Exercise 7

1a) Let M be an R-mod & $N \leq M$. Then
M is Noetherian $\Rightarrow$ N is Noeth. & M/N is Noeth
If M is Noeth so is M^n.

1b) Let M, N be Noetherian. Prove that $N \oplus M$ is Noetherian

2) Show that the ring of cts functions
$\mathbb{R} \longrightarrow \mathbb{R}$ is not Noetherian.
(Hint: consider functions which are eventually zero)

3) An ideal I is irreducible if it cannot be written as $I = A \cap B$ for ideals $I \subset A, B \subset R$.
Prove that in a Noetherian ring each ideal $I = I_1 \cap \dots \cap I_n$ for $I_1, \dots, I_n$ irreducible.

4) Relate 3) to decomposition of integers into prime powers when $R = \mathbb{Z}$.

4) Describe the free k-algebra and free commutative k-algebra on a set?

5) Prove the converse to 1a).