = 0.5$, $\sigma_2 = 0.1$, $\gamma = 0.1$, $\varphi = 50$

$\lambda = 1.8658$





Parametry závislé na velikosti složek populace

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \qquad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\sigma_1 = \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2}$$
$$\sigma_2 = \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$$
$$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2}$$
$$\varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$$

$$\lambda = \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \lambda(n_1, n_2) = \lambda(\boldsymbol{n})$$

$$\lambda_0 = \lambda(\boldsymbol{o})$$
$$\lambda_\infty = \lim_{\|\boldsymbol{n}\| \to \infty} \lambda(\boldsymbol{n})$$

Parametry závislé na velikosti složek populace

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix}\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \qquad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\sigma_1 = \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2}$$
$$\sigma_2 = \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$$
$$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2}$$
$$\varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$$

$$\lambda = \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \lambda(n_1, n_2) = \lambda(\boldsymbol{n})$$

$$\lambda_0 = \lambda(\boldsymbol{o})$$
$$\lambda_\infty = \lim_{\|\boldsymbol{n}\| \to \infty} \lambda(\boldsymbol{n})$$

$$\lim_{\sigma_1 \to 0} \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \sigma_2$$
$$\lim_{\sigma_2 \to 0} \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \tfrac{1}{2}\left(\sigma_1(1-\gamma) + \sqrt{\sigma_1^2(1-\gamma)^2 + 4\sigma_1\gamma\varphi}\right)$$
$$\lim_{\gamma \to 0} \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \sigma_1$$
$$\lim_{\varphi \to 0} \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \sigma_1(1-\gamma)$$

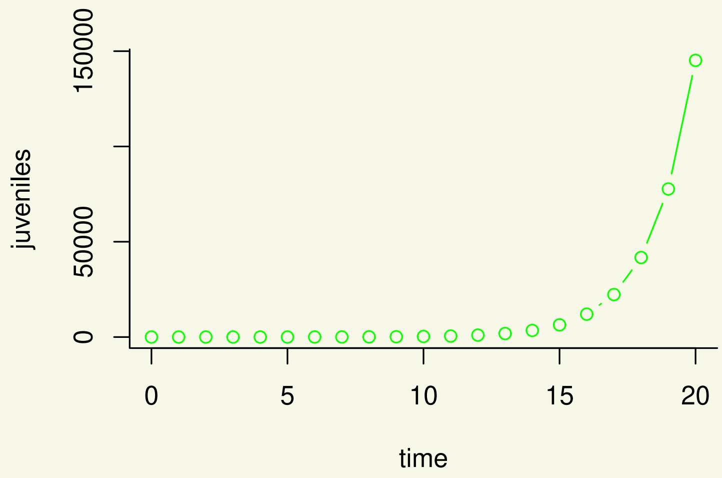
Parametry závislé na velikosti složek populace

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix}\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \qquad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\sigma_1 = \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2}$$
$$\sigma_2 = \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$$
$$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2}$$
$$\varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$$

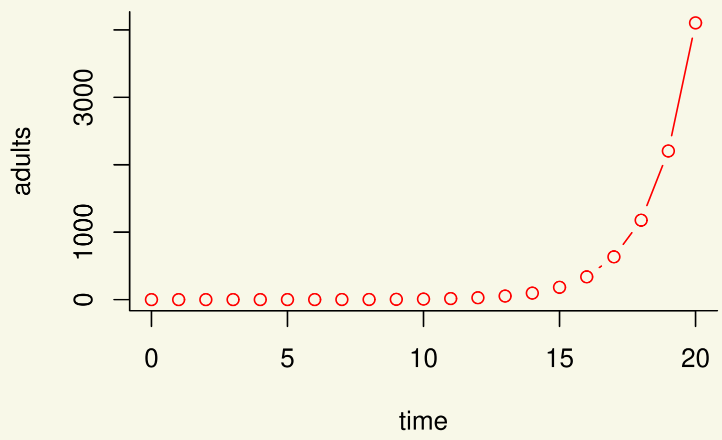
$$\lambda = \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \lambda(n_1, n_2) = \lambda(\boldsymbol{n})$$

$$\lambda_0 = \lambda(\boldsymbol{o}) = 1.8658$$
$$\lambda_\infty = \lim_{\|\boldsymbol{n}\| \to \infty} \lambda(\boldsymbol{n}) = 1.8658$$



$$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 50$$

$$s_{11} = s_{12} = 0,\ s_{11} = s_{12} = 0,\ g_1 = g_2 = 0,\ f_1 = f_2 = 0$$



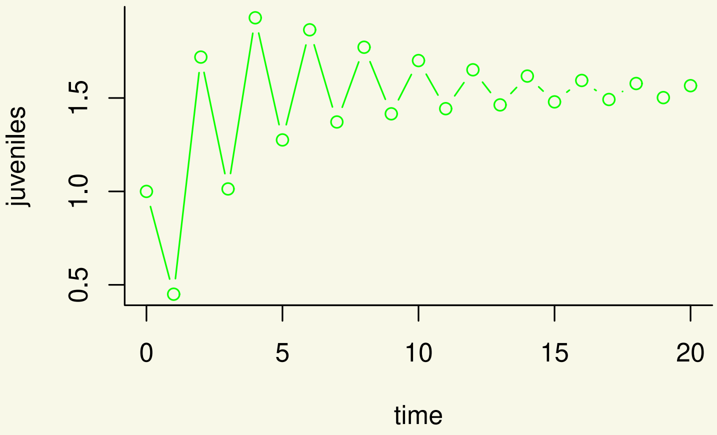
Parametry závislé na velikosti složek populace

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix}\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \qquad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\sigma_1 = \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2}$$
$$\sigma_2 = \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$$
$$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1n_1 - g_2n_2}$$
$$\varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1n_1 - f_2n_2}$$

$$\lambda = \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \lambda(n_1, n_2) = \lambda(\boldsymbol{n})$$

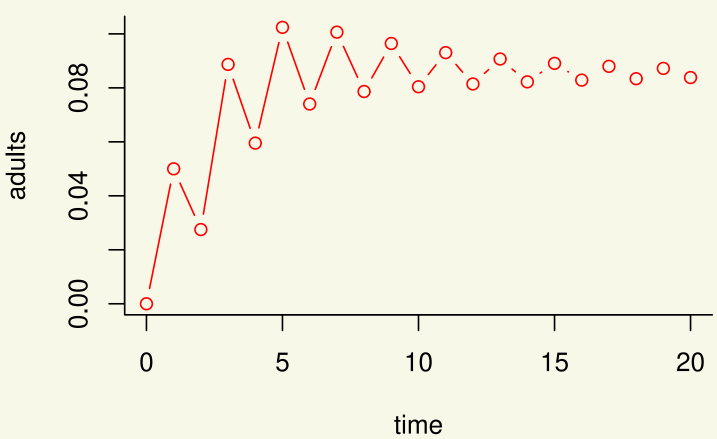
$$\lambda_0 = \lambda(\boldsymbol{o}) = 1.8658$$
$$\lambda_\infty = \lim_{\|\boldsymbol{n}\|\to\infty} \lambda(\boldsymbol{n}) = 0.45$$



$$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 50$$

$$s_{11} = s_{12} = 0,\ s_{11} = s_{12} = 0,\ g_1 = g_2 = 0,\ f_1 = f_2 = 1$$

Stabilizace populace omezením plodnosti



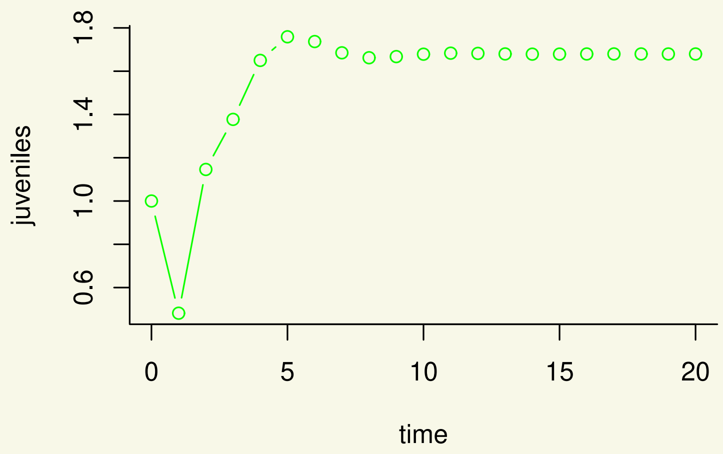
Parameztry závislé na velikosti složek populace

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \qquad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\sigma_1 = \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2}$$
$$\sigma_2 = \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$$
$$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2}$$
$$\varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$$

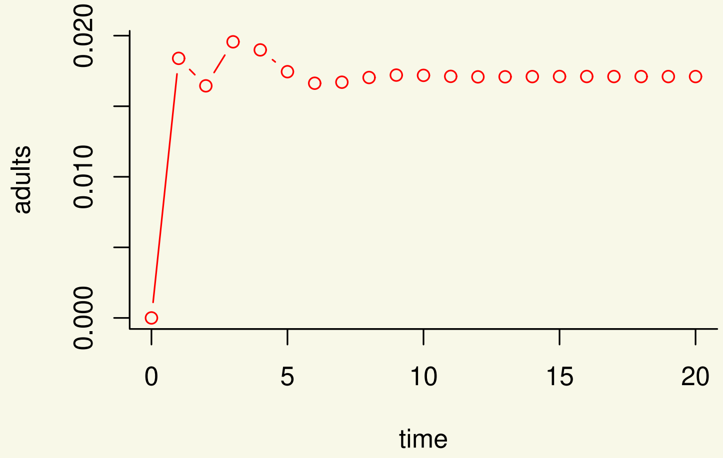
$$\lambda = \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \lambda(n_1, n_2) = \lambda(\boldsymbol{n})$$

$$\lambda_0 = \lambda(\boldsymbol{o}) = 1.8658$$
$$\lambda_\infty = \lim_{\|\boldsymbol{n}\| \to \infty} \lambda(\boldsymbol{n}) = 0.5$$



$$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 50$$

$$s_{11} = s_{12} = 0,\ s_{11} = s_{12} = 0,\ g_1 = g_2 = 1,\ f_1 = f_2 = 0$$



Stabilizace populace odložením reprodukce

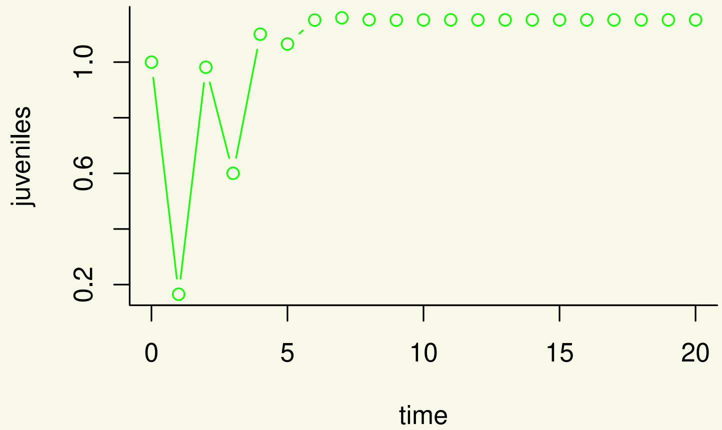
Parametry závislé na velikosti složek populace

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \qquad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\sigma_1 = \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2}$$
$$\sigma_2 = \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$$
$$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2}$$
$$\varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$$

$$\lambda = \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \lambda(n_1, n_2) = \lambda(\boldsymbol{n})$$

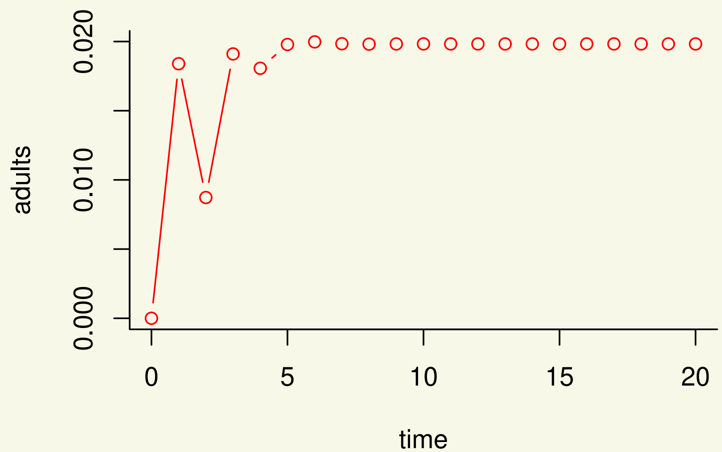
$$\lambda_0 = \lambda(\boldsymbol{o}) = 1.8658$$
$$\lambda_\infty = \lim_{\|\boldsymbol{n}\| \to \infty} \lambda(\boldsymbol{n}) = 0.1$$



$$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 50$$

$$s_{11} = s_{12} = 1,\ s_{11} = s_{12} = 0,\ g_1 = g_2 = 0,\ f_1 = f_2 = 0$$

Stabilizace populace zvětšením úmrtnosti juvenilních jedinců (infanticidou)



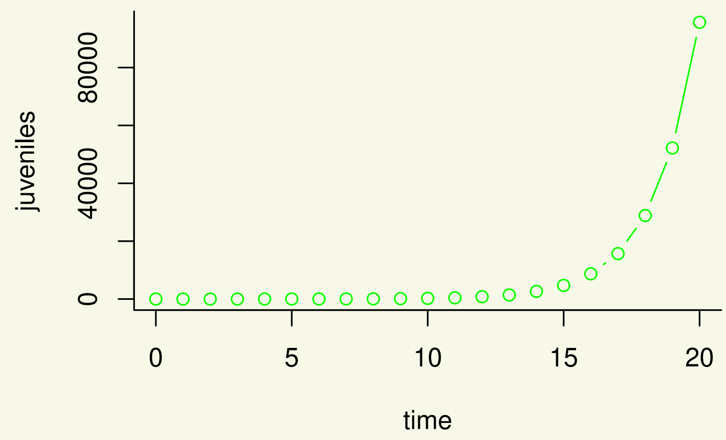
Parametry závislé na velikosti složek populace

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \qquad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\sigma_1 = \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2}$$
$$\sigma_2 = \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$$
$$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2}$$
$$\varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$$

$$\lambda = \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \lambda(n_1, n_2) = \lambda(\boldsymbol{n})$$

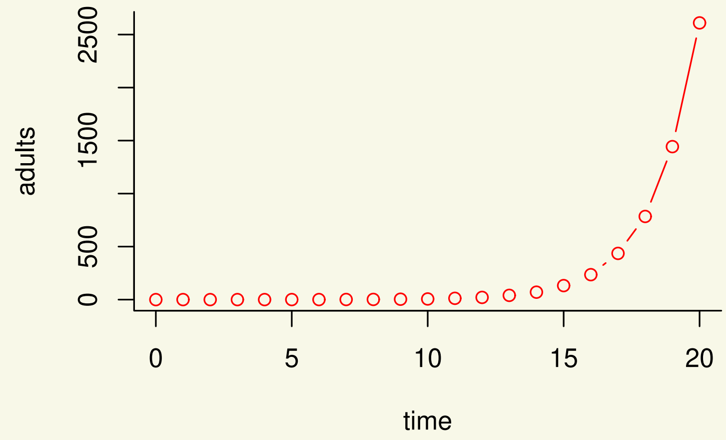
$$\lambda_0 = \lambda(\boldsymbol{o}) = 1.8658$$
$$\lambda_\infty = \lim_{\|\boldsymbol{n}\| \to \infty} \lambda(\boldsymbol{n}) = 1.8221$$



$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 50$

$s_{11} = s_{12} = 0,\ \color{red}{s_{11} = s_{12} = 1},\ g_1 = g_2 = 0,\ f_1 = f_2 = 0$

Zpomalení růstu populace zvětšením úmrtnosti plodných jedinců (při velké plodnosti)



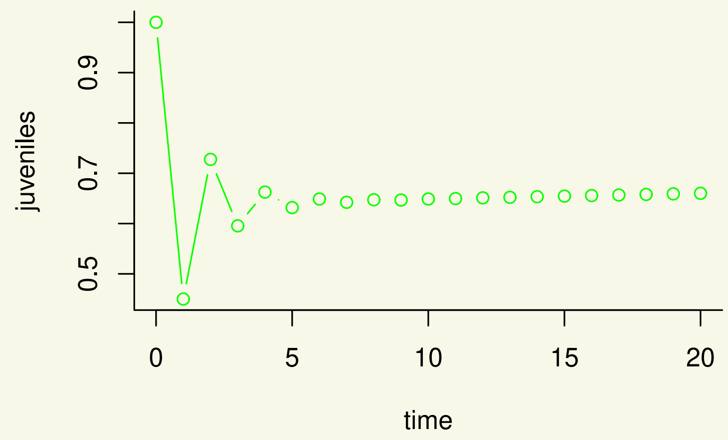
Parametry závislé na velikosti složek populace

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \quad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} \sigma_1 &= \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2} \\ \sigma_2 &= \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2} \\ \gamma &= \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2} \\ \varphi &= \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2} \end{aligned}$$

$$\lambda = \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \lambda(n_1, n_2) = \lambda(\boldsymbol{n})$$

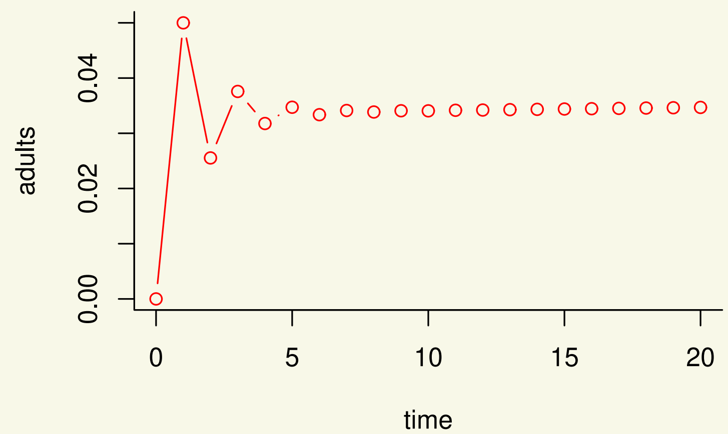
$$\lambda_0 = \lambda(\boldsymbol{o}) = 1.0204$$
$$\lambda_\infty = \lim_{\|\boldsymbol{n}\| \to \infty} \lambda(\boldsymbol{n}) = 0.9837$$



$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 10.5$

$s_{11} = s_{12} = 0,\ s_{11} = s_{12} = 1,\ g_1 = g_2 = 0,\ f_1 = f_2 = 0$

Stabilizace populace zvětšením úmrtnosti plodných jedinců (při malé plodnosti)

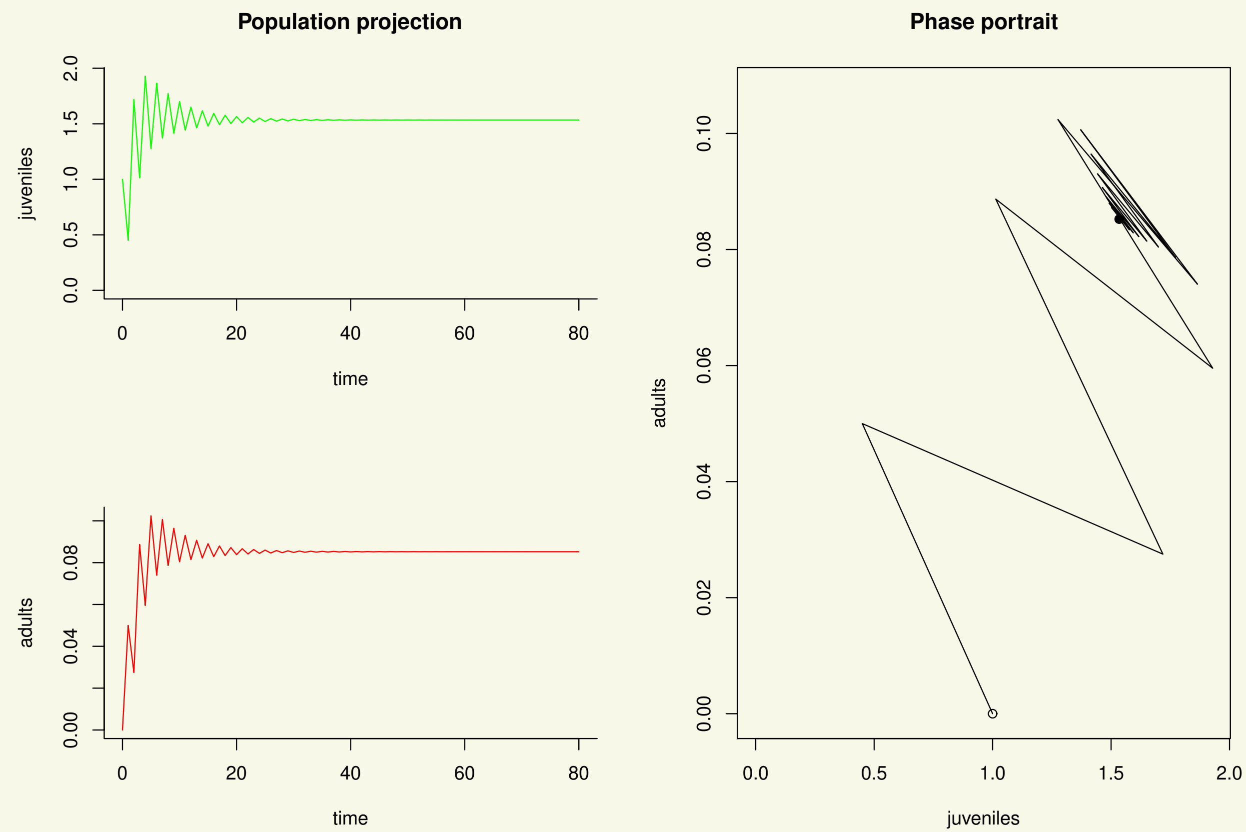


Trajektorie a atraktory

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \quad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\sigma_1 = \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2} \qquad \sigma_2 = \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$$
$$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2} \qquad \varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$$

$$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 50,$$
$$s_{11} = s_{12} = s_{21} = s_{22} = 0,\ g_1 = g_2 = 0,\ f_1 = f_2 = 1$$



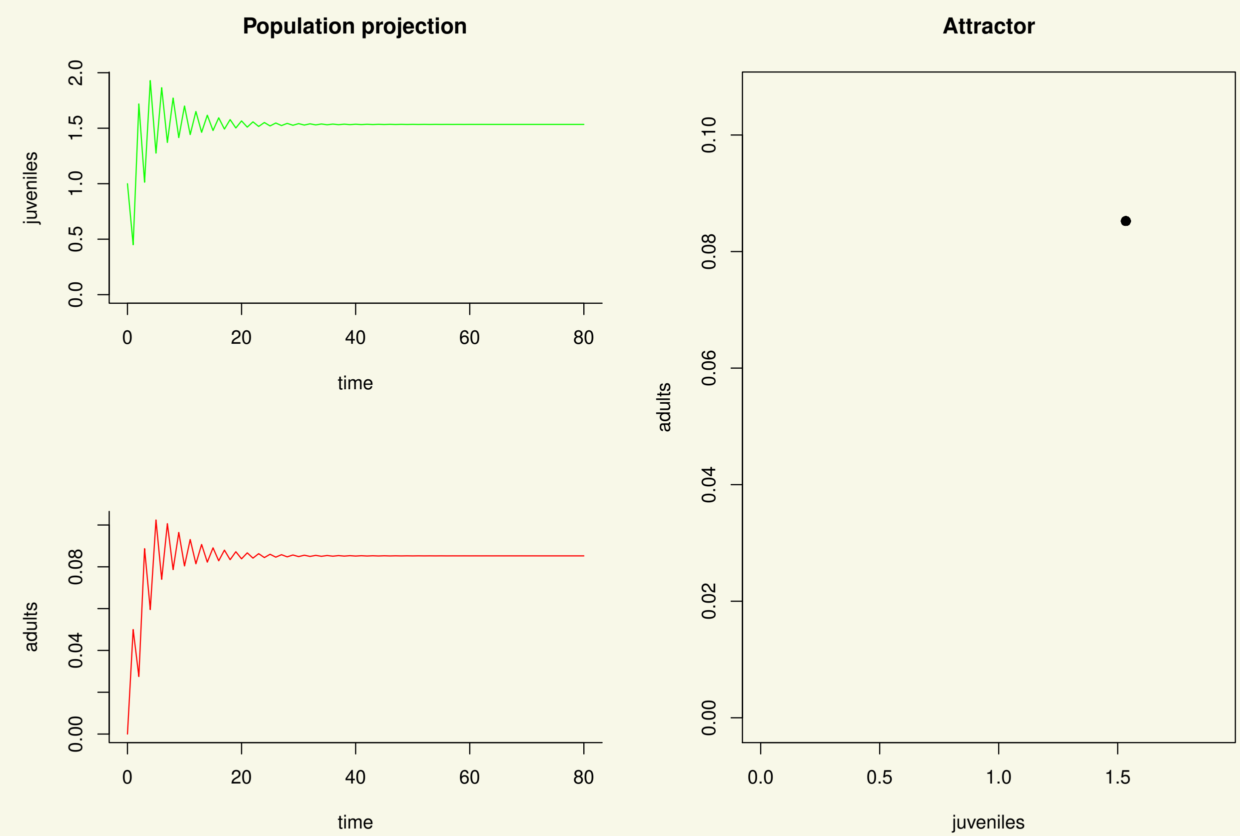
Trajektorie a atraktory

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix}\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \quad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\sigma_1 = \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2} \qquad \sigma_2 = \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$$

$$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2} \qquad \varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$$

$$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 50,$$
$$s_{11} = s_{12} = s_{21} = s_{22} = 0,\ g_1 = g_2 = 0,\ f_1 = f_2 = 1$$



Rovnovážný bod

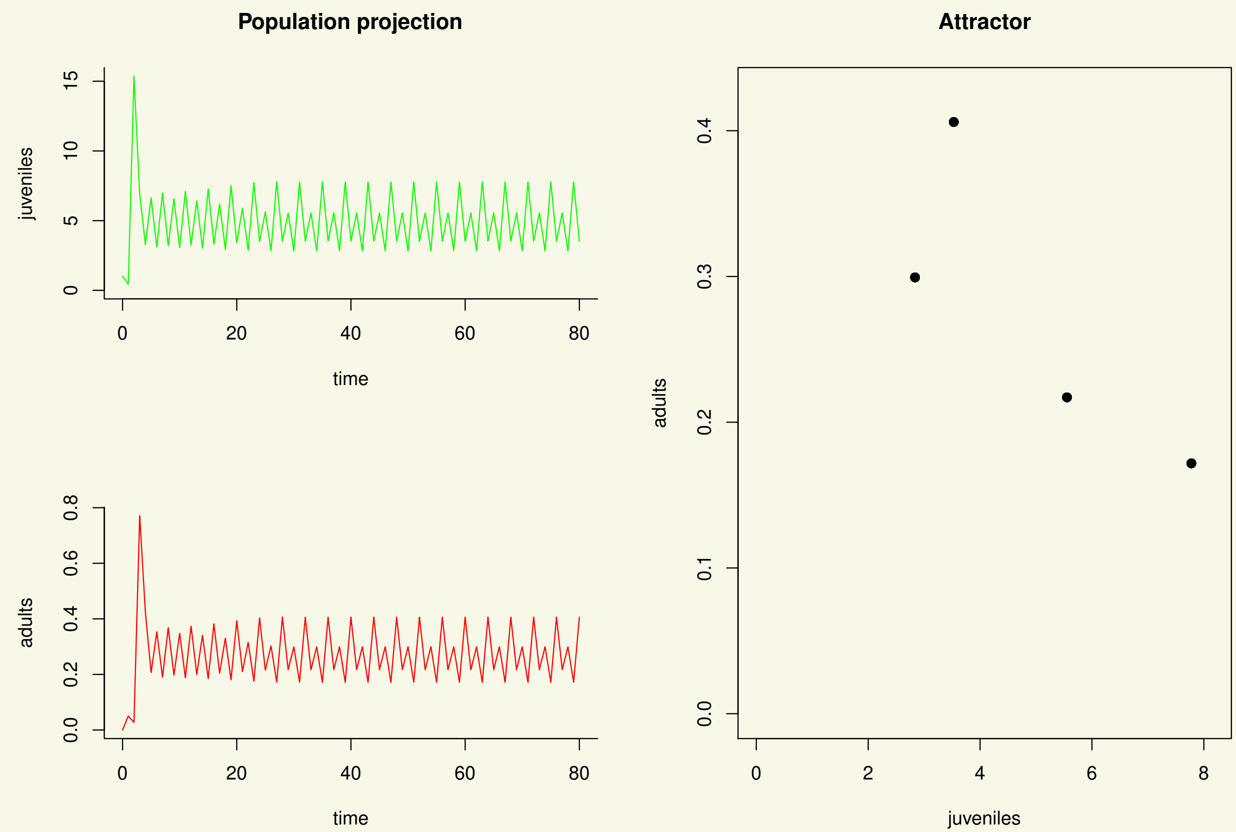
Trajektorie a atraktory

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \quad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\sigma_1 = \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2} \qquad \sigma_2 = \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$$

$$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2} \qquad \varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$$

$$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 500,$$
$$s_{11} = s_{12} = s_{21} = s_{22} = 0,\ g_1 = g_2 = 0,\ f_1 = f_2 = 1$$



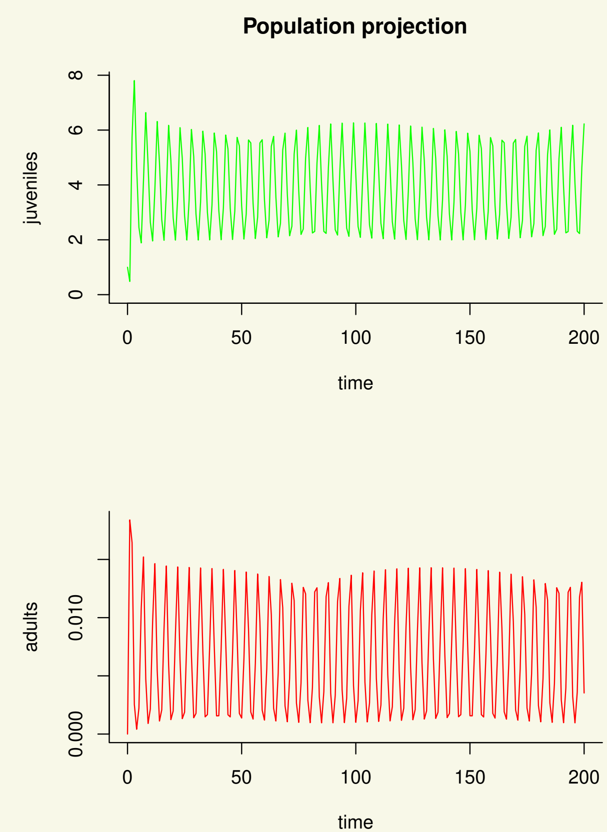
Cyklus délky 4

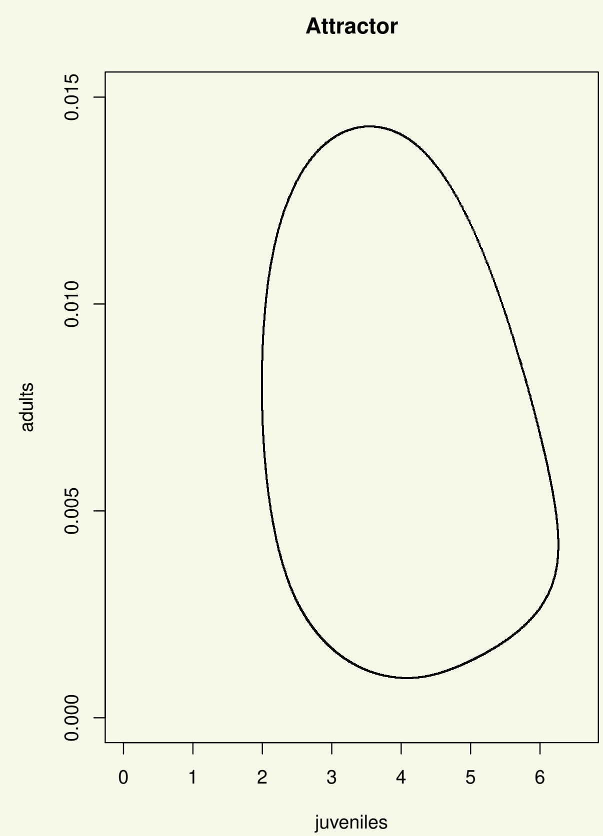
Trajektorie a atraktory

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \qquad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\sigma_1 = \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2} \qquad \sigma_2 = \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$$
$$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2} \qquad \varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$$

$$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 300,$$
$$s_{11} = s_{12} = s_{21} = s_{22} = 0,\ g_1 = g_2 = 1,\ f_1 = f_2 = 0$$





Invariantní smyčka

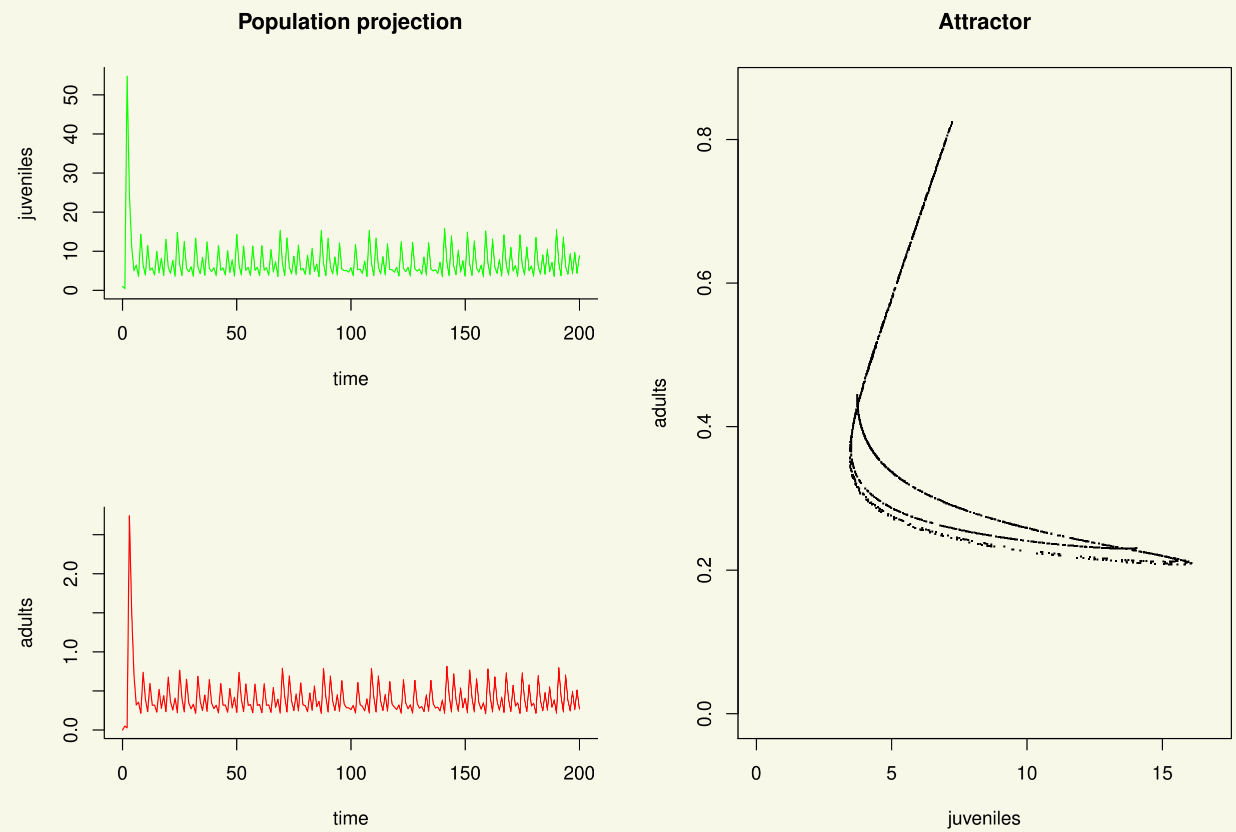
Trajektorie a atraktory

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \quad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\sigma_1 = \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2} \qquad \sigma_2 = \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$$

$$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2} \qquad \varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$$

$$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 1\,800,$$
$$s_{11} = s_{12} = s_{21} = s_{22} = 0,\ g_1 = g_2 = 0,\ f_1 = f_2 = 1$$



Podivný atraktor

Trajektorie a atraktory

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix}\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \qquad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$\sigma_1 = \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2}$

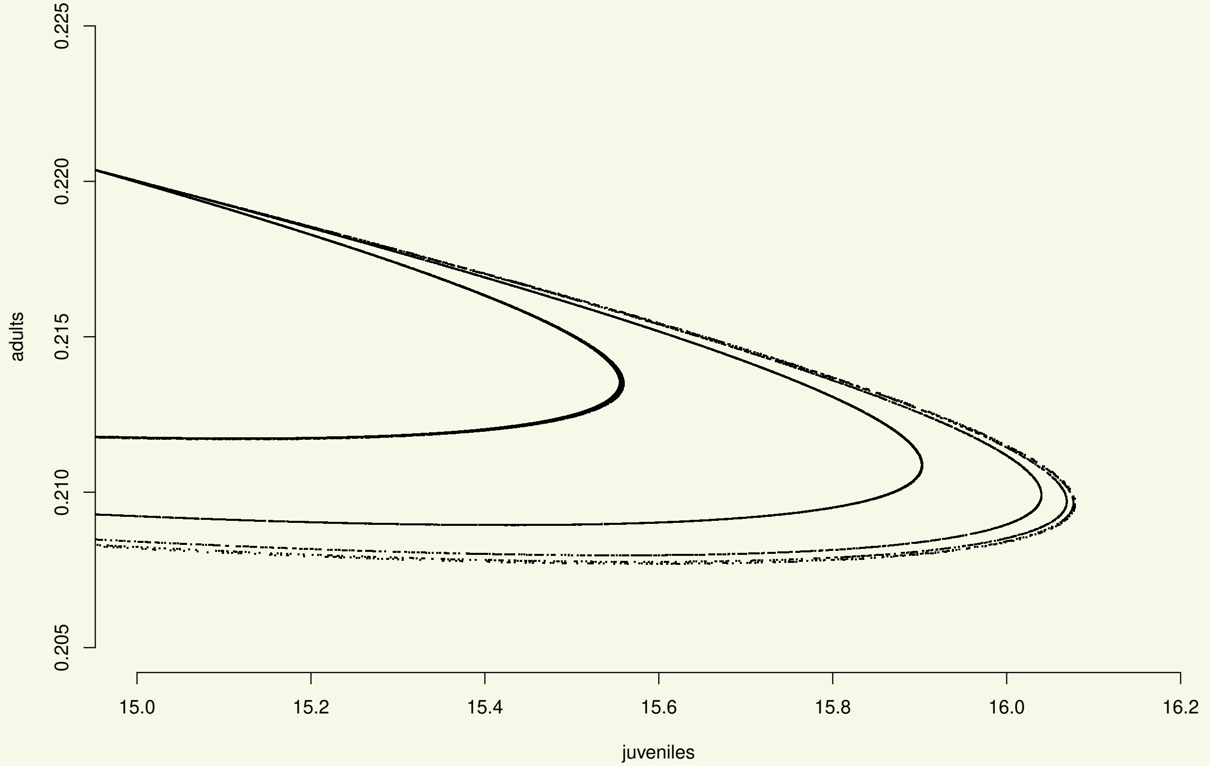
$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2}$

$\sigma_2 = \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$

$\varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$

$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 1\,800,$

$s_{11} = s_{12} = s_{21} = s_{22} = 0,\ g_1 = g_2 = 0,\ f_1 = f_2 = 1$



Podivný atraktor

Leslieho model s plodností závislou na velikosti populace

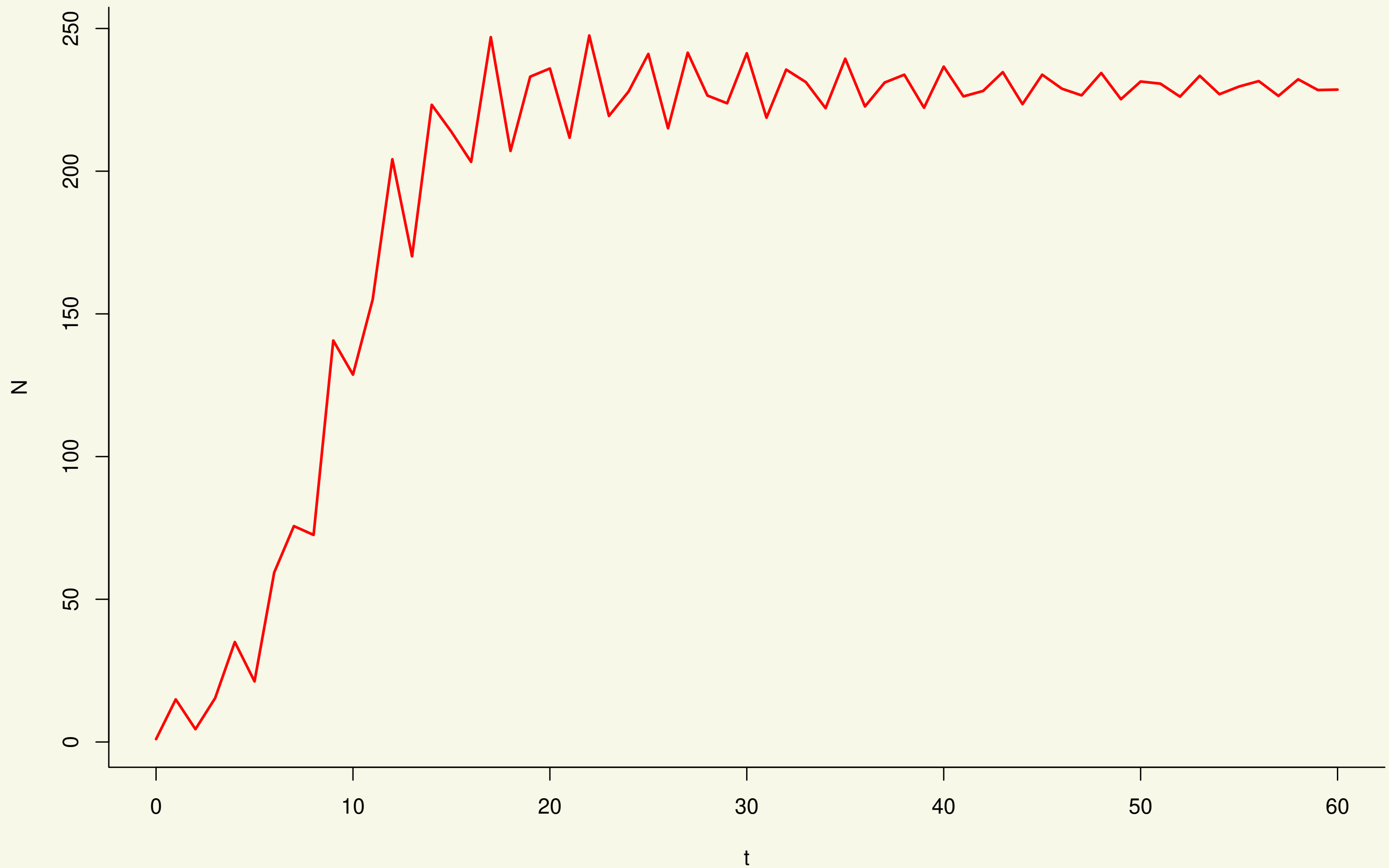
$$\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}(t+1) = \begin{pmatrix} 0 & g\big(N(t)\big) & 5g\big(N(t)\big) \\ 0.3 & 0 & 0 \\ 0 & 0.5 & 0 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}(t),$$

$$N = n_1 + n_2 + n_3, \quad g(N) = R\mathrm{e}^{-0.005N}$$

Leslieho model s plodností závislou na velikosti populace

$$\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}(t+1) = \begin{pmatrix} 0 & g\big(N(t)\big) & 5g\big(N(t)\big) \\ 0.3 & 0 & 0 \\ 0 & 0.5 & 0 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}(t),$$

$$N = n_1 + n_2 + n_3, \quad g(N) = R\mathrm{e}^{-0.005N}$$

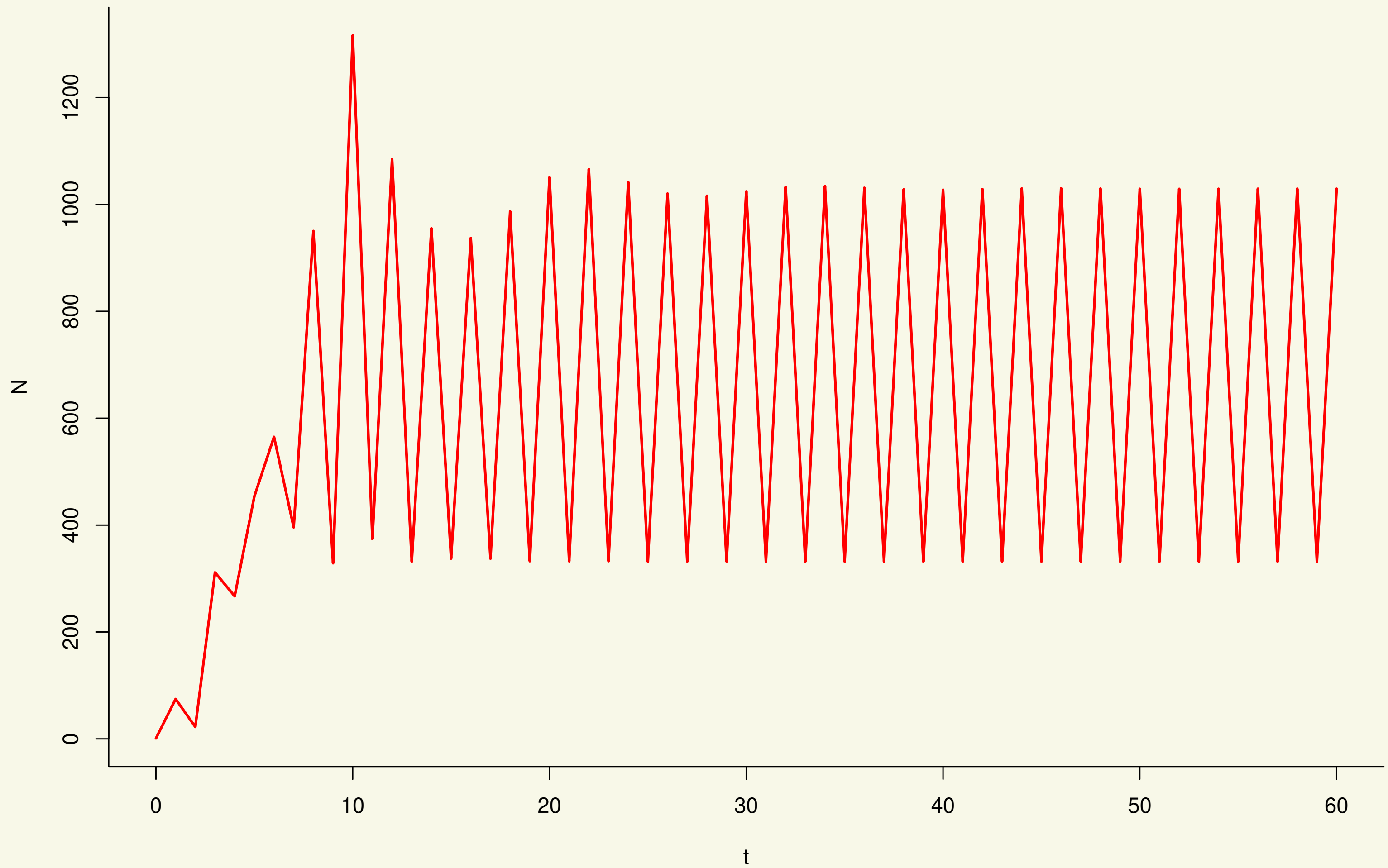


$R = 3$

Leslieho model s plodností závislou na velikosti populace

$$\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}(t+1) = \begin{pmatrix} 0 & g\big(N(t)\big) & 5g\big(N(t)\big) \\ 0.3 & 0 & 0 \\ 0 & 0.5 & 0 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}(t),$$

$$N = n_1 + n_2 + n_3, \quad g(N) = R\mathrm{e}^{-0.005N}$$

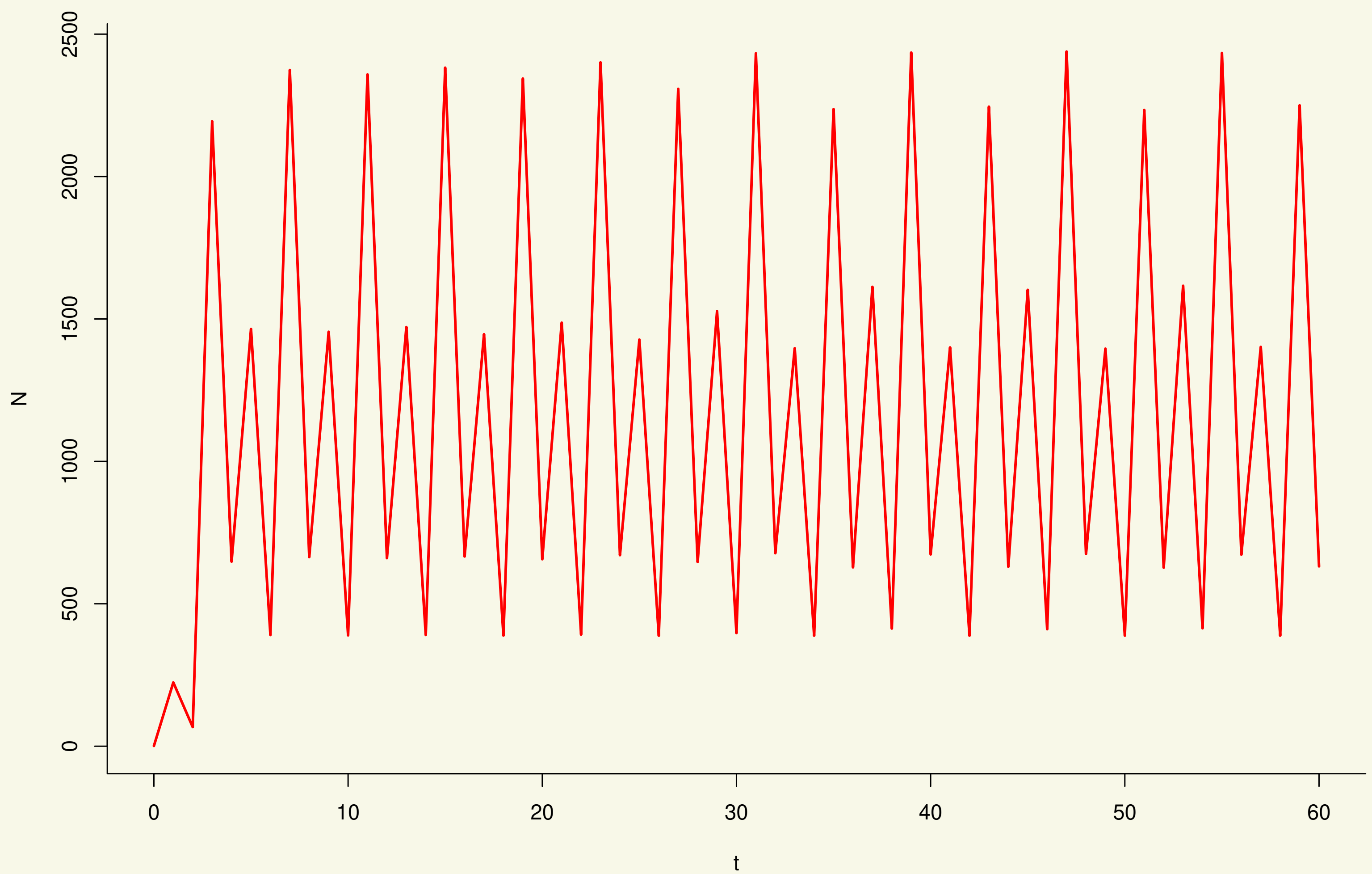


$R = 15$

Leslieho model s plodností závislou na velikosti populace

$$\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}(t+1) = \begin{pmatrix} 0 & g\big(N(t)\big) & 5g\big(N(t)\big) \\ 0.3 & 0 & 0 \\ 0 & 0.5 & 0 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}(t),$$

$$N = n_1 + n_2 + n_3, \quad g(N) = R\mathrm{e}^{-0.005N}$$

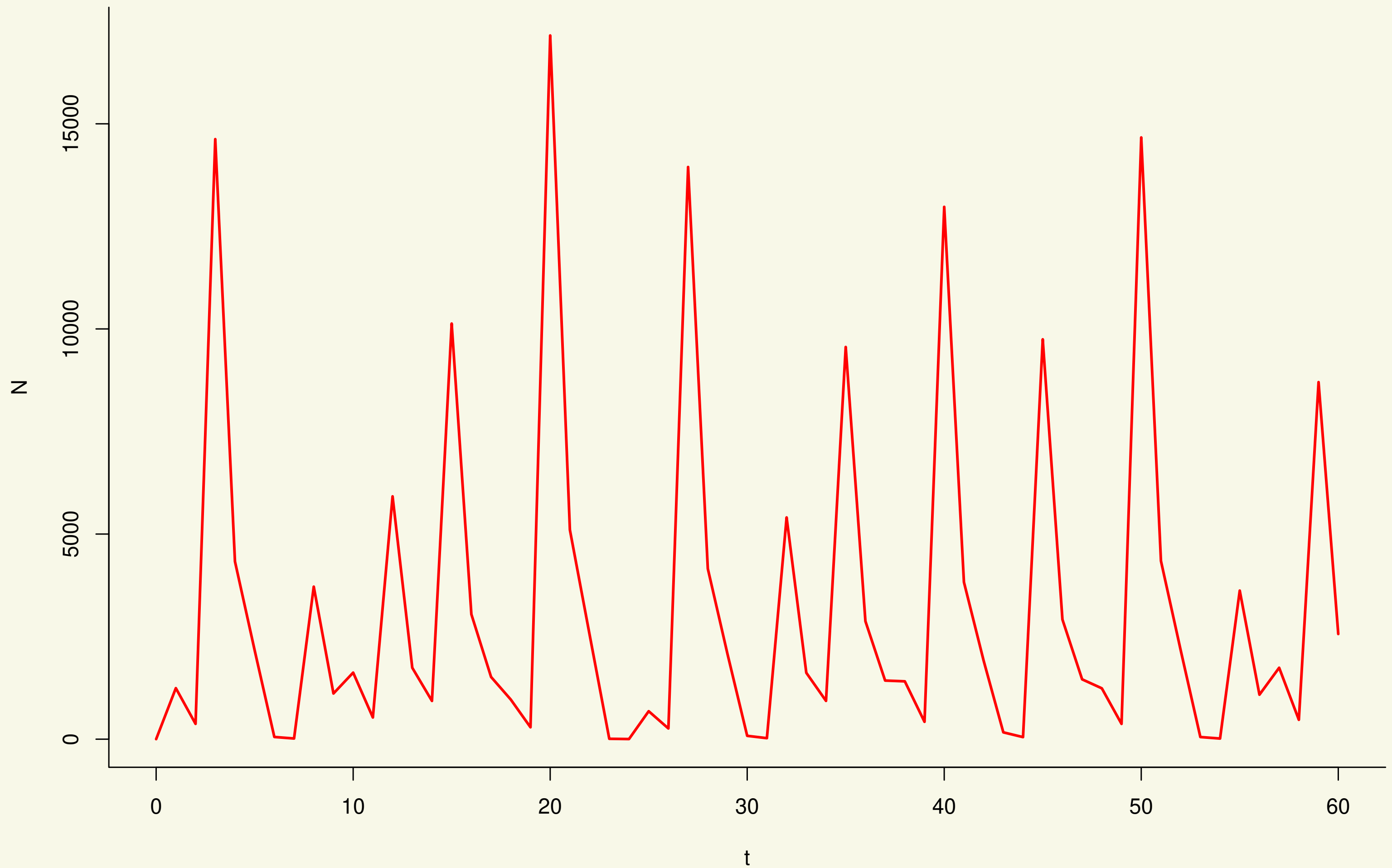


$R = 45$

Leslieho model s plodností závislou na velikosti populace

$$\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}(t+1) = \begin{pmatrix} 0 & g\big(N(t)\big) & 5g\big(N(t)\big) \\ 0.3 & 0 & 0 \\ 0 & 0.5 & 0 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}(t),$$

$$N = n_1 + n_2 + n_3, \quad g(N) = R\mathrm{e}^{-0.005N}$$

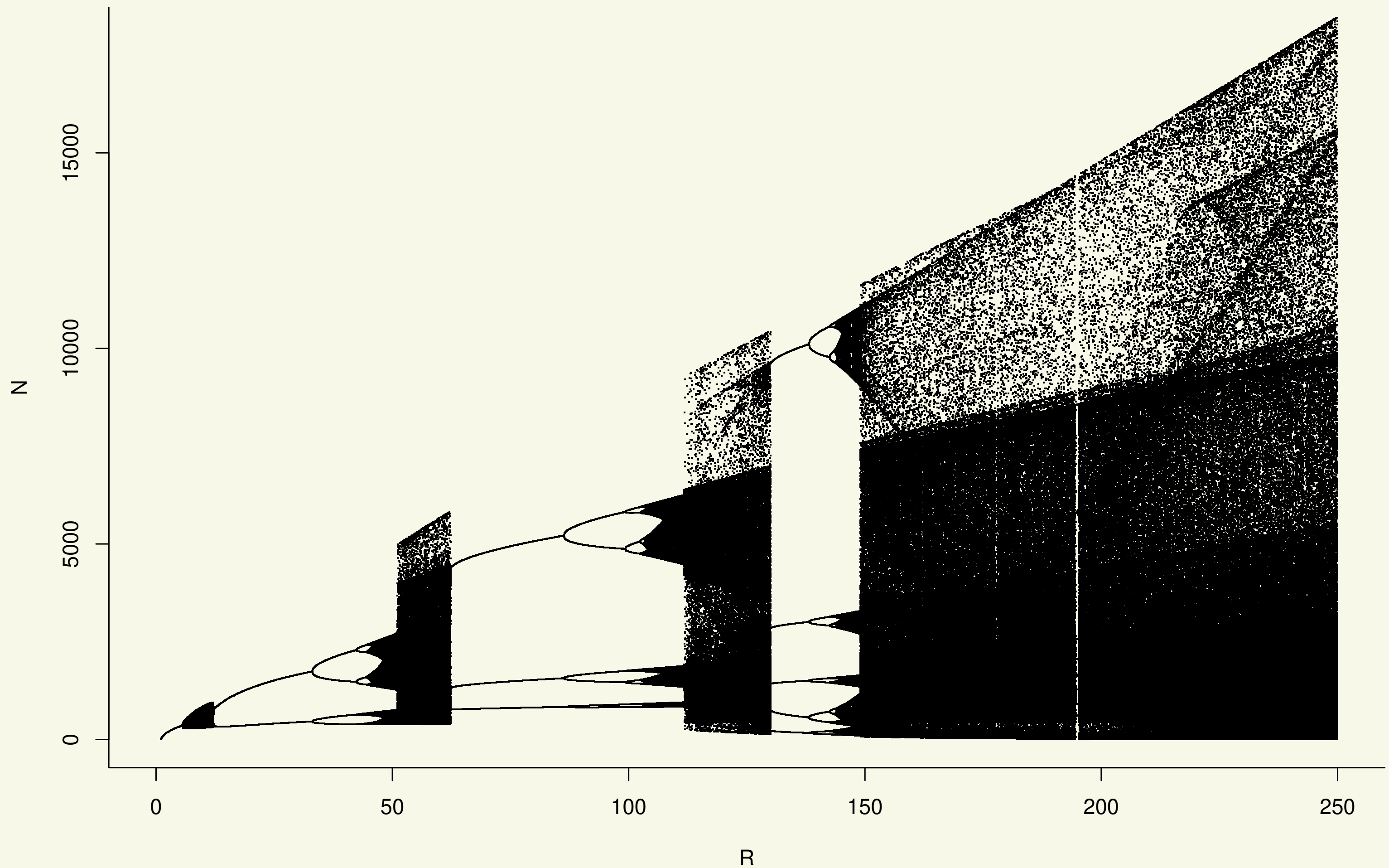


$R = 250$

Leslieho model s plodností závislou na velikosti populace

$$\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} (t+1) = \begin{pmatrix} 0 & g\big(N(t)\big) & 5g\big(N(t)\big) \\ 0.3 & 0 & 0 \\ 0 & 0.5 & 0 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} (t),$$

$$N = n_1 + n_2 + n_3, \quad g(N) = R\mathrm{e}^{-0.005N}$$



Model kanibalismu

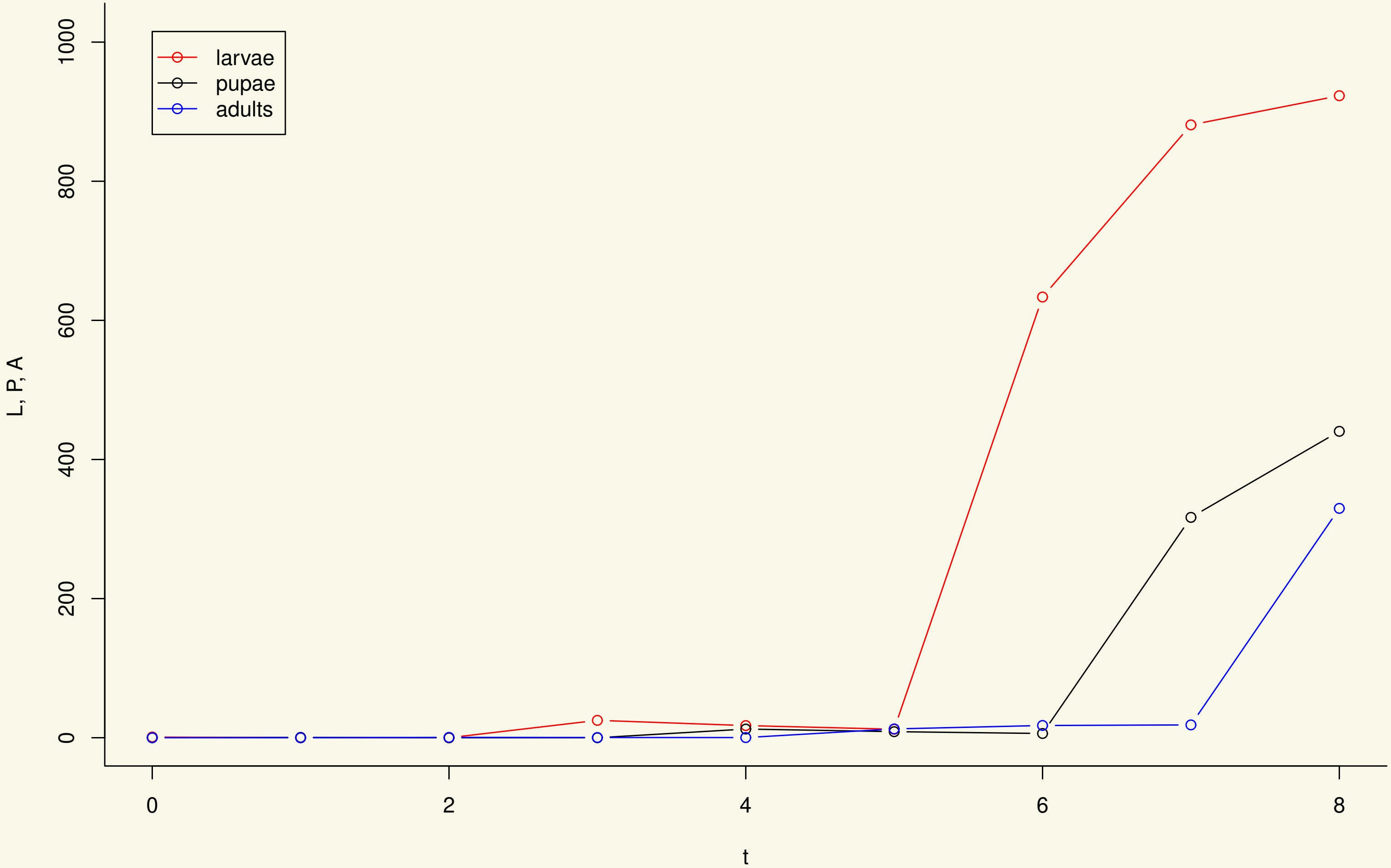
$$
\begin{aligned}
L(t+1) &= bA(t)\exp\{-c_{ea}A(t) - c_{el}L(t)\}\\
P(t+1) &= (1-\mu_l)L(t)\\
A(t+1) &= \exp\{-c_{pa}A(t)\}P(t) + (1-\mu_a)A(t)
\end{aligned}
$$

L, P, A ... množství larev, kukel a dospělců
b ... počet vajíček jedné dospělé samice za projekční interval
μ_l, μ_a ... přirozená úmrtnost larev a dospělců
c_{ea}, c_{el}, c_{pa} ... „míry kanibalismu“

Model kanibalismu

$$L(t+1) = bA(t)\exp\{-c_{ea}A(t) - c_{el}L(t)\}$$
$$P(t+1) = (1-\mu_l)L(t)$$
$$A(t+1) = \exp\{-c_{pa}A(t)\}P(t) + (1-\mu_a)A(t)$$

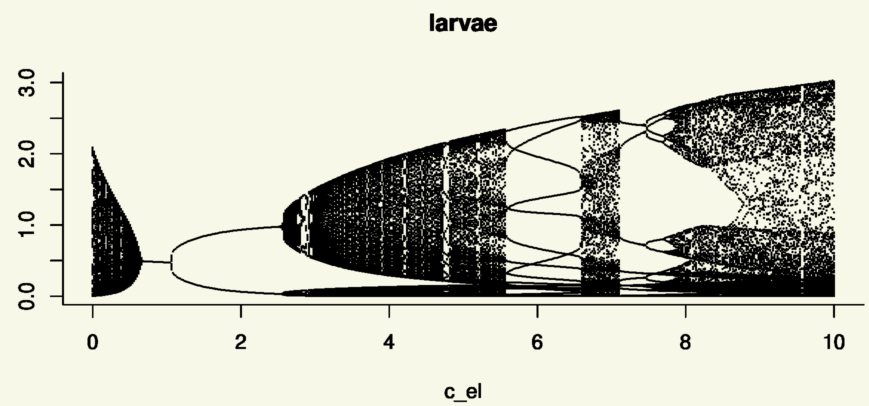
$b = 50$, $\mu_l = 0.5$, $\mu_a = 0.3$, $c_{ea} = 0$, $c_{pa} = 0$, $c_{el} = 0$

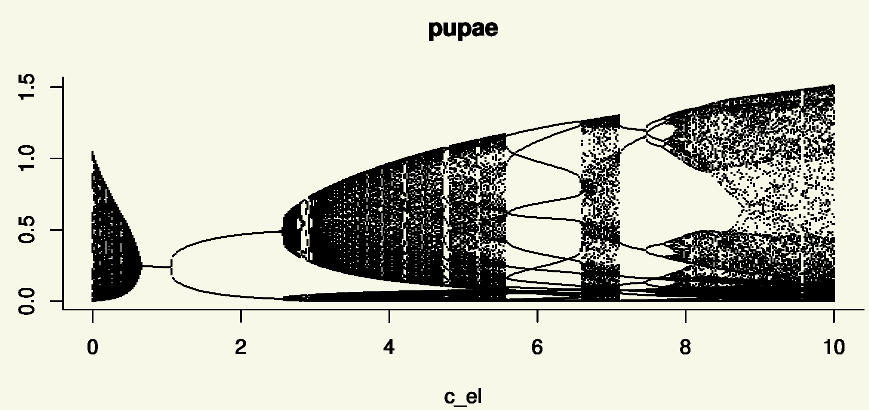


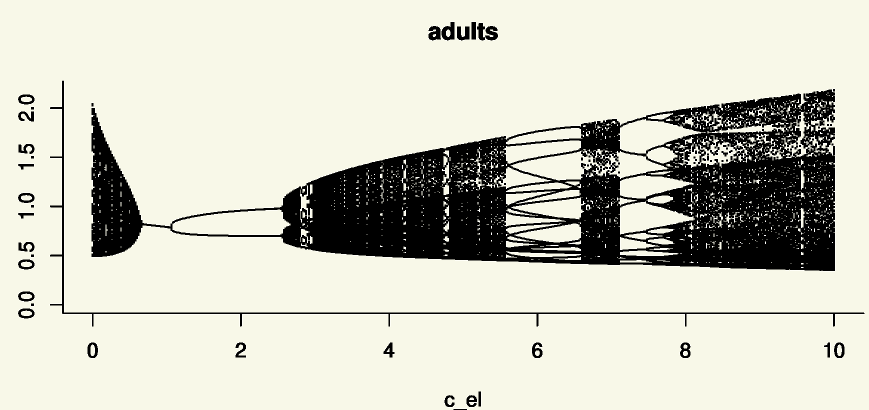
Model kanibalismu

$$L(t+1) = bA(t)\exp\{-c_{ea}A(t) - c_{el}L(t)\}$$
$$P(t+1) = (1-\mu_l)L(t)$$
$$A(t+1) = \exp\{-c_{pa}A(t)\}\,P(t) + (1-\mu_a)A(t)$$

$b = 50$, $\mu_l = 0.5$, $\mu_a = 0.3$, $c_{ea} = 5$, $c_{pa} = 0$,
$c_{el} \in [0, 10]$



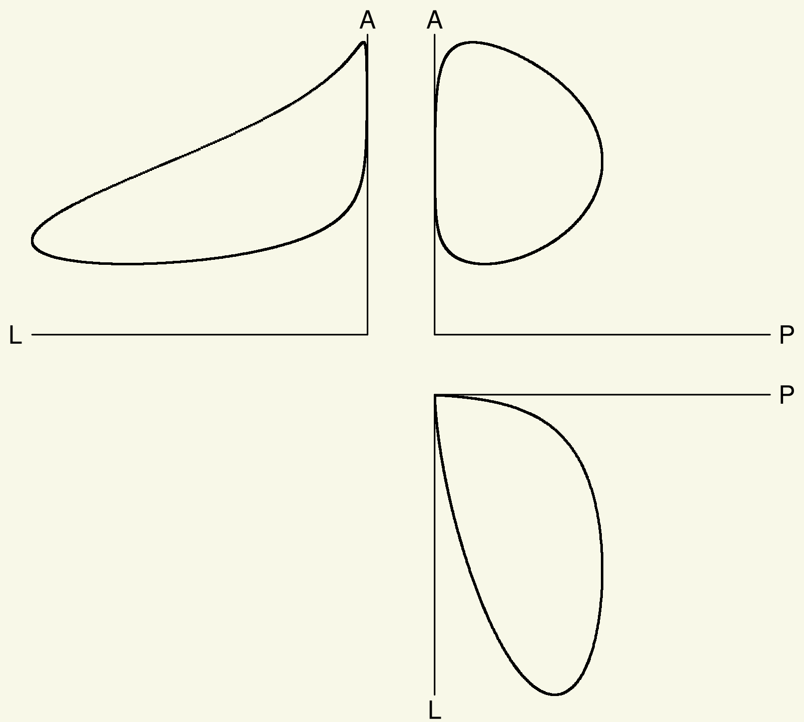


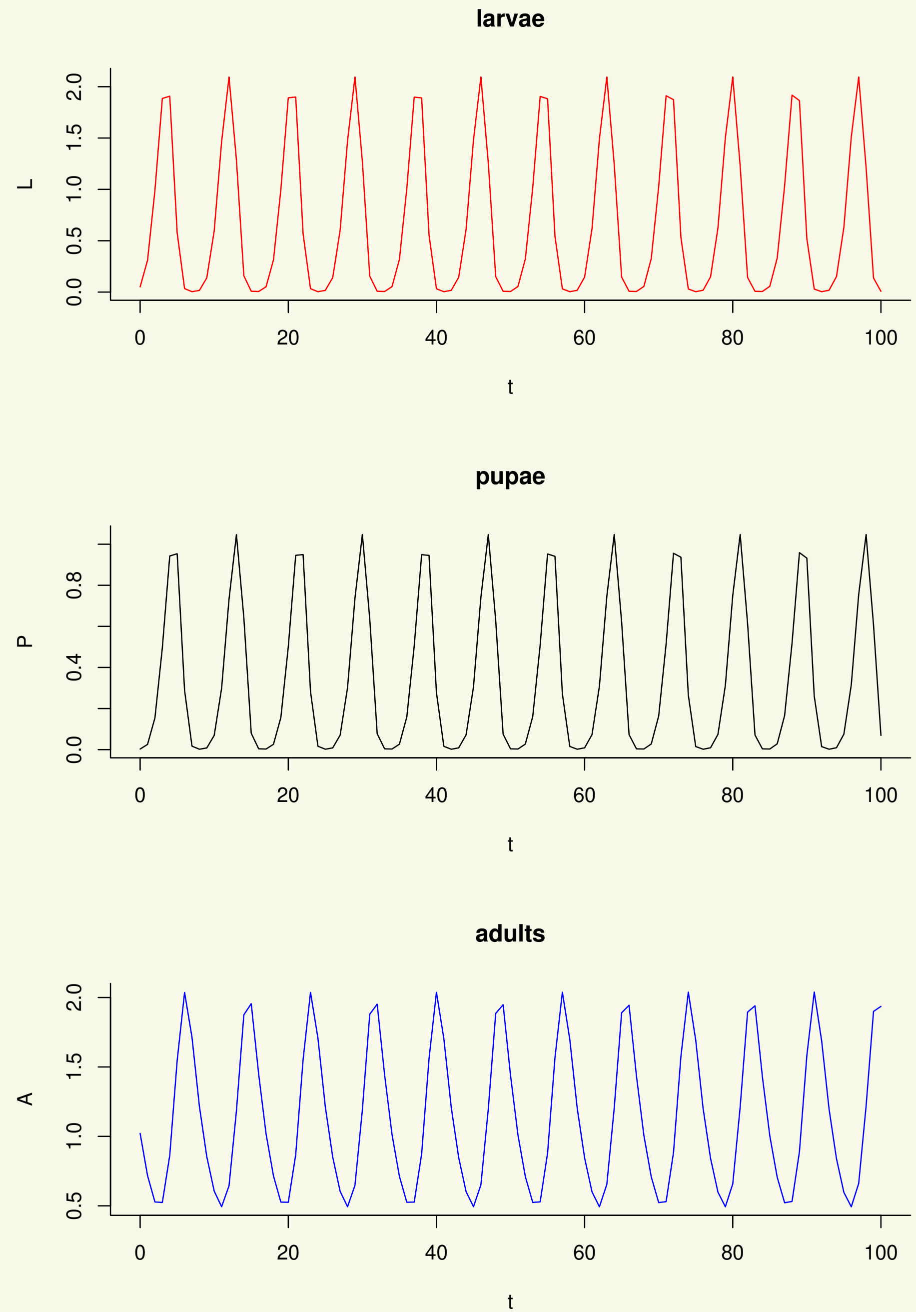


Model kanibalismu

$$L(t+1) = bA(t)\exp\{-c_{ea}A(t) - c_{el}L(t)\}$$
$$P(t+1) = (1-\mu_l)L(t)$$
$$A(t+1) = \exp\{-c_{pa}A(t)\}P(t) + (1-\mu_a)A(t)$$

$b = 50$, $\mu_l = 0.5$, $\mu_a = 0.3$, $c_{ea} = 5$, $c_{pa} = 0$, $c_{el} = 0$

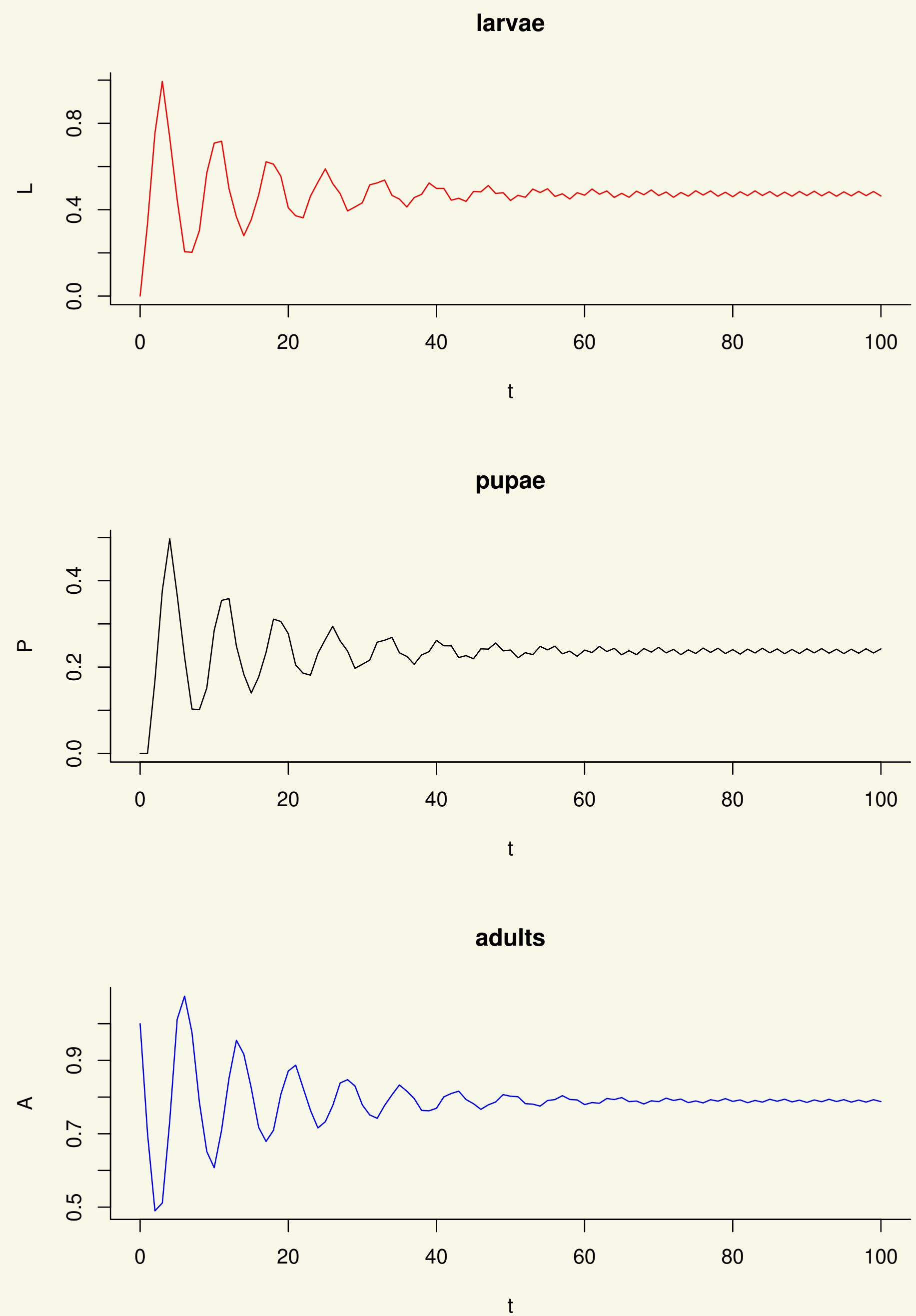




Model kanibalismu

$$L(t+1) = bA(t)\exp\{-c_{ea}A(t) - c_{el}L(t)\}$$
$$P(t+1) = (1-\mu_l)L(t)$$
$$A(t+1) = \exp\{-c_{pa}A(t)\}P(t) + (1-\mu_a)A(t)$$

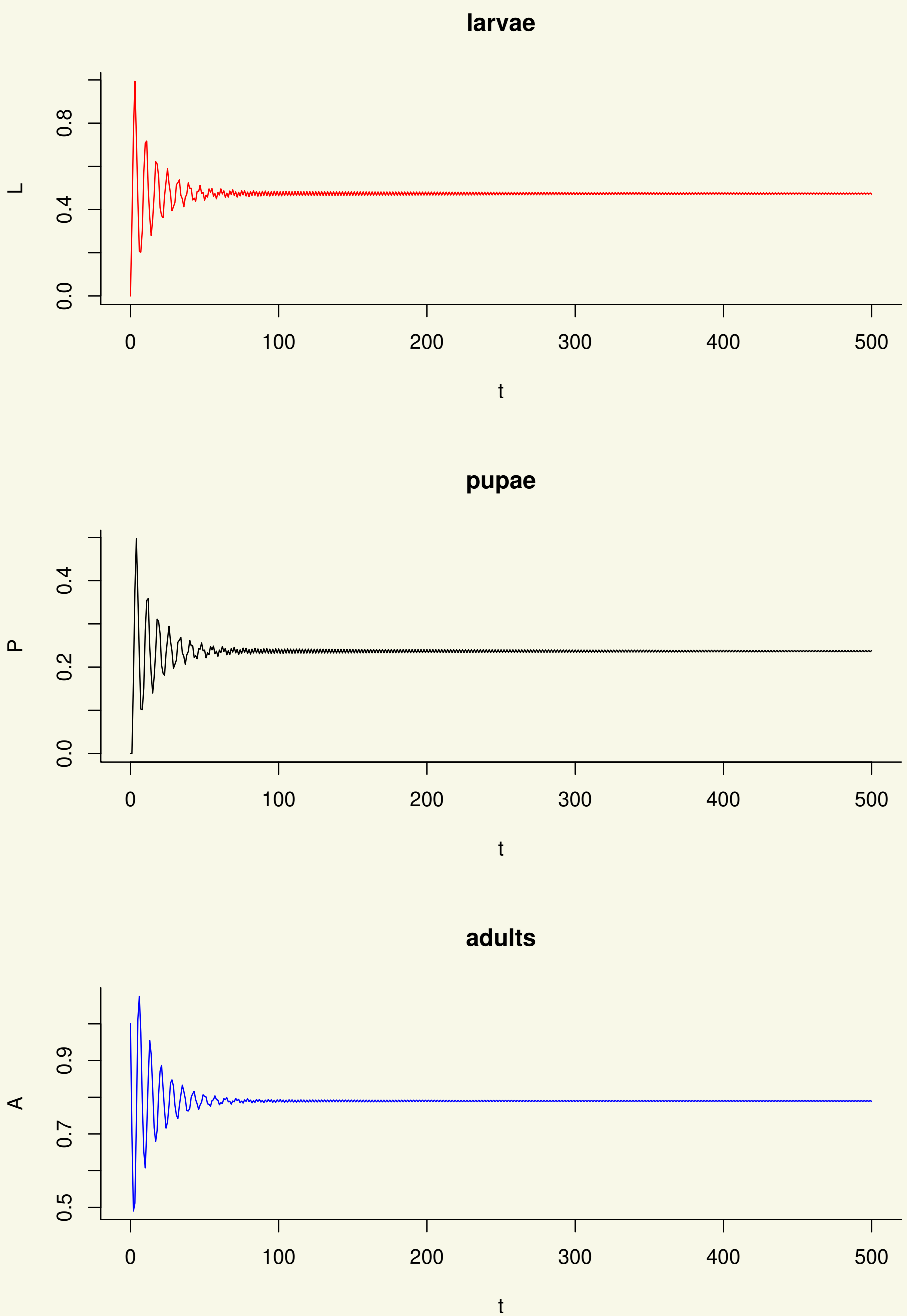
$b = 50$, $\mu_l = 0.5$, $\mu_a = 0.3$, $c_{ea} = 5$, $c_{pa} = 0$, $c_{el} = 1$



Model kanibalismu

$$L(t+1) = bA(t)\exp\{-c_{ea}A(t) - c_{el}L(t)\}$$
$$P(t+1) = (1-\mu_l)L(t)$$
$$A(t+1) = \exp\{-c_{pa}A(t)\}P(t) + (1-\mu_a)A(t)$$

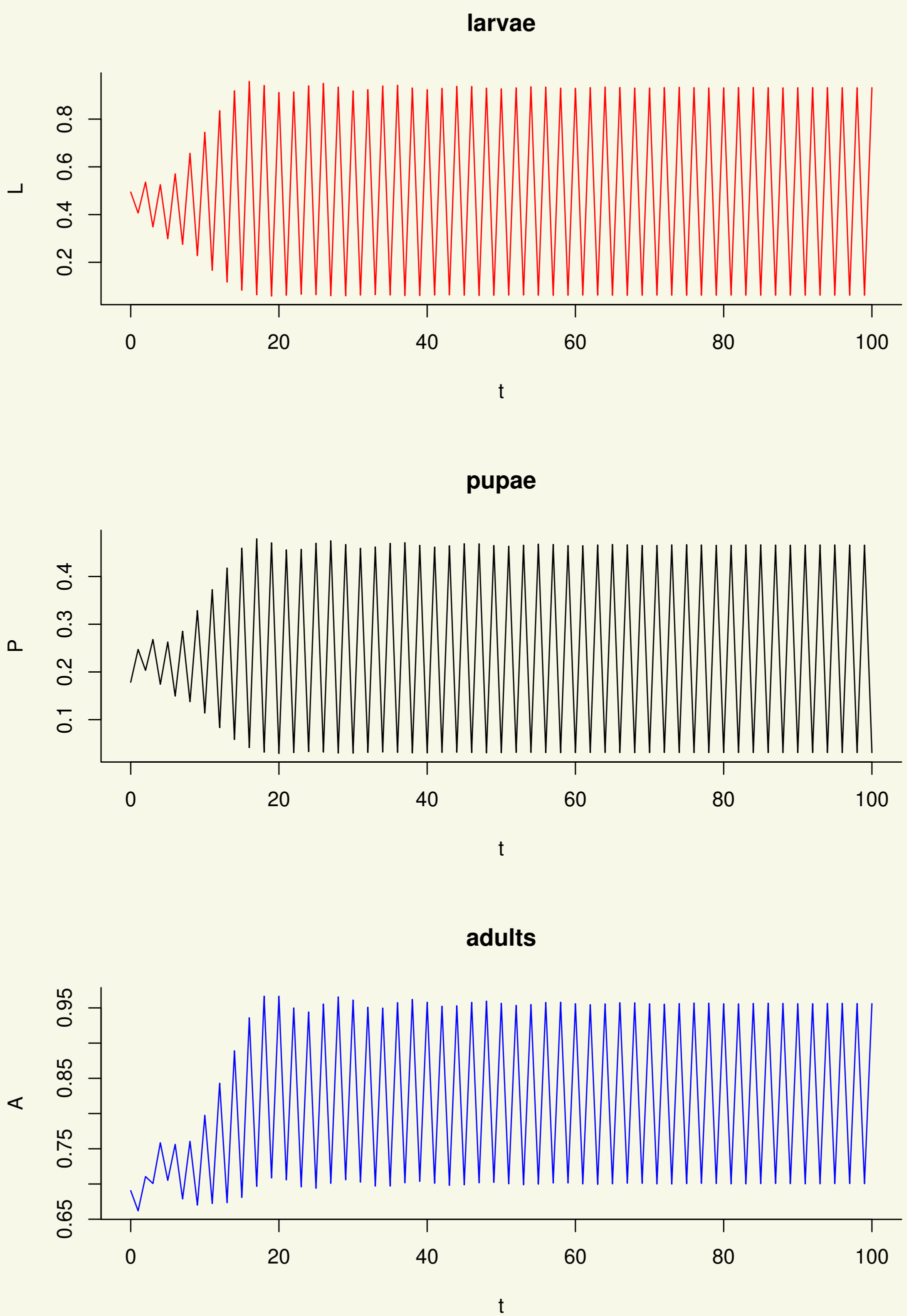
$b = 50$, $\mu_l = 0.5$, $\mu_a = 0.3$, $c_{ea} = 5$, $c_{pa} = 0$, $c_{el} = 1$



Model kanibalismu

$$L(t+1) = bA(t)\exp\{-c_{ea}A(t) - c_{el}L(t)\}$$
$$P(t+1) = (1-\mu_l)L(t)$$
$$A(t+1) = \exp\{-c_{pa}A(t)\}P(t) + (1-\mu_a)A(t)$$

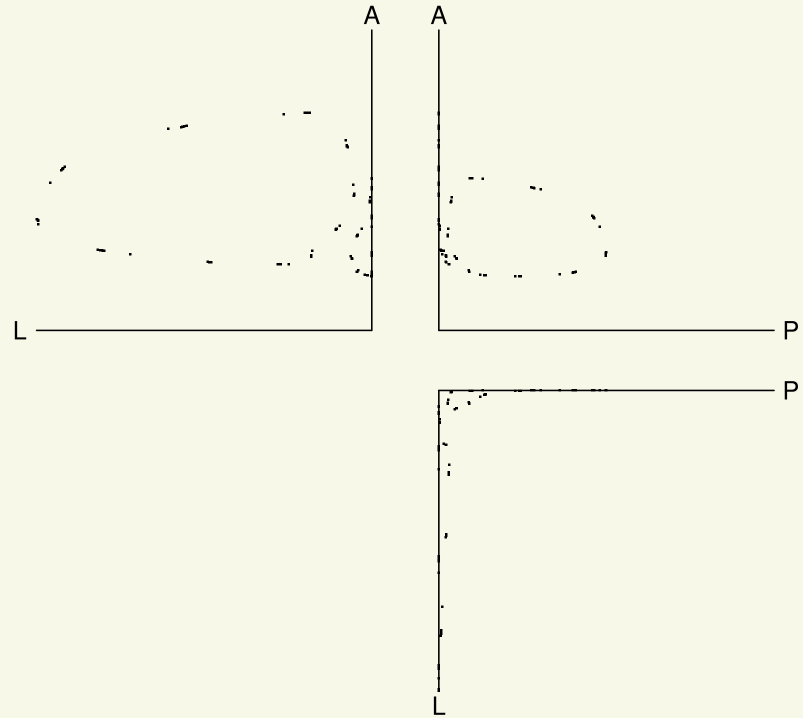
$b = 50$, $\mu_l = 0.5$, $\mu_a = 0.3$, $c_{ea} = 5$, $c_{pa} = 0$, $c_{el} = 2$

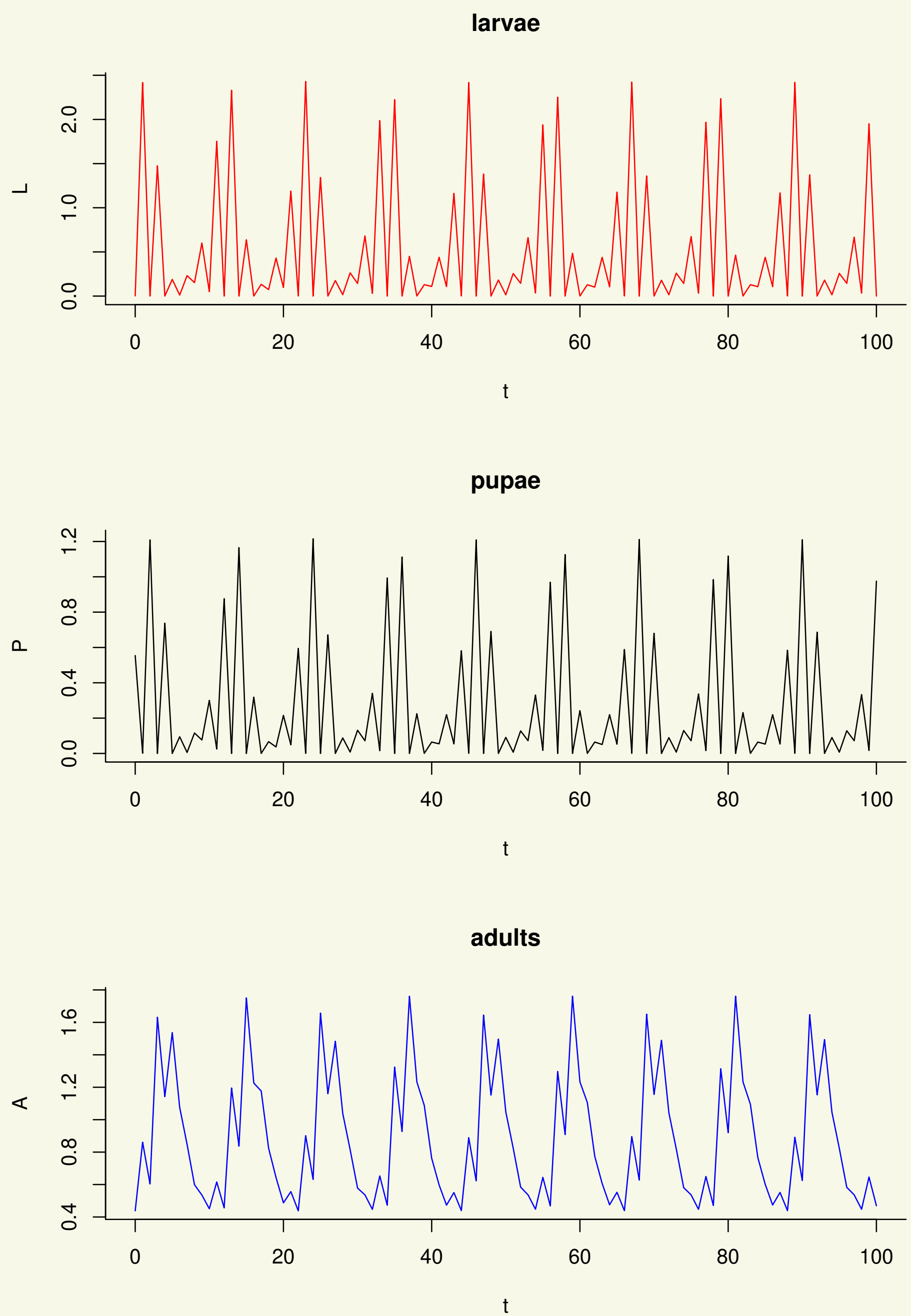


Model kanibalismu

$$L(t+1) = bA(t)\exp\{-c_{ea}A(t) - c_{el}L(t)\}$$
$$P(t+1) = (1-\mu_l)L(t)$$
$$A(t+1) = \exp\{-c_{pa}A(t)\}P(t) + (1-\mu_a)A(t)$$

$b = 50$, $\mu_l = 0.5$, $\mu_a = 0.3$, $c_{ea} = 5$, $c_{pa} = 0$, $c_{el} = 6$





Model kanibalismu

$$L(t+1) = bA(t)\exp\{-c_{ea}A(t) - c_{el}L(t)\}$$
$$P(t+1) = (1-\mu_l)L(t)$$
$$A(t+1) = \exp\{-c_{pa}A(t)\}P(t) + (1-\mu_a)A(t)$$

$b = 50$, $\mu_l = 0.5$, $\mu_a = 0.3$, $c_{ea} = 5$, $c_{pa} = 0$, $c_{el} = 9$