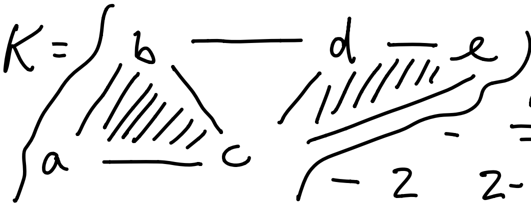
- This is a quick intro to simplicial complexes & their homology. Simp. complexes give an abstract description of spaces that can be built from points, lines, triangles & higher dimensional triangles (simplices).

$\boxed{\text{Def}}$ A simplicial complex K consists of a set K_0 of vertices + a set $S(K)$ of non-empty finite subsets of $U(K)$ called simplices such that:

- if X is a simplex & $Y \subseteq X$ is a non empty subset then Y is a simplex.

$\boxed{\text{Notation}}$ The simplices with $n+1$-elements are called n-simplices.

$\boxed{\text{Example 1}}$ - Triangulated spaces can be viewed as simplicial complexes:

$K =$ 

is a simp. complex with
- 5 vertices $a, \dots, e$
- 7 1-simplices $\{a,b\}, \{b,c\}, \dots$
- 2 2-simplices $\{a,b,c\}, \{c,d,e\}$.

- This is a 2-dimensional simplicial complex.
- A 1-dimensional simplicial complex is a graph.

Remark: Spaces "describable" by simp. complexes include most of interest: tori, ...

- Given K a simplicial complex, with an ordering on its set of vertices we have a function
$$d_i : K_n \longrightarrow K_{n-1} \text{ for } 0 \le i \le n$$
which omits the i'th elt. of a n-simplex.

Example) - consider Ex 1 & the Ordering $a<b<c<d<e$

Then $K_1 \xrightarrow{d_1} K_0$ sends
$$ab \longmapsto a$$
$$cd \longmapsto b$$
write $ab := \{a, b\}$ etc

Then $K_2 \xrightarrow{d_0} K_1$ sends
$$abc \longmapsto bc$$
$$cde \longmapsto de$$

- Let $C(K)_n$ = free abelian group on K_n - elements are sums $\sum \underbrace{n_j}_{\text{integer}} \underbrace{x_j}_{n\text{-simplex.}}$

- We obtain
$$C(K)_n \xrightarrow{d_i} C(K)_{n-1} \text{ by extension}$$
$$\sum n_j x_j \longmapsto \sum n_j d_i(x_j)$$
& take the sum

$$C(K)_n \xrightarrow{\;d_n = \sum_{i=0}^{n} (-1)^i d_i\;} C(K)_{n-1}$$

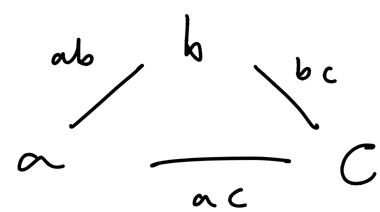
& this is a chain complex

& the n'th simplicial homology is $H_n(C_K)$.

Exercises

1) Consider the 1-dimensional simplicial complex

$K =$



Calculate the chain complex $C(K)$ & its homology.