Exercise 11

Let S_3 be the symmetric group on 3 elements 1, 2 & 3.
Our goal is to understand its irreducible complex representations.

Consider $\mathbb{C}[1,2,3]$ the permutation G-module.

a) Show that $\langle 1+2+3 \rangle \leq \mathbb{C}[1,2,3]$ is a submodule iso to the trivial module.

b) Show that $\langle 2-1, 3-2 \rangle \leq \mathbb{C}[1,2,3]$ is also a S_3-submodule & that

$$\langle 2-1, 3-2 \rangle \oplus \langle 1+2+3 \rangle = \mathbb{C}[1,2,3]$$

c) Prove that $\langle 2-1, 3-2 \rangle$ is an irreducible S_3-module.

d) Calculate all irreducible S_3-modules up to iso.