Exercises 9

(1) Show that the category Var of varieties & polynomial maps has finite products.

(2) An algebraic group is a group in Var: that is, a variety V equipped with a group structure such that the multiplication $V \times V \xrightarrow{\cdot} V$, inverse $(-)^{-1} : V \to V$ & unit $1 \xrightarrow{e} V$ are polynomial.

Consider the set of invertible 2×2-matrices as the variety

$$Gl(2,k) = \langle a,b,c,d,t : (ad-bc)t = 1 \rangle \subseteq k^5$$

& show that matrix multiplication gives it the structure of an algebraic group.

(3) For a commutative ring R, the tensor product of R-algebras $A \otimes B$, is simply defined as the tensor product $A \otimes_R B$ of R-modules, with the mult.

$$A \otimes_R B \otimes_R A \otimes_R B \xrightarrow{1 \otimes s \otimes 1} A \otimes_R A \otimes_R B \otimes_R B \xrightarrow{\cdot_A \otimes_R \cdot_B} A \otimes_R B,$$

ie. $(x \otimes y).(x' \otimes y') = x.x' \otimes y.y'$

Show that this makes $A \otimes B$ an R-algebra.

④ Show, furthermore, that if A, B are <u>commutative</u> algebras, then $A \otimes B$ is their <u>coproduct</u> in the category of <u>commutative algebras</u>!